Bayesian Prediction in M/M/1 Queue with Quasi and Beta Prior Distribution

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***Abstract*—Under quasi- and beta-prior distributions, Bayesian prediction of the traffic intensity for the M/M/1 queue is taken into consideration. Some of the performance metrics are computed and evaluated in comparison to traditional methods. In order to demonstrate the approach, certain numerical results are performed last.**

***Index Terms*—Bayesian Prediction, M/M/1 queue, Traffic Intensity, Quasi prior distribution, Beta prior distribution.**

I. INTRODUCTION

Nowadays Queueing theory is important in many different domains. Entities in the system may be served individually or in batches, which is one of a queuing system's key properties in the service process. Research on behavioural issues is the foundation for a significant portion of queuing theory's findings. The inferences in stochastic processes, particularly in queuing processes, have drawn more attention from scholars in recent years. The factors of the queuing process, such as the arrival rate, service rate, and traffic intensity, are crucial for the estimation and hypothesis testing problems from the perspective of queue design.

The traditional M/M/1 queue strategy focuses on using the most recent data for the parameters μ and λ. Two more sorts of information are often useful in addition to the most recent information. The first is being aware of the potential outcomes of the decision, and the second is having knowledge from before. Thomas Baye's [1] was the first to employ prior knowledge in inductive inference, and the method of statistics that officially aims to do so is known as Bayesian analysis.

In Queueing theory, the statistical analysis is an important field of research. Lehoczky [2], and Bhat et al.[3] is good reviews of the subject. In recent years, the interest in Bayesian methods are increasing. Jeyabharathi and Latha [4,5,6] derived probability of acceptance of Bayesian sampling plans with various reference plans. A detailed exposition of the Bayesian approach to queues were given by McGrath et al. [7] and McGrath and Singpurwalla [8] . Armero and Bayarri [9,10,11] and Sohn [12] deal with *M/M/*1 and *M/M/c* queues; Ganesh et al. [13] and Tebaldi and West [14] analysed queueing networks; Conti [15] with *Geo/G/*1 discrete-time queueing models; Rios Insua et al. [16] with *M/Er/*1 and *M/Hk/*1 queues; Wiper [17] analyzed *Er/M/*1 and *Er/M/c* queues.

The statistical analysis is a key area of study in queueing theory. The reviews by Lehoczky [2] and Bhat et al.[3] are also favourable. The popularity of Bayesian techniques has grown recently. Jeyabharathi and Latha [4,5,6] calculated the likelihood that Bayesian sampling plans would be accepted using various reference plans. The Bayesian method to queues was thoroughly explained by McGrath et al. [7] and McGrath and Singpurwalla [8]. M/M/1 and M/M/c queues are discussed by Armero and Bayarri [9, 10, 11, and 12], Sohn [12], Tebaldi and West [14], Ganesh et al. [13], Conti [15], Geo/G/1 discrete-time queueing models [14], Rios Insua et al. [16] ,M/Er/1 and M/Hk/1 queues [14], and Wiper [17].

In McGrath et al. [7], a generic Bayesian technique was used to study queue theory. Optimal control of M/M/1 and M/Ek/1 queueing systems with server startup, breakdown, and baulking under N-policy was discussed by Jayachitra and James Albert [18,19, and 20].

Bayesian analysis is a relatively new research field in the queueing system. As far as we are aware, Bagchi and Cunningham [21] and Muddapur [22] are the first studies on Bayesian estimation in queuing models. There has been a lot of research on Bayesian inference for the straightforward Markovian M/M/c queue in the 1980s and 1990s; see Armero [23], McGrath et al. [7],

and McGrath and Singpurwalla [8]. Modern numerical integration techniques have more recently made it possible to build inference and prediction in more general queueing systems; for additional information, see Armero and Conesa [24]. H. Pen and G. Wang [25] examined the Bayesian estimation of traffic intensity in the M/M/1 queue under the cautious loss function.
Finding Bayesian estimates of the traffic intensity for the M/M/1 queue under the Quasi and Beta prior distributions is the goal of the current work. A few performance metrics are acquired and contrasted with traditional methods. This essay is structured as follows; some helpful introductions are included in section 2. In sections 3 and 4, some performance measurements are produced using Bayesian estimators of the traffic intensity under quasi- and beta-prior distributions. Numerical examples are provided in section 5 to demonstrate our findings. Finally, section 6 provides a conclusion.

II PREMILINARIES

Queueing theory has many applications in communication theory, computer design and manufacturing process etc. Let us consider a M/M/1 queueing system with mean arrival rate λ and mean service rate μ . Let X be the random variable representing the number of customers in the system under the steady state distribution specified by the probability mass function

$$P(x/\rho) = (1-\rho)\rho^{x} \quad (1)$$

Where $\rho = \frac{\lambda}{\mu}, 0 < \rho < 1$ represent the traffic intensity for the given M/M/1 queueing system.
Let $T = \sum_{i=0}^{n} X_i$

The likelihood function corresponding to equation (2.1) is given by
$L(\rho) = (1-\rho)^{n}$ Where

$$t = \sum_{i=0}^{n} X_i \quad (2)$$

III BAYESIAN ESTIMATION FOR TRAFFIC INTENSITY

When using the Bayesian technique, we also presumptively know something about the queueing parameter from prior experiences. After that, the preceding knowledge can be condensed in terms of. A prior probability distribution, also referred to as the prior, in Bayesian statistical inference is the probability distribution that would describe one's assumptions about an uncertain variable before certain data is taken into consideration. We can assume quasi- and beta-prior distributions in the following scenarios.

