Transient Free Convective MHD Flow in Slip Regime Under Inclined Magnetic Field

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Abstract: The study investigates effect of slip parameter under inclined magnetic field on unsteady free MHD flow of two-dimensional viscous incompressible conducting fluid through porous medium in slip flow regime near vertical wall with variable suction velocity in a boundary layer under inclined magnetic field. Assuming that the flow is due to time dependent variable suction velocity, governings equation in non-dimensional are solved by perturbation method about a small perturbation parameter &. The characteristics of fluid flow, temperature-profile, rate of heat-transfer and skin friction for various values of magnetic field parameter, slip-parameter, ratio of viscosity, and inclination of the field are discussed graphically. The observations conclude significant effects of slip parameter, magnetic field-inclination and ratio of viscosity on heat transfer and nature of flow.

Keywords: MHD, Convective flow, Vertical insulated-wall, Slip-flow regime, inclined-magnetic field.

1. INTRODUCTION

In fluid dynamics, when buoyancy force are induced free convective flow occurs due to variation of density gradient caused by change of temperature. Such kind of flow are common in nature (e.g. atmospheric flow), our everyday life as well where flow is controlled by the temperature gradient. Such type of flow through porous medium has special importance in many industrial and scientific areas, as it is in the process of solidification of alloys and metals, in process of cooling in a nuclear reactor, dissemination and managing the chemical waste and pollutants in nuclear reactor, designing a MHD generator etc.

Over the years, many researchers have contributed to this field of fluid-dynamics. Berezovsky et al. [2] had studied on Heat transfer on the vertical semi-infinite plate. Chandran et al. [3,4] had discussed on unsteady free convection flow and accelerated boundary motion and heat transfer past a continuous

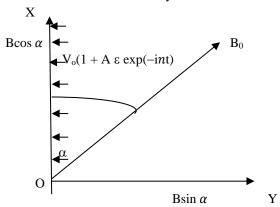
moving porous boundary. Das et al. [5] have discussed on transient free convection flow past a semi-infinite vertical plate with the temperature variation. Kim [6] has discussed on unsteady MHD free Convective heat transfer with variable suction which past a semi-infinite vertical porous plate. Mishra et al. [7] studied on transient free convection flow passing a vertical plate through the porous medium in the slip flow regime. Sharma et al. [8] have studied the effect of viscous incompressible flow which passes through a vertical plate in slipflow regime. Shahin et al. [9] have studied on heat transfer and mass transfer and also on three dimensional in the presence of periodic suction through the porous medium. Singh et al.[10] discussed about MHD flow past a semi-infinite vertical permeable wall. Uwanta et al. [11] have discussed Heat and Mass Transfer Flow through a Porous Medium with Variable Permeability and Periodic Suction, Ostrach [12,13,14] had studied various aspects of convective flow which passes through a vertical plate under various physical conditions. Drake [15], Soundalgekar [16] have studied free convective effects on the Stokes problems for the infinite vertical plate. Helmy [17], Chen et al. [18], Sahin[19] have studied on MHD flow passing along vertical plate. Sugunamma et al. [20] have studied about slight change in velocity, temperature, Mass Grashof number, Thermal Grashof number, Inclined magnetic field parameter, Radiation parameter along an inclined plate embedded in porous medium. Sahoon [21] has studied that in slip flow regime a visco-elastic fluid passing through a vertical plate packed with uniform porous matrix in presence of a magnetic field. Garg et al.[22] have studied about convective unsteady flow in slip regime which passes through a vertical plate with the convective boundary MHD flow. Recently, Patel et al. [1] have discussed effect of magnetic field in viscous incompressible fluid of a unsteady convective heat transfers and transient flow.

Encouraging with the above works and looking to its numerous applications, we have tried to investigate on unsteady free convective viscous incompressible MHD flow which passes through a vertical porous plate with boundary condition in slip flow regime and under the influence of inclined magnetic field. We assume that the flow is induced by time dependent suction velocity $[V_o(1 + A \epsilon)]$ $\exp(-int)$ in the boundary layer slip flow regime in the insulated wall in the porous medium. The two dimensional equation equations in non-dimensional system are solved by perturbation method about a small perturbation parameter ϵ that converts the non-linear equations into ordinary equations. The characteristics such as velocity

temperature profile, skin friction, rate heat transfer for various values of various values of magnetic field parameter, slip-parameter, ratio of viscosity, and inclination of the field are discussed graphically and thereby conclusions are drawn.

