

FIR Filter Design using Different Window Techniques

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Abstract- Digital filter are widely used in the world of communication and computation. On the other hand to design a digital finite impulse response (FIR) filter that satisfying all the required conditions is a challenging one. In this paper, design techniques of low pass FIR filters using Blackman window method, Optimal Parks McClellan method and Genetic Algorithm method are presented. The stability, number of components required and filter coefficients are demonstrated for different design techniques. It is shown that filter design by using GA is best because the numbers of components required are less and stability is more as compare with other techniques. Design comparisons are presented to show the effectiveness of GA optimization method

Index Terms– Rectangular, Hanning and Hamming windows, FIR Filter.

I. INTRODUCTION

Digital filters are actually the discrete time systems which are mainly used for filter purpose. And these include filtering of arrays or sequences. These arrays or sequences can be obtained by sampling the input signal which is analog in nature. The digital filters perform the frequency related operations such as low pass, high pass, band reject, band pass and all pass, etc. also the digital specifications include cut off frequency, sampling frequency of input signal, pass band variation, stop band attenuation, approximation, type of filter. So these filters are proficient of performing that condition which are extremely complicated, to accomplish with an analog implementation. Also the major advantage of the digital filter is that all the characteristics of a digital filter can be easily upgraded under software control. Actually the digital filter may be realized through hardware as well as software. But the software digital filters requires digital hardware for their operation.

There are two types of filters which provides these functions are Finite Impulse Response(FIR) and Infinite Impulse Response(IIR) filters.

FIR Filter Systems are those systems for which the unit sample response $h(n)$ has finite number of terms.

IIR Filter Systems are those systems for which such infinite number of unit sample response terms are to be considered are called Infinite Impulse Response.

A. Difference Between FIR and IIR Filter

- FIR filter is Finite in nature while IIR Filter if infinite.
- Feedback system is not involved in FIR so it is non recursive while IIR filter is recursive in nature.
- The impulse response of an FIR filter will finally reaches zero. The impulse response of an IIR filter may very well keep "ringing" infinitely.
- IIR filters can be accurately simulate analog filter response while FIR can't do this
- FIR filter has a linear phase where as IIR filter has no particular phase.
- FIR filter is more stable than IIR filter.
- FIR filter consist of only zeroes and IIR filter consists of both poles and zeroes.

FIR filters are filters having a transfer function of a polynomial in z - and is an all-zero filter in the logic that the zeroes in the z -plane resolve the frequency response magnitude characteristic [4]. The z transform of a N -point FIR filter is given by

$$H(z) = \sum_{k=0}^{N-1} h(n)z^{-n}$$

FIR filters are particularly useful for applications where exact linear phase response is required. The FIR filter is generally implemented in a non-recursive way which guarantees a stable filter. FIR filter design essentially consists of two parts

- Approximation problem
- Realization problem

The approximation stage takes the measurement and gives a transfer function through four steps. They are as follows:

- A desired or ideal response is chosen, usually in the frequency domain.
- An allowed class of filters is chosen (e.g. the length N for a FIR filters).
- A measure of the quality of approximation is chosen.

- A method or algorithm is selected to find the best filter transfer function. The realization part deals with choosing the structure to implement the transfer function which may be in the form of circuit diagram or in the form of a program.

There are essentially three well-known methods for FIR filter design namely:

- (1) The window method
- (2) The frequency sampling technique
- (3) Optimal Filter Design Method

B. Applications

Traditionally, most digital filter applications have been limited to audio and high-end image processing. With advances in process technologies and digital signal processing methodologies, digital filters are now cost-effective in the IF range and in almost all video markets. Digital filters are commonly used for audio frequencies for two reasons. First, digital filters for audio are superior in price and performance to the analog alternative. Second, audio Analog-to-Digital Converters (A/Ds) and Digital-to-Analog Converters (DACs) can be manufactured with high accuracy and are available at low cost. Thus, the combined cost of filtering and conversion (if necessary) is low. The cost trades are much more difficult in the 1MHz to 100MHz signal range, such as the IF ranges of many radio receivers.

II. THE WINDOW METHOD

The method the most used in digital filter design is Fourier series method. However, there is a problem in this method. The problem is that Fourier series method causes to Gibb’s oscillations at cut-off frequency region. In this the desired frequency response $H(e^{jw})$. The Fourier transform of the weighting function consists of the main lobe, which contains most of the energy of the window function and side lobes which decays rapidly.

