SET THEORY

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***Abstract*- The modern study of set theory was initiated by Georg Cantor and Richard Dedekind in the 1870s. After the discovery of paradoxes innaive set theory, numerous axiom systems were proposed in the early twentieth century. The basic concepts and notations, applications and area of study is being discussed in the paper.This paper is about the basics of the set theory.**

I. INTRODUCTION

Set theory is the branch of mathematical logic that studies sets, which are collections of objects. Although any type of object can be collected into a set, set theory is applied most often to objects that are relevant to mathematics.The language of set theory can be used in the definitions of nearly all mathematical objects.The objects of study of set theory are sets . As sets are fundamental objects that can be used to define all other concepts in terms of more fundamental.

The language of set theory is based on a single fundamental relation, called membership. We say that A is a member of B (in symbolsA ∈B), or that the set B contain A as its element. The understanding is that a set is determined by its elements.

II. BASIC CONCEPTS AND NOTATIONS

Set theory begins with a fundamental binary relation between an object o and a set A. If o is a member (or element) of A. A derived binary relation also called set inclusion.

- Subset: If all the members of set A are also the members of set B, then A is a subset of B, denoted by A ⊆ B. For example, {1,2} is a subset of {1,2,3}, but {1,4} is not.

- Proper subset : A is called a proper subset of B if and only if A is a subset of B , but B is not a subset of A.

- Union: Union of the sets A and B , denoted A ∪ B, is the set of all objects that are a member of A, or B, or both . The union of {1,2,3} and {2,3,4} is the set {1,2,3,4}.

- Intersection: Intersection of the sets A, and B,denoted A ∩ B , is the set of all objects that are a members of both A an.d B. the intesection of {1,2,3} and {2,3,4} is the set {2,3}.

- Set difference: set difference of U and A, denoted U\A, is the set of all members of U that arre not members of A. The set difference {1,2,3} \ {2,3,4} is {1},while coversely, the set difference {2,3,4}\ {1,2,3,} is {4} . When A is a subset of U, the set difference U\A is also called the complement of A in U. In this case, if the choice of U is clear from the context, the notation A^c is sometimes used instead of U\A , particularly if U is a universal set as in study of Venn diagrams.
- Symmetric difference: of sets *A* and *B*, denoted $A \vartriangle B$ or $A \ominus B$, is the set of all objects that are a member of exactly one of *A* and *B* (elements which are in one of the sets, but not in both). For instance, for the sets {1,2,3} and {2,3,4} , the symmetric difference set is {1,4} . It is the set difference of the union and the intersection, $(A \cup B) \setminus (A \cap B)$ or $(A \setminus B) \cup (B \setminus A)$.

- Cartesian product of *A* and *B*,denoted $A \times B$, is the set whose members are all possible ordered pairs (*a*,*b*) where *a* is a member of *A* and *b* is a member of*B*. The cartesian product of {1, 2} and {red, white} is {(1, red), (1, white), (2, red), (2, white)}.

- Power set of a set *A* is the set whose members are all possible subsets of *A*. For example, the power set of {1, 2} is { {}, {1}, {2}, {1,2} } .

- Empty set :the unique set containing no elements

III. APPLICATIONS

Many mathematical concepts can be defined precisely using only set theoretic concepts. For example, mathematical structures as diverse as graphs,manifolds, rings, and vector spaces can all be defined as sets satisfying various (axiomatic) properties. Equivalence and order relations are ubiquitous in mathematics, and the theory of mathematical relations can be described in set theory.

Set theory is also a promising foundational system for much of mathematics. Since the publication of the first volume of *Principia Mathematica*, it has been claimed that most or even all mathematical theorems can be derived using an aptly designed set of axioms for set theory, augmented with many definitions, using first or second order logic. For example, properties of the natural and real numbers can be derived within set theory, as each number system can be identified with a set of equivalence classes under a suitable equivalence relation whose field is some infinite set.

Set theory as a foundation for mathematical analysis, topology, abstract algebra, and discrete mathematics is likewise uncontroversial; mathematicians accept that (in principle) theorems in these areas can be derived from the relevant definitions and the axioms of set theory. Few full derivations of complex mathematical theorems from set theory have been formally verified, however, because such formal derivations are often much longer than the natural language proofs mathematicians commonly present. One verification project, Metamath, includes derivations of more than 10,000 theorems starting from the ZFC axioms and using first order logic.

IV. AREA OF STUDY

Set theory is a major area of research in mathematics, with many interrelated subfields.

I. Combinatorial set theory
II. Descriptive set theory
III. Fuzzy set theory
IV. Inner model theory
V. Large cardinals
VI. Cardinal invariants
VII. Set-theoretic topology

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