2. MATHEMATICAL FORMULATION

Let us assume an unsteady free two dimensional flow through porous medium conducting a incompressible electrically fluid within a certain region bound by a vertical wall under slip boundary condition. The surface of the wall is considered along X-axis in upward direction while Y-axis is along perpendicular to X-axis. A uniformly magnetic field B₀ has been applied that makes an angle α with the surface of the wall i.e. the Xaxis(as shown below).



It is assumed that the flow is induced by time dependent variables suction velocity $[V_0(1 + A \epsilon \exp(-int))]$ in the boundary layer slip regime on the vertically insulated wall. It is also considered that the Reynolds number is so little that the effect of magnetic field is neglected. After neglecting viscous and magnetic dissipation due to fluid flow, under Boussinesq approximation the governing equation (Patel et al. [1]) are

$$\frac{\partial u'}{\partial t'} - V_0' (1 + \varepsilon e^{-n't'}) \frac{\partial u'}{\partial y'} = g\beta (T' - T_\infty') + \frac{\mu_{eff}}{\rho} \frac{\partial^2 u'}{\partial y'^2} - \frac{\mu_f u}{\rho k} - \frac{\sigma B^2 \cos^2 \alpha u'}{\rho}$$
(1)
$$\frac{\partial T'}{\partial t'} - V_0' (1 + \varepsilon e^{-n't'}) \frac{\partial T'}{\partial y'} = \frac{K}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2}$$
(2)

$$\frac{\partial T'}{\partial t'} - V_0' (1 + \varepsilon e^{-n't'}) \frac{\partial T'}{\partial y'} = \frac{K}{\rho c_n} \frac{\partial^2 T'}{\partial y'^2}$$
 (2)

The boundary conditions are

$$u'=L'(\frac{\partial u'}{\partial y'}), \quad T'=T'_w \quad \text{at} \quad y'=0$$

$$u'\to 0, \qquad T'\to T'_m \quad \text{at} \quad y'\to \infty \tag{3}$$

Following dimensionless quantities are considered to convert equations (1) & (2) into non-dimensional format.

$$y = \frac{y'v'_0}{v}, t = \frac{t'v'_0{}^2}{v}, u = \frac{u'}{v_0}, n = \frac{un'}{v_0{}^2}, \theta = \frac{T'-T'_\infty}{T'_w-T'_\infty}, G_r = \frac{\beta gv(T'_w-T'_\infty)}{v'_0{}^2}, R_v = \frac{\mu_{eff}}{\mu_f}, K_p = \frac{k'v'_0{}^2}{v^2}, P_r = \frac{\mu c_p}{k}, M = \frac{\sigma B^2 v}{\rho V'_0{}^2}$$
(4)

using (4) in equation (1) & (2), the governing equation are as follows.

$$\frac{\partial u}{\partial t} - (1 + \epsilon e^{-nt}) \frac{\partial u}{\partial y} = G_r \theta + R_v \frac{\partial^2 u}{\partial y^2} - u(\frac{1}{k_p} + M) \cos^2 \alpha$$
 (5)

$$\frac{\partial \theta}{\partial t} - (1 + \epsilon e^{-nt}) \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} \tag{6}$$

Boundary conditions for the above equation are as follows.

$$u = h_1 \left(\frac{\partial u}{\partial y}\right), \ \theta = 1, \ at \ y = 0$$

$$u \to 0, \ \theta \to 0, \ at \ y \to \infty$$

$$where \ h_1 = L' \frac{V'_o}{\upsilon}$$
(7)

3. SOLUTION OF THE GOVERNING EQUATIONS

Using the boundary conditions (7) we solved equation (5) ad (6) using perturbation technique about a small parameter ($\varepsilon <<1$). The temperature θ and the velocities u in front of the wall are assumed as

$$u(y, t) = u_0(y) + \varepsilon e^{-nt} u_1(y) + \varepsilon^2 e^{-2nt} u_2(y)$$
(8)

$$\theta(y, t) = \theta_0(y) + \varepsilon e^{-nt} \theta_1(y) + \varepsilon^2 e^{-2nt} \theta_2(y)$$
(9)

Substituting equations (8) & (9) in equations (5) & (6), solutions are obtained for zeroth order, first order and second order with respect to respective boundary conditions are as given below.

