RELATIONS AND FUNCTIONS

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***Abstract*- A relation is any association between elements of one set, called the domain or (less formally) the set of inputs, and another set, called the range or set of outputs. Some people mistakenly refer to the range as the codomain, but as we will see, that really means the set of all possible outputs—even values that the relation does not actually use. Formally, R is a relation if**

$R \subseteq X \times Y = \{(x, y) \mid x \in X, y \in Y\}$

for the domain X and codomain Y. In mathematics, a function is a relation between a set of inputs and a set of permissible outputs with the property that each input is related to exactly one output. Functions of various kinds are "the central objects of investigation" in most fields of modern mathematics. There are many ways to describe or represent a function. Some functions may be defined by a formula or algorithm that tells how to compute the output for a given input. Others are given by a picture, called the graph of the function. There are many operations on relations are inversion , concatenation , diagonal of set etc. There are three types of functions : injective functions , surjective functions , bijective functions . There are many applications of relations and functions . Relations can be transitive , symmetric and reflexive. In this research paper we will discuss how relations are different and related from the functions in discrete mathematics .

***Index Terms*- domain , range , codomain , image**

I. INTRODUCTION

Relation

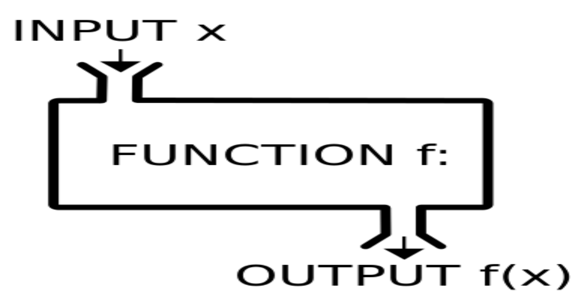
A relation is any association between elements of one set, called the domain the set of inputs, and another set, called the range or set of outputs. Some people mistakenly refer to the range as the codomain(range), but as we will see, that really means the set of all possible outputs—even values that the relation does not actually use. A relation is allowed to have the object x in the first set to be related to more than one object in the second set. So a relation may not be represented by a function machine, because, given the object x to the input of the machine, the machine couldn't spit out a unique output object that is paired to x.

For example, if the domain is a set Fruits = {apples, oranges, bananas} and the codomain(range) is a set Flavors = {sweetness, tartness, bitterness}, the flavors of these fruits form a relation: we might say that apples are related to (or associated with) both sweetness and tartness, while oranges are related to tartness only and bananas to sweetness only. (We might disagree somewhat, but that is irrelevant to the topic of this book.) Notice that "bitterness", although it is one of the possible Flavors (codomain)(range), is not really used for any of these relationships; so it is not part of the range (or image) {sweetness, tartness}. Another way of looking at this is to say that a relation is a subset of ordered pairs drawn from the set of all possible ordered pairs (of elements of two other sets, which we normally refer to as the Cartesian product of those sets). Formally, R is a relation if

$R \subseteq X \times Y = \{(x, y) \mid x \in X, y \in Y\}$

Function

A function is a relationship between two sets of numbers. We may think of this as a mapping; a function map a number in one set to a number in another set. Notice that a function maps values to one and only one value. Two values in one set could map to one value, but one value must never map to two values: that would be a relation, not a function.



A composite function g(f(x)) can be visualized as the combination of two "machines". The first takes input x and outputs f(x). The second takes f(x) and outputs g(f(x)).

Types of relation

When we are looking at relations, we can observe some special properties different relations can have.

1. Reflexive Relation:

R is a relation in A and for every a Î A, (a,a) Î R then R is said to be a reflexive relation.
Example:
Every real number is equal to itself. Therefore "is equal to " is a reflexive relation in the set of real numbers.

2. Symmetric Relation:

R is a relation in A and (a,b) Î R implies (b,c) Î R then R is said to be a symmetric relation.
Example:
In the set of all real numbers "is equal to" relation is symmetric.

3. Anti-Symmetric Relation:

   R is a relation in A. If (a,b) Î R and (b,a) Î R implies a = b, then R is said to be an anti-symmetric relation.

Example:
In set of all natural numbers the relation R defined by "x divides y if and only if (x,y) Î R" is anti-symmetric. For x|y and y|x then x = y.

4. Transitive Relation:
   R is a relation in A if (a,b) Î R and (b,c) Î R implies (a,c) Î R is called a transitive relation.

Example:
In the set of all real numbers the relation "is equal to" is a transitive relation. For a = b, b = c implies a = c.

5. Equivalence Relation:

A relation R in a set A is said to be an equivalence relation if it is reflexive, symmetric and transitive.
Example:
In the set of all real numbers the relation "is equal to" is an equivalence relation for a Î R, a = a, b = a implies b = a and a = b, b = c implies a = c.

