

# Numerical Analysis of Nanofluid Flow and Conjugative Heat Transfer Through a 3-D Rectangular Duct

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**Abstract**— In the present study, conjugative heat transfer of nano fluid in a horizontal rectangular duct using CuO-Water is numerically investigated. The fluid flow is assumed to be single phase, laminar, incompressible, steady and three dimensional in nature. A constant wall heat flux has been allowed through the wall and corresponding overall heat transfer coefficient and temperature field along with the Nusselt number variation have been obtained. The detail velocity field, temperature field, friction factors have been derived, validated and presented for different Reynolds number and particle volume fraction of the nano particle. The most significant achievement is the enhancement of the overall heat transfer coefficient and Nusselt number indicating those parameter increases with the increase in the nano particle percentage. The Reynolds number has an important bearing on the result since an increase in *Re* also indicates the improvement of heat transfer parameters.

**Index Terms**- Conjugative heat transfer, horizontal rectangular duct, nano fluid, particle volume fraction.

## NOMENCLATURE

List of symbols

$C_p$	Specific Heat at Constant Pressure(J/Kg-K)
$D_h$	Hydraulic Diameter (m)
$h$	Heat Transfer Coefficient (W/K-m <sup>2</sup> )
$K$	Thermal conductivity (W/m-K)
$Nu$	Nusselt Number
$P$	Pressure (N/m <sup>2</sup> )
$Re$	Reynolds Number
$T_w$	Temperature at wall (K)
$T_{nf}$	Temperature of nano fluid (K)
$u$	Velocity along X-direction (m/s)
$v$	Velocity Y-direction (m/s)
$w$	Velocity in Z-direction (m/s)

Greek Symbols

$\beta$	ratio of nano layer thickness(dimensionless)
$\theta$	Dimensionless Temperature(dimensionless)

$\mu$	Coefficient of viscosity (N-s/m <sup>2</sup> )
$\rho_{nf}$	Density of nano fluid (Kg/m <sup>3</sup> )
$\rho_s$	Density of nano particles (Kg/ m <sup>3</sup> )
$\phi$	Particle volume fraction of nano fluid (%)

Subscripts

in	Inlet conditions
nf	Nano fluid
s	Nano particles
w	Water

## I. INTRODUCTION

The nano fluid flow and heat transfer research has been started recently and it has got strong momentum because it is emerging and new topic in fluid flow and heat transfer. Vasefi and Alizadeh [1] numerically investigated CuO-Water nano fluid in different geometries by two-phase Euler-Lagrange method. They have thoroughly estimated the velocity field and found that with increase in percentage of nano particle the heat transfer rate increases as the overall heat transfer coefficient increases. Bouhalleb and Abbassi [2] also estimated numerically the heat transfer rate using CuO-Water nano fluid in rectangular geometries. Irmawati and Mohammed [3] later analysed mixed convective heat transfer in water-Based Al<sub>2</sub>O<sub>3</sub> nano fluid in horizontal rectangular duct. Ziaei-Rad and Elyasi [4] considered laminar pulsating nano fluid flow and heat transfer in a rectangular channel. They considered sinusoidal inlet flow and their results are very much useful. Mahian et al [5] made an elaborate discussion on the entropy generation in nano fluid for different geometries.

The most important findings of all of the researchers are that the heat transfer rate is enhanced since by increasing percentage of nano fluid particle the overall heat transfer coefficient increases. Present authors are motivated to do research on nano fluid anticipating the fact that such an analysis would be useful in heat

exchanger design and development in industrial applications.

## II. MATHEMATICAL FORMULATION

### A. Problem Description

In the figure 1 the schematic geometry of the physical problem has been described. The inlet flow is uniform with constant velocity  $U_{in}$ . The walls are fixed and impermeable and a no-slip condition is valid there. The length to diameter is sufficient to consider a fully developed flow at the outlet. The wall thermal conditions are considered as constant heat flux flowing through them. However the side walls are kept as adiabatic for the present case of study. The governing three dimensional equations in rectangular co-ordinate are given below.