In this method the desired frequency response specification $H_d(w)$, corresponding unit sample response $h_d(n)$ is determined using the following relation

$$h_d(n) = 1/2\pi \int_{-\pi}^{\pi} H_d(w) e^{jwn} dw$$

$$H_d(w) = \sum_{n=-\infty}^{\infty} h_d(n) e^{-jwn}$$

In general, unit sample response $h_d(n)$ obtained from the above relation is infinite in duration, so it must be truncated at some point say $n = M-1$ to yield an

FIR filter of length (i.e. 0 to M-1). This truncation of $h_d(n)$ to length M-1 is same as multiplying $h_d(n)$ by the rectangular window defined as

$$W(n) = \begin{cases} a - b \cos(2\pi(n+1)/(N+1)) + c \cos(4\pi(n+1)/(N+1)) & n = 0, 1, \dots, N-1 \\ = 0 & \text{otherwise} \end{cases}$$

Thus the unit sample response of the FIR filter becomes

$$h(n) = \begin{cases} h_d(n) w(n) & 0 \leq n \leq M-1 \\ = 0 & \text{otherwise} \end{cases}$$

Now, the multiplication of the window function $w(n)$ with $h_d(n)$ is equivalent to convolution of $H_d(w)$ with $W(w)$, where $W(w)$ is the frequency domain representation of the window function

$$W(w) = \sum_{n=0}^{M-1} w(n) e^{-jwn}$$

Thus the convolution of $H_d(w)$ with $W(w)$ yields the frequency response of the truncated FIR filter

$$H(w) = 1/2\pi \int_{-\pi}^{\pi} H_d(v) W(w - v) dw$$

The frequency response can also be obtained using the following relation

$$H(w) = \sum_{n=0}^{M-1} h(n) e^{-jwn}$$

Direct truncation of $h_d(n)$ to M terms to obtain $h(n)$ leads to the Gibbs phenomenon effect which manifests itself as a fixed percentage overshoot and ripple before and after an

approximated discontinuity in the frequency response due to the non-uniform convergence of the Fourier series at a discontinuity. Thus the frequency response obtained by using above equation contains ripples in the frequency domain. In order to reduce the ripples, instead of multiplying $h_d(n)$ with a rectangular window $w(n)$, $h_d(n)$ is multiplied with a window

function that contains a taper and decays toward zero gradually, instead of abruptly as it occurs in a rectangular window. As multiplication of sequences $h_d(n)$ and $w(n)$ in time domain is equivalent to convolution of $H_d(w)$ and $W(w)$

in the frequency domain, it has the effect of smoothing $H_d(w)$. The several effects of windowing

the Fourier coefficients of the filter on the result of the frequency response of the filter are as follows:

- (i) A major effect is that discontinuities in $H(w)$ become transition bands between values on either side of the discontinuity.
- (ii) The width of the transition bands depends on the width of the main lobe of the frequency response of the window function, $w(n)$ i.e. $W(w)$.
- (iii) Since the filter frequency response is obtained via a convolution relation, it is clear that the resulting filters are never optimal in any sense.
- (iv) As M (the length of the window function) increases, the main lobe width of $W(w)$ is reduced which reduces the width of the transition band, but this also introduces more ripple in the frequency response.
- (v) The window function eliminates the ringing effects at the band edge and does result in lower side lobes at the expense of an increase in the width of the transition band of the filter.

The FIR filter design process via window functions can be split into several steps:

1. Defining filter specifications;
2. Specifying a window function according to the filter specifications;
3. Computing the filter order required for a given set of specifications;
4. Computing the window function coefficients;
5. Computing the ideal filter coefficients according to the filter order;
6. Computing FIR filter coefficients according to the obtained window function and ideal filter coefficients;
7. If the resulting filter has too wide or too narrow transition region, it is necessary to change the filter order by increasing or decreasing it according to needs, and after that steps 4, 5 and 6 are iterated as many times as needed.

The final objective of defining filter specifications is to find the desired normalized frequencies ($\omega_c, \omega_{c1}, \omega_{c2}$), transition width and stopband attenuation. The window function and filter order are both specified according to these parameters.

Some of the windows commonly used are as follows:

1. Bartlett triangular window

$$W(n) = \frac{2(n+1)}{N+1} ; n=0,1,2,\dots,(N-1)/2$$

$$= \frac{2-(2(n+1))/N+1}{N+1} ; n= (N-1)/2 \dots N-1$$

$$= 0, \text{ otherwise}$$

2-5. Generalized cosine windows (Rectangular, Hanning, Hamming and Blackman)

$$W(n) = a - b \cos(2p(n+1)/(N+1)) + c \cos(4p(n+1)/(N+1))$$

$$; n=0,1,\dots,N-1$$

$$= 0 \quad ; \text{ otherwise}$$

II. Simulation and Results:-

In simulation number of sample point (N)=65, Filter Order=61, $W_s=.4580, W_p=.341$