Zeroth order Solution

$$Rv u_0''(y) - \left(\frac{1}{K_P} + M\right) \cos^2 \alpha \ u_0(y) + G_r \ \theta_0(y) + u_0'(y) = 0 \tag{10}$$

$$\frac{1}{P_r}\theta_0''(y) + \theta_0'(y) = 0 \tag{11}$$

The boundary condition for the above equation are

$$u_0 = h \left(\frac{\partial u_0}{\partial y}\right)$$
 $\theta_0 = 1 \text{ at } y = 0$

$$u_0 \to 0 \text{ and } \theta_0 \to 0, \text{ at } y \to \infty$$
 (12)

Using the boundary condition the solution of the zeroth order equation is

$$u_0 = C_4 e^{-k_4 y} + B_1 e^{-pry} (13)$$

$$\theta_0 = e^{-pry} \tag{14}$$

First order equations

$$Rvu_1''(y) + \{n - (\frac{1}{K_n} + M)cos^2\alpha\}u_1(y) + G_r\theta_1(y) + u_1'(y) = c_4k_4e^{-k_4y} + B_1Pre^{-Pry}$$
(15)

$$\theta_1''(y) + \Pr \theta_1'(y) + \Pr n\theta_1(y) = \Pr^2 e^{-\Pr y}$$
(16)

Corresponding boundary conditions

$$u_1 = h\left(\frac{\partial u_0}{\partial y}\right),$$
 $\theta_1 = 1 \text{ at } y = 0$

$$u_1 \to 0, \qquad \theta_1 \to 0, \text{ at } y \to \infty \tag{17}$$

Using the boundary condition the solution of the first order equation is

$$u_1 = C_5 e^{-K_5 y} + B_5 e^{-K_4 y} + B_6 e^{-Pry} + B_7 e^{-k_2 y}$$
(18)

$$\theta_1 = m_2 e^{-K_2 y} + \frac{Pre^{-Pry}}{n} \tag{19}$$

Second order equations

$$R_v u_2''(\mathbf{y}) + u_2'(\mathbf{y}) + (2\mathbf{n} - (\frac{1}{k_p} + M)\cos^2\alpha)u_2(\mathbf{y}) + G_r\theta_2(\mathbf{y}) = c_5k_5e^{-k_5y} + B_5k_4e^{-k_4y} + B_6\mathrm{Pr}e^{-Pry} + B_7K_2e^{-k_2y}(20)$$

$$\theta_2''(y) + \theta_2'(y) \Pr + 2n \Pr \theta_2(y) = \frac{Pr^3}{n} e^{-Pry} - \Pr k_2 m_2 e^{-k_2 y}$$
(21)

Second order boundary condition are

$$u_2 = h(\frac{\partial u_0}{\partial y})$$
, $\theta_2 = 1$ at $y = 0$

$$u_2 \to 0,$$
 $\theta_2 \to 0, \text{ at y } \to \infty$ (22)

Using the boundary condition the solution of the Second order equation is

$$u_2 = c_6 e^{-k_6 y} + B_{13} e^{-k_5 y} + B_{14} e^{-k_4 y} + B_{15} e^{-Pry} + B_{16} e^{-k_2 y} + B_{17} e^{-k_3 y}$$
(23)

$$\theta_2 = C_3 e^{-k_3 y} + A_1 e^{-Pry} - A_2 e^{-k_2 y} \tag{24}$$

The Nusselt number is

$$Nu = -(\frac{\partial \theta}{\partial y})_{y=0}$$

$$= \Pr + \varepsilon e^{-nt} \left(\frac{Pr^2}{n} - m_2 k_2 \right) + \varepsilon^2 e^{-2nt} \left(k_3 c_3 - A_2 k_2 \right)$$
 (25)