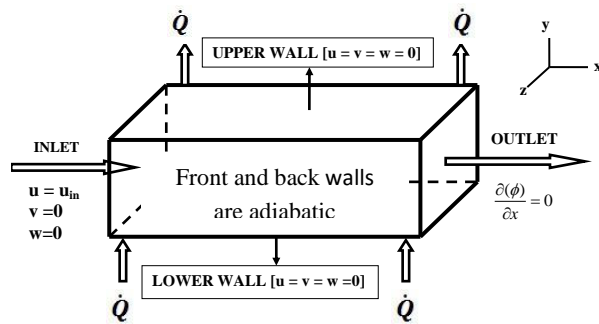


Fig. 1: The physical geometry and flow configuration of the horizontal rectangular duct

### B. Governing Equations

Continuity Equation:

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} = 0$$

X-Momentum Equation:

$$U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} + W \frac{\partial U}{\partial z} = - \frac{\rho_f}{\rho_{nf}} \frac{\partial P}{\partial x} + \frac{1}{Re} \frac{\rho_f}{\rho_{nf}} \frac{\mu_{nf}}{\mu_f} \left[ \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} \right]$$

Y-Momentum Equation:

$$U \frac{\partial V}{\partial x} + V \frac{\partial V}{\partial y} + W \frac{\partial V}{\partial z} = - \frac{\rho_f}{\rho_{nf}} \frac{\partial P}{\partial y} + \frac{1}{Re} \frac{\rho_f}{\rho_{nf}} \frac{\mu_{nf}}{\mu_f} \left[ \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} \right]$$

Z-Momentum Equation:

$$U \frac{\partial W}{\partial x} + V \frac{\partial W}{\partial y} + W \frac{\partial W}{\partial z} = - \frac{\rho_f}{\rho_{nf}} \frac{\partial P}{\partial z} + \frac{1}{Re} \frac{\rho_f}{\rho_{nf}} \frac{\mu_{nf}}{\mu_f} \left[ \frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2} + \frac{\partial^2 W}{\partial z^2} \right]$$

Energy Equation:

$$U \frac{\partial \theta}{\partial x} + V \frac{\partial \theta}{\partial y} + W \frac{\partial \theta}{\partial z} = \frac{\alpha_{nf}}{\alpha_f} \frac{1}{Re \cdot Pr} \left[ \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} + \frac{\partial^2 \theta}{\partial z^2} \right]$$

### C. Mathematical Modelling

Nano fluids with different concentrations of CuO nano particle including 0.0%, 0.01%, 0.02%, 0.05% and 0.1% volume fractions in distilled water were used to study heat transfer characteristics in laminar flow. The following are relevant equations adopted from [5].

The density of a nano fluid

$$\rho_{nf} = \phi \cdot \rho_s + (1 - \phi) \rho_w$$

Viscosity of the nano fluid

$$\mu_{nf} = \mu_w \cdot (1 + 2.5\phi)$$

The equation of specific heat of the nano fluid

$$C_{p_{nf}} = \frac{\phi \cdot (\rho_s \cdot C_{p_s}) + (1 - \phi) \cdot (\rho_w \cdot C_{p_w})}{\rho_{nf}}$$

Effective thermal conductivity of the nano fluid

$$k_{nf} = \left[ \frac{k_s + 2k_w + 2(k_s - k_w)(1 + \beta)^3 \phi}{k_s + 2k_w - (k_s - k_w)(1 + \beta)^3 \phi} \right] k_w$$

Convective heat transfer coefficient

$$\overline{h}_{nf} = \frac{q}{A \cdot (T_w - T_b)_{nf}}$$

Where  $T_b$  is the bulk mean temperature at a cross section.