3.1 Ideal Magnitude Response

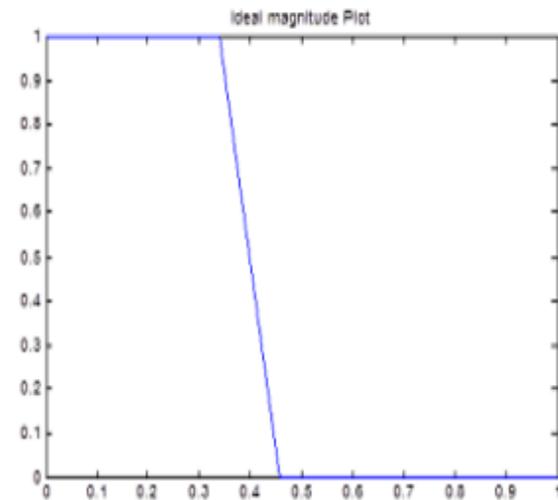


Fig:-1

3.2 Magnitude response of rectangular window

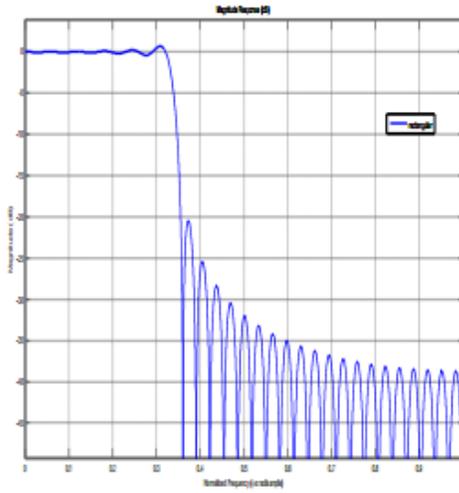


Fig-2

3.4 Magnitude response of hamming window

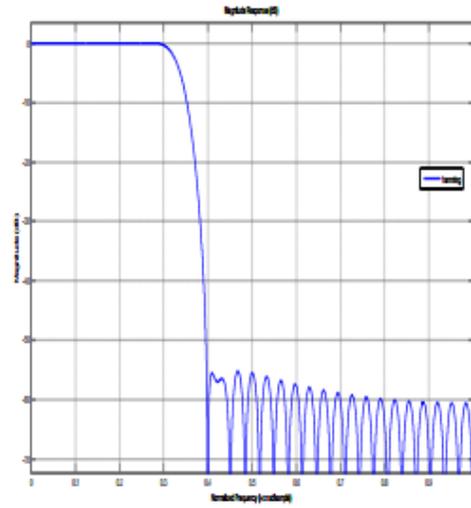


Fig-4

3.3 Magnitude response of hanning window

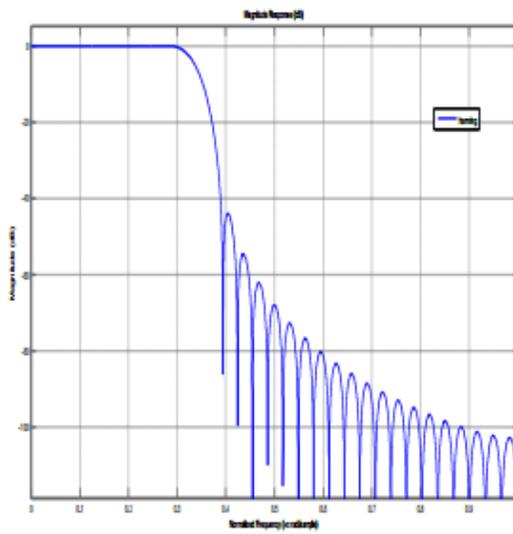


Fig-3

3.5 Phase Response of Rectangular window

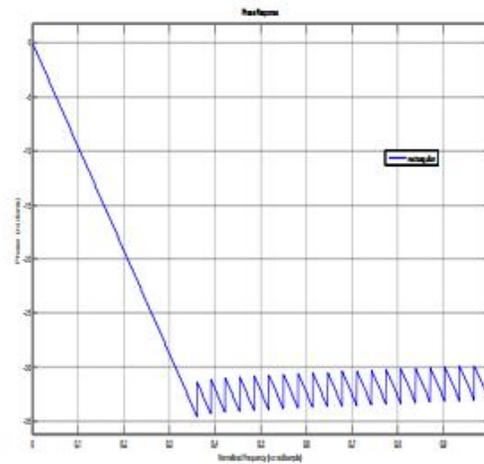


Fig-5

3.6 Phase Response of Hanning window