Operations on Relations.

There are some useful operations one can perform on relations, which allow to express some of the above mentioned properties more briefly.
1.Inversion
Let R be a relation, then its inversion, R-1 is defined by
R-1 := {(a,b) | (b,a) in R}.

2.Concatenation

Let R be a relation between the sets A and B, S be a relation between B and C. We can concatenate these relations by defining
R • S := {(a,c) | (a,b) in R and (b,c) in S for some b out of B}

3.Diagonal of a Set

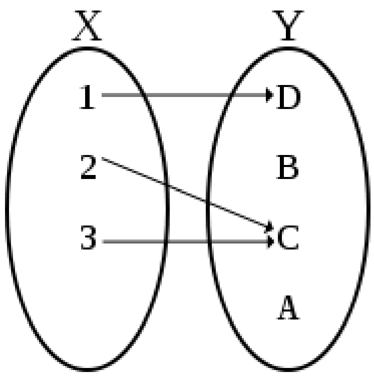
Let A be a set, then we define the diagonal (D) of A by
D(A) := {(a,a) | a in A}
Shorter Notations

Using above definitions, one can say (lets assume R is a relation between A and B):
R is transitive if and only if R • R is a subset of R.
R is reflexive if and only if D(A) is a subset of R.
R is symmetric if R-1 is a subset of R.
R is antisymmetric if and only if the intersection of R and R-1 is D(A).
R is asymmetric if and only if the intersection of D(A) and R is empty.
R is a function if and only if R-1 • R is a subset of D(B).
In this case it is a function A → B. Let's assume R meets the condition of being a function, then
R is injective if R • R-1 is a subset of D(A).
R is surjective if {b | (a,b) in R} = B.

II. TERMS IN FUNCTION

Range, image, codomain

If D is a set, in which x forms a new set, called the range of f. D is called the domain of f, and represents all values that f takes.
In general, the range of f is usually a subset of a larger set. This set is known as the codomain of a function. For example, with the function f(x)=cos x, the range of f is [-1,1], but the codomain is the set of real numbers



the above function diagram shows domain{1,2,3} and codomain{A,B,C,D} and the image is {C,D}.

III. TYPES OF FUNCTIONS

There are a number of general basic properties and notions in a function with domain X and codomainY.

Functions can either be one to one (injective), onto (surjective), or bijective.

INJECTIVE : Functions are functions in which every element in the domain maps into a unique elements in the codomain.

SURJECTIVE : Functions are functions in which every elements in the codomain is mapped by an element in the domain.

BIJECTIVE: Functions are functions that are both injective and surjective.

---onto functions a function f form A to B is onto

When f and f-1 are both functions, they are called one-to-one, injective, or invertible functions. This is one of two very important properties a function f might (or might not) have; the other property is called onto or surjective, which means, for any y ∈ Y (in the codomain), there is some x ∈ X (in the domain) such that f(x) = y. In other words, a surjectivefunction f maps onto every possible output at least once.

A function can be neither one-to-one nor onto, both one-to-one and onto (in which case it is also called bijective or a one-to-one correspondence), or just one and not the other. (As an example which is neither, consider f = {(0,2), (1,2)}. It is a function, since there is only one y value for each x value; but there is more than one input x for the output y = 2; and it clearly does not "map onto" all integers.)

Shorter notation

When we have a function f, with domain D and range R, we write:If we say that, for instance, x is mapped to x2.

Notice that we can have a function that maps a point (x,y) to a real number, or some other function of two variables -- we have a set of ordered pairs as the domain. Recall from set theory that this is defined by the Cartesian product - if we wish to represent a set of all real-valued ordered pairs we can take the Cartesian product of the real numbers with itself to obtain

.When we have a set of n-tuples as part of the domain, we say that the function is n-ary (for numbers n=1,2 we say unary, and binary respectively).

Other function notation

Functions can be written as above, but we can also write them in two other ways. One way is to use an arrow diagram to represent the mappings between each element. We write the elements from the domain on one side, and the elements from the range on the other, and we draw arrows to show that an element from the domain is mapped to the range.

For example, for the function f(x)=x3, the arrow diagram for the domain {1,2,3} would be: Another way is to use set notation. If f(x)=y, we can write the function in terms of its mappings. This idea is best to show in an example.

Let us take the domain D={1,2,3}, and f(x)=x2. Then, the range of f will be R={f(1),f(2),f(3)}={1,4,9}. Taking the Cartesian product of D and R we obtain F={(1,1),(2,4),(3,9)}.So using set notation, a function can be expressed as the Cartesian product of its domain and range f(x),This function is called f, and it takes a variable x. We substitute some value for x to get the second value, which is what the function maps x to.