Nusselt Number of the nano fluid

$$\overline{Nu}_{nf} = \frac{\overline{h}_{nf} \cdot D_h}{k_{nf}}$$

D. Boundary Conditions

WALLS

$$U = V = W = 0, \quad \dot{q} = -k \frac{\partial \theta}{\partial y}$$

INLET

$$U = 1, \quad W = V = \theta = 0 \quad (\text{at the entrance, } z=0)$$

EXIT

$$\frac{\partial(\phi)}{\partial x} = 0, \quad (\text{Fully developed condition}),$$

Where  $\phi = f(U, V, W, \theta)$  etc.

The dimensionless forms are interpreted as follows:

$$X = x / D_h; \quad Y = y / D_h; \quad Z = z / D_h; \quad U = u / U_{in}; \quad V = v / U_{in}; \quad W = w / U_{in} \quad \theta = (T - T_{in}) / (T_w - T_{in}),$$

$$P = \frac{p}{\rho_f} \frac{U_{in}^2}{\mu_f} \quad Re = \frac{U_{in} D_h \rho_f}{\mu_f}$$

$$Pr = \frac{\mu_f}{\rho_f \alpha_f}$$

### III. SOLUTION METHODOLOGY

A fully staggered grid system has been adopted for the velocity components and the scalar variables and these equations were discretized using a control volume formulation. The numerical solution in the present work is accomplished by using SIMPLER algorithm and the power-law scheme proposed by Patankar [6].

### IV. RESULTS AND DISCUSSION

The results are presented for different Reynolds number 100, 500, 1000 and 2000 and for different particle volume concentration of 0 to 10%. The nano fluid has been considered as a homogenous mixture which enters the duct with uniform temperature and velocity.

#### A. Grid Independent Study and Validation

For the authenticity of the computer code the grid independence study has to be performed. A grid independence study has been performed with two grid sizes 101 x 15 x 15 and 201 x 15 x 15 and it has been found that the results almost collapse on each other shown in the figure 2. However to avoid any complexities the higher grid size (201 X 15 X 15) is used. Results are presented for Reynolds number  $Re = 500$  and concentration of nano fluid  $\phi = 0.1\%$ . The results are very good and a pure collapse of the result have been observed.

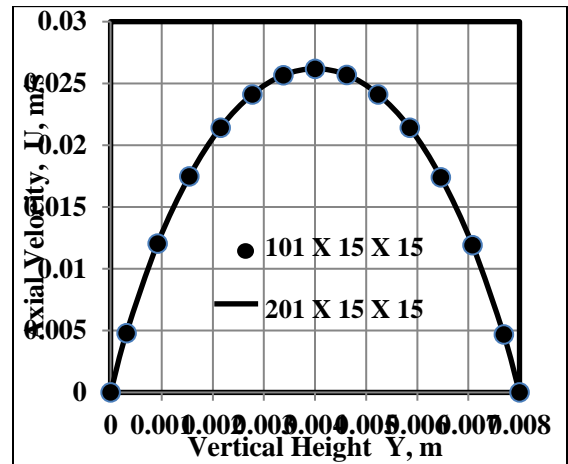


Fig. 2: The Grid Independent Study

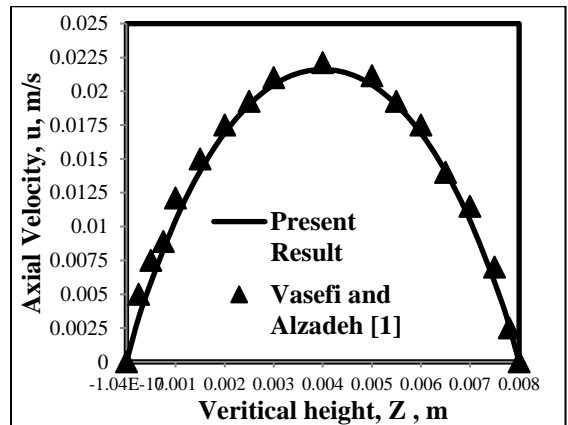


Fig. 3: Validation of Present Study

The validation of the present result is carried out by comparing results of [1] shown in Figure 3. Centreline Velocity Distribution