The skin friction is

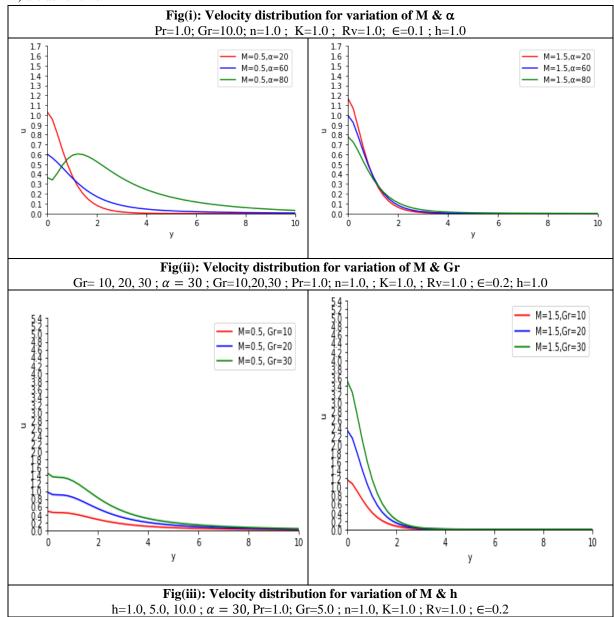
$$\tau = \left(\frac{du_0}{dy}\right)_{y=0} + \varepsilon \left(\frac{du_1}{dy}\right)_{y=0} e^{-nt}
= -(k_4c_4 + B_1\text{Pr}) - \varepsilon^2 e^{-nt} (k_5C_5 + B_5k_4 + B_6Pr + K_2B_7) - \varepsilon^2 e^{-2nt} (k_6C_6 + B_{13}k_5 + B_{15}Pr + B_{16}k_2 + B_{17}k_3)$$
(26)

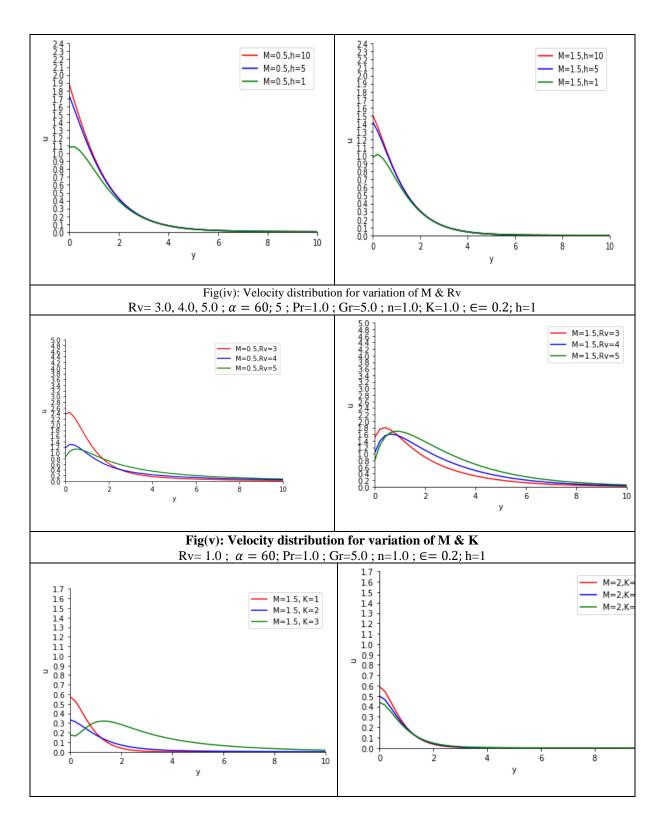
4. RESULT AND ANALYSIS

To analysis the flow characteristics of the problem, numericals result are obtained from solution (13-26) given above using Python programing and thereby plots are obtained for fluid velocity distribution and Skin Friction in slip flow regime for various values of inclination of the field (α) , ratio of viscosity (Rv), magnetic parameter(M), flow slip parameter (h), permeability (K), and the Grashof

number (Gr). We have considered M≈0.5 & 1.5 as smaller and higher values of magnetic field, $\alpha \approx 20$ & 80 as smaller higher values of field inclination, h≈1& 10 as smaller and higher values of flow slip parameter, Rv ≈ 3.0 & 5.0 as smaller and higher values of ratio of viscosity, K≈ 1.0 &3.0 as smaller and higher values of medium permeability, Gr =10.0 & 30.0 as smaller and higher values of Grashof Number respectively. We have considered \in =0.15

& 0.55 as smaller and larger values of the perturbation parameter. The analysis of plots, Fig (iviii) are as follows.