Case (i)

If the experiment has no prior information about the parameters, one may use the quasi density as given be

$$\pi_1(\rho; c) = \frac{1}{\rho^{c}} \quad , \rho > 0, c > 0 \quad (3)$$

Here c=0, represents to a diffuse prior and c=1 to a non-informative prior.

We consider the case when the prior density of ρ can be obtained by using Baye's theorem as given by

$$h_1(\rho/x) = (1-\rho)^{n}\rho^{t-c} \quad (4)$$

Case (ii)

The most widely used prior distribution of ρ is the Beta prior distribution with parameter $\alpha > 0$ and $\beta > 0$ with p.d.f is given by

$$\pi_2(\rho; \alpha, \beta) = \frac{1}{\beta(s,t)}\rho^{\alpha-1}(1-\rho)^{\beta-1} \quad ,$$

$$0 < \rho < 1, \ \alpha, \beta > 0 \quad (5)$$

We consider the case when the prior density of ρ is Beta distribution with parameters α (>0) and β (>0). The posterior density of ρ can be obtained by using Bayes theorem as

$$h_2(\rho/x) = \frac{(1-\rho)^{n+\beta-1}\rho^{t+\alpha-1}}{\int_0^1 (1-\rho)^{n+\beta-1}\rho^{t+\alpha-1} d\rho} \quad (6)$$

IV PERFORMANCE MEASURES

Let N denote the steady state number of customers in the system. Then for M/M/1 queue, the density of N is geometric with parameters $(1-\rho)$ and if the prior distribution is Quasi then the probability of number of customers of (4) is given by

$$P\,[N = {}^{n}/_{x}] = \sum_{n=0}^{\infty}\left[P\,[N = {}^{n}/_{\rho}]\right] h_1(\rho/x)$$

$$P_0 = (1-\rho)\rho^{T-c}$$

$$P_n = (1-\rho)^n \rho^n P_0$$

Hence Expected number of customers in the system is given by

$$L_s = \frac{(1-\rho)^2}{1-\rho+\rho^2}\rho^{T-c}$$

If the prior distribution is Beta then the probability of number of customers of (6) is given by

$$P(N = n/x) = \int_0^1 \rho\left(N = {}^{n}/_{\rho}\right) h_2(\rho/x)\, d\rho$$

$$= \int_0^1 \frac{(1-\rho)\rho^n (1-\rho)^{n+\beta-1}\rho^{T+\alpha-1}\, d\rho}{\int_0^1 (1-\rho)^{n+\beta-1}\rho^{T+\alpha-1} d\rho}$$

$$P_n = \frac{(n+\beta)\Gamma T+\alpha+n}{\Gamma T+\alpha} \cdot \frac{\Gamma T+\alpha+n+\beta}{\Gamma T+\alpha+2n+\beta+1}$$

Thus the steady state probability that the system is empty is given by

$$P[N = 0/x] = \frac{\beta}{T+\alpha+\beta}$$

Hence Expected number of customers in the system is given by

$$L_s = \sum_0^\infty n \frac{(n+\beta)\Gamma T+\alpha+n}{\Gamma T+\alpha} \cdot \frac{\Gamma T+\alpha+n+\beta}{\Gamma T+\alpha+2n+\beta+1}$$

V NUMERICAL ANALYSIS

A numerical example is presented for illustrative purpose. Suppose a small textile show room with a single salesman wants to establish their showroom and increase the salesman during the festival time. So Bayesian Estimation is used for different values of the parameters ρ under the Quasi prior density and Beta prior density are given below in Table [1] and Table [2] respectively to take the optimum decision based on the current year along with past year probability distribution.

Table 1

	$\rho = 0.5$		$\rho = 0.8$	
	C=0	C=1	C=0	C=1
P_0	0.125	0.5	0.1673	0.2091
P_1	0.0315	0.125	0.0346	0.0418
P_2	0.0078	0.03125	0.0066	0.0083

Table 2

		$\rho = 0.5$			$\rho = 0.8$		
α	B	P_0	P_1	P_2	P_0	P_1	P_2
1.0	1.0	0.5	0.190	0.314	0.357	0.197	0.079
1.0	1.5	0.33	0.187	0.069	0.48	0.197	0.072
1.0	2.0	0.57	0.181	0.062	0.526	0.197	0.065
1.5	1.0	0.4	0.047	0.047	0.303	0.201	0.093
2.0	1.5	0.43	0.208	0.039	0.348	0.205	0.097
2.0	3.0	0.66	0.205	0.072	0.517	0.211	0.079

From the Table (1 to 2), it is observed that there are significant changes in the Bayesian approach.

VI INFERENCE

Under quasi and beta prior distributions, Bayesian prediction of the traffic intensity for the M/M/1 queue has been studied. Calculations have been made for some of the performance indicators. These numerical values are important for decision-making and practical queueing system analysis. As a result, the Bayesian approach is adaptable to modelling a range of previous knowledge.

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