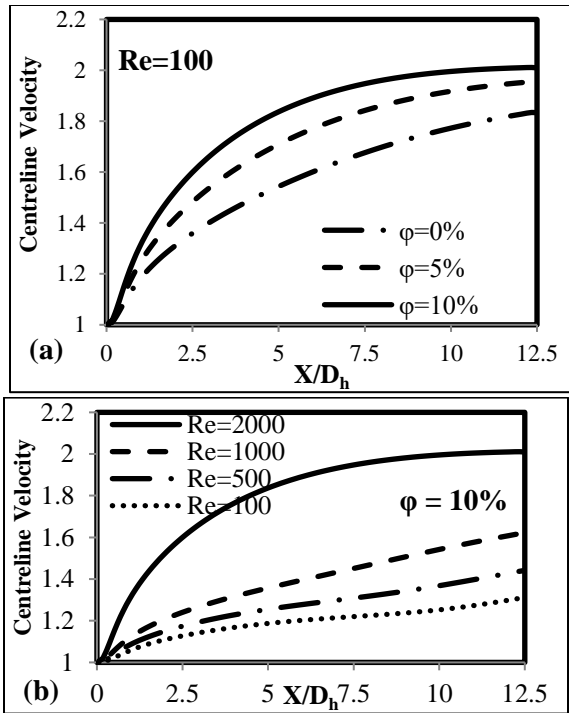


Fig. 4: Variation of Centreline Velocity with, (a) Different % of nano particle, (b) Different Reynolds number

In the figure 4(a) it is observed that for a Reynolds number of 100, the centreline velocity increases with the increase of the % of nano particle mixed with the pure water. In the figure 4(b) the same variations for the Reynolds number corresponding to a particular percentage of CuO in pure water has been shown.

**B. Friction Factor Variation**

Friction factor is given by: 
$$f = \frac{\mu_{nf} \frac{\partial U}{\partial y}}{0.5 \rho_f U_{in}^2}$$

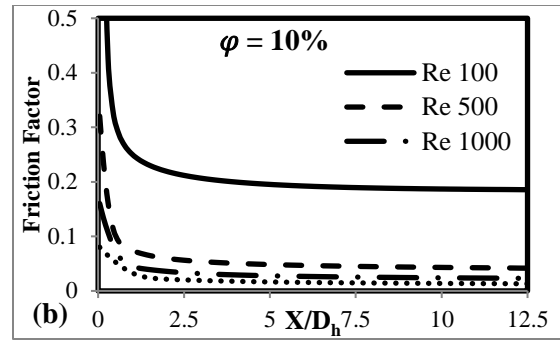
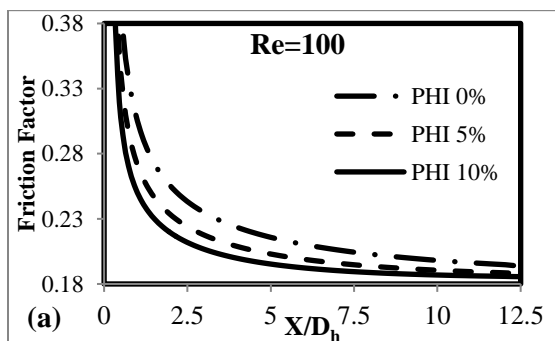


Fig. 5: Variations of Friction Factor with, (a) Different % of nano particle, (b) Different Reynolds number

In figures 5(a) it has been observed that the increase in % of CuO for a particular Reynolds number(100) decreases the friction factor. This is also true for the variations of friction factors with respect to Re keeping the % of CuO constant as revealed in the figures 5(b).