IV. FUNCTION COMPOSITION

The function composition of two functions takes the output of one function as the input of a second one. More specifically, the composition of f with a functiong: Y → Z is the function defined by:

That is, the value of x is obtained by first applying f to x to obtain y = f(x) and then applying g to y to obtain z = g(y). In the notation , the function on the right, f, acts first and the function on the left, g acts second. The notation can be memorized by reading the notation as "g of f. The composition is only defined when the codomain of f is the domain of g. Assuming that, the composition in the opposite order need not be defined. Even if it is, i.e., if the codomain off is the codomain of g, it is not in general true ,

That is, the order of the composition is important. For example, supposef(x) = x2 and g(x) = x+1. Then g(f(x)) = x2+1, while f(g(x)) = (x+1)2, which isx2+2x+1, a different function.

Identity function

The unique function over a set *X* that maps each element to itself is called the *identity function* for *X*. Each set has its own identity function, so the subscript cannot be omitted unless the set can be inferred from context. Under composition, an identity function is "neutral": if *f*is any function from *X* to *Y*.

Restrictions and extensions

A restriction of a function f is the result of trimming its domain. More precisely, if S is any subset of X, the restriction of f to S is the function f|S from S to Y such that f|S(s) = f(s) for all s in S. If g is a restriction of f, then it is said that f is an extension of g.

The overriding of f: X → Y by g: W → Y (also called overriding union) is an extension of g denoted as (f ⊕ g): (X ∪ W) → Y. Its graph is the set-theoretical union of the graphs of g and f|X \ W. Thus, it relates any element of the domain of g to its image under g, and any other element of the domain off to its image under f.

Inverse function

An inverse function for f, denoted by f−1, is a function in the opposite direction, from Y to X, satisfying:

That is, the two possible compositions of f and f−1 need to be the respective identity maps of X and Y.

As a simple example, if f converts a temperature in degrees Celsius C to degrees Fahrenheit F, the function converting degrees Fahrenheit to degrees Celsius would be a suitable f−1.

Such an inverse function exists if and only if f is bijective. In this case, f is called invertible. The notation and f−1 are taking to multiplication and reciprocal notation.

With this analogy, identity functions are like the multiplicative , and inverse functions are like reciprocal for the domain X and codomain(range) Y. The inverse relation of R, which is written as R-1, is what we get when we interchange the X and Y values:

R-1 = {(y, x) | (x, y) ∈ R}

V. DIFFERENCE BETWEEN RELATION AND FUNCTION

- Using the example we can write the relation in set notation: {(apples, sweetness), (apples, tartness), (oranges, tartness), (bananas, sweetness)}. The inverse relation, which we could describe as "fruits of a given flavor", is {(sweetness, apples), (sweetness, bananas), (tartness, apples), (tartness, oranges)}.

One important kind of relation is the function. A function is a relation that has exactly one output for every possible input in the domain. (The domain does not necessarily have to include all possible objects of a given type. In fact, we sometimes intentionally use a restricted domain in order to satisfy some desirable property.) The relations discussed above (flavors of fruits and fruits of a given flavor) are not functions: the first has two possible outputs for the input "apples" (sweetness and tartness); and the second has two outputs for both "sweetness" (apples and bananas) and "tartness" (apples and oranges).

The main reason for not allowing multiple outputs with the same input is that it lets us apply the same function to different forms of the same thing without changing their equivalence. That is, if f is a function with a (or b) in its domain, then a = b implies that f(a) = f(b). For example, z - 3 = 5 implies that z = 8 because f(x) = x + 3 is a function defined for all numbers x.

The converse, that f(a) = f(b) implies a = b, is not always true. When it is never more than one input x for a certain output y = f(x). This is the same as the definition of function, but with the roles of X and Y interchanged; so it means the inverse relation f-1 must also be a function. In general—regardless of whether or not the original relation was a function—the inverse relation will sometimes be a function, and sometimes not.

VI. CONCLUSION

In this paper relation and function will be the focus of most of the rest of algebra, as well as pre-calculus and calculus.it is an important stepping stone to the rest of algebra.relation and function will be used to solve many different types of problems .however ,one must first learn the basics-how to recognize a function ,and how to determine its domain and function.it explains how to represent relation and function using both mapping diagram and graphs.

REFERENCES

[1] http://en.wikipedia.org/wiki/Function_(mathematics)

[2] http://en.wikibooks.org/wiki/Discrete_Mathematics/Functions_and_relations

[3] http://www1.cs.columbia.edu/~zeph/3203s04/lectures.html

[4] http://www.csie.ndhu.edu.tw/~rschang/dmath.html

[5] http://www.slideshare.net/uyar/discrete-mathematics-relations-and-functions