Fig(vi) : Skin Friction Variation with M &h ; $\alpha = 70$; K=1.0 Rv= 1.0 ; Pr=1.0 ; Gr=10.0 ; n=1.0 ; $\epsilon = 0.2$

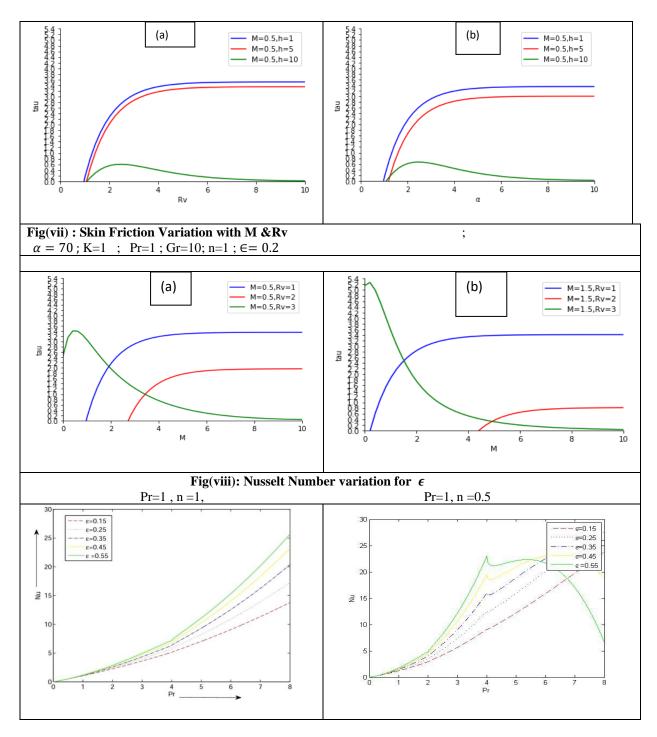


Figure (i) shows velocity distribution not towards the plate for M & α , for the small values of M, rate of decrease of flow is higher for smaller values of α the rate of decrease of flow is uniform for all values of α when M is higher. Figure(ii) Shows velocity distribution for M & Gr, for smaller values of M, the rate of decrease of velocity is less in compare to when M higher , for all the value of Grashof number. Velocity increases with the increase of Gr. Figure(iii) Shows velocity

distribution with M & h, higher the values of h magnitude of fluid velocity is higher for all values of M. Figure(iv) Shows the velocity distribution with M & Rv, the effect of Rv on u depends upon applied magnetic field. For smaller values of magnetic field within the neighbourhood of vertical wall flow velocity is higher for smaller values of Rv, however when field is higher the flow of velocity sharply decreases with increase of Rv. Figure(v) shows flow distribution M & K, in case of M=1.5

within the neighbourhood of plate $0 \le y \le 1.0$ (approx.) fluid velocity is less for higher values of K which is almost opposite away from the plate $y \ge 15$. In case of M=2.0 fluid velocity decreases with increase of K and decreases exponentially away from the plate. Fig(vi(a)) shows τ increases with increase of Rv, rate of increase is more for lower range of Rv. This variation is more prominent when h is higher. Figure(vi(b)) shows τ increases with increase of α . The rate of increase τ is higher for smaller range of α . Figure(vii))shows τ decreases exponentially with M when Rv=3 while increases exponentially when Rv =1&2 Figure(viii) shows the value of Nusselt number increases exponentially with Prandtl number.

5. CONCLUSIONS

 Higher the field inclination, flow is slow; smaller the inclination, rate of decrease of flow away from the plate is higher.