**C. Variations of Heat Transfer Coefficient**

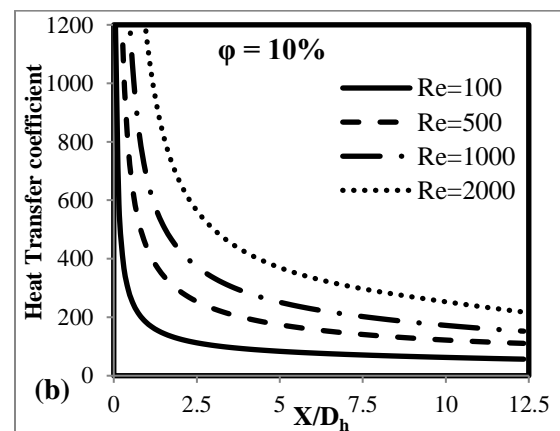
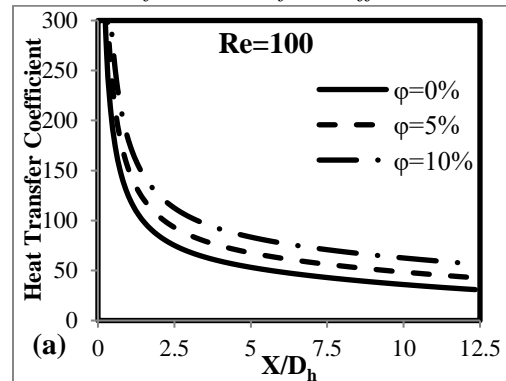


Fig. 6: Variation of Heat Transfer Coefficient with, (a) Different % of nano particle, (b) Different Reynolds number

The most important outcome heat transfer coefficient of nano fluid increases with the increase in the % of CuO in pure water shown in fig. 6(a). We also observe the enhancement of the overall heat transfer coefficient by increasing the Re in fig. 6(b). These two diagrams actually suggest that using the nano fluid the heat transfer can be enhanced with a great degree.

D. Variations of Nusselt Number with  $\phi$  & Re

Similarly the increase in Nusselt number of the nano fluid has been achieved with the increase of nano particle percentage and Reynolds number as shown in fig. 7(a) and (b).

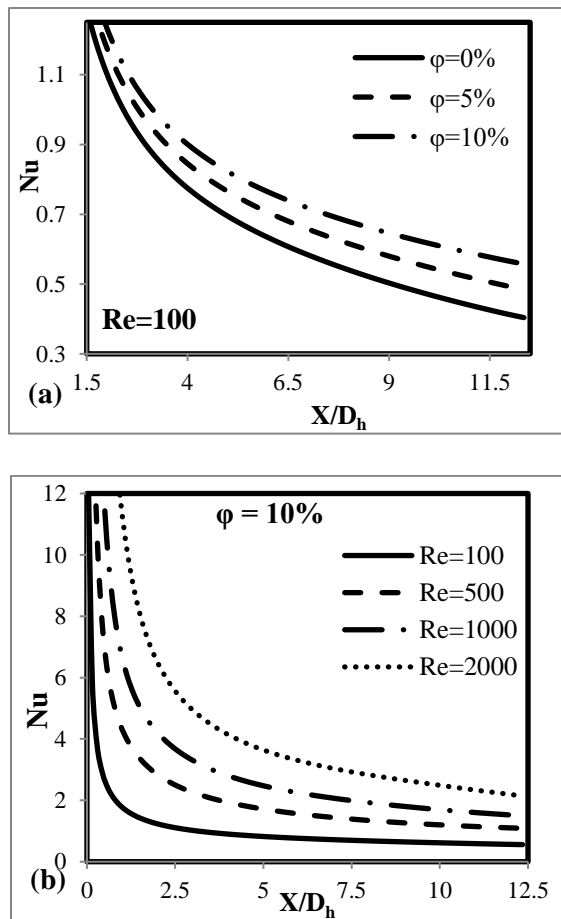


Fig. 7: Variation of Nusselt Number with, (a) Different % of nano particle, (b) Different Reynolds number

CONCLUSION

The effect of different Reynolds number and nano particle volume concentration are intensively

interpreted. It is found that the heat transfer coefficient increases with increase in percentage of nano particle. This is a great boost in heat transfer. This has also been observed that the Nusselt number also increases with the increase in percentage of nano particle. The same is also true for the changes in Reynolds number in similar fashion.

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