- Flow is faster for higher values of slip parameter irrespective of field.
- Flow is faster when both magnetic field and ratio of viscosity is smaller. When field is higher, flow decreases sharply with the increase of ratio of viscosity.
- Within the neighbourhood of the wall flow velocity is less for higher values of permeability which is opposite in case of away from the plate.
- Skin friction increase as ratio of viscosity increases. Rate of increase is higher when slip parameter is higher.
- Skin friction increase as inclination of field increase.
- Skin friction deceases exponentially with magnetic field when ratio of viscosity is higher, but increases exponentially when ratio of viscosity is smaller.

$$\begin{array}{c} \text{Appendices} \\ k_2 = \frac{\Pr + \sqrt{(Pr^2 - 4nPr)}}{2} \; ; \; \; m_2 = 1 - \frac{Pr}{n} \; ; \; k_3 = \frac{\Pr + \sqrt{(Pr^2 - 8nPr)}}{2} \; ; \; c_3 = \frac{Prk_2m_2}{k_2^2 - Prk_2 + 2nPr} - \frac{Pr^2}{n} \; ; \\ A_1 = \frac{Pr^2}{2n^2} \; , A_2 = \frac{Prk_2m_2}{k_2^2 - k_2 Pr + 2nPr} \; ; \; \; k_4 = \left(\frac{-1}{Rv} + \sqrt{\left(\frac{1}{Rv^2} + \frac{4\left(\frac{1}{k_p} + M\right)\cos^2\alpha}{Rv}\right)}\right) / 2 \; ; B_1 = \frac{-G_re^{-Pry}}{Rv^{Pr^2 - Pr - \left(\frac{1}{k_p} + M\right)\cos^2\alpha}} B_2 = \\ \frac{nB_1Pr - G_rPr}{nR_v} \; ; \; B_3 = \frac{G_4k_4}{R_v} \; ; \; B_4 = \frac{G_rm_2}{R_v} \; ; \; k_5 = \left(\frac{1}{R_v} + \sqrt{\left(\frac{1}{R_v^2} + \frac{4\left(\left(\frac{1}{k_p} + M\right)\cos^2\alpha\alpha - n\right)}{Rv}\right)}\right) / 2 \; ; \\ C_4 = \frac{-B_1(1 + hPr)}{1 + hk_4} \; ; \; C_5 = -h(k_4c_4 + B_1Pr) - m_4 \; ; B_5 = \frac{B_3}{k_4^2 - \frac{k_4}{R_v} + \frac{n - \left(\frac{1}{k_p} + M\right)\cos^2\alpha}{Rv}} \; ; \quad \xi = \left(\frac{1}{k_p} + M\right) \; ; \\ B_6 = \frac{B_2}{Pr^2 - \frac{Pr}{R_v} + \frac{n - \left(\frac{1}{k_p} + M\right)\cos^2\alpha}{Rv}} \; ; \; B_7 = \frac{B_2}{Pr^2 - \frac{Pr}{R_v} + \frac{n - \left(\frac{1}{k_p} + M\right)\cos^2\alpha}{Rv}} \; ; \quad B_8 = C_5k_5 \; ; \quad B_9 = B_5k_4 \; ; \quad B_{10} = \\ B_6Pr - G_rA_1 \; ; \; B_{11} = B_7k_2 + G_rA_2 \; ; \; B_{12} = G_rC_3 \; ; \\ k_6 = \left(\frac{-1}{R_v} - \sqrt{\left(\frac{1}{R_v^2} + \frac{4\left(\left(\frac{1}{k_p} + M\right)\cos^2\alpha\alpha - 2n\right)}{Rv}\right)}\right) / 2 \; ; \quad B_{13} = \frac{B_8}{k_5^2 - \frac{k_5}{R_v} + \frac{(2n - \left(\frac{1}{k_p} + M\right)\cos^2\alpha}{Rv}} \; ; \quad B_{14} = \frac{B_9}{k_4^2 - \frac{k_4}{R_v} + \frac{(2n - \left(\frac{1}{k_p} + M\right)\cos^2\alpha}{Rv}} \; ; \\ B_{15} = \frac{B_{10}}{Pr^2 - \frac{Pr}{R_v} + \frac{n - \left(\frac{1}{k_p} + M\right)\cos^2\alpha}{Rv}} \; ; \quad B_{16} = \frac{B_{11}}{k_2^2 - \frac{k_2}{R_v} + \frac{(2n - \left(\frac{1}{k_p} + M\right)\cos^2\alpha}{Rv}} \; ; \\ B_{17} = \frac{B_{12}}{k_3^2 - \frac{k_3}{R_v} + \frac{(2n - \left(\frac{1}{k_p} + M\right)\cos^2\alpha}{Rv}} \; ; \quad C_6 = -h(k_4C_4 + B_1Pr) - m_5 \\ \end{array}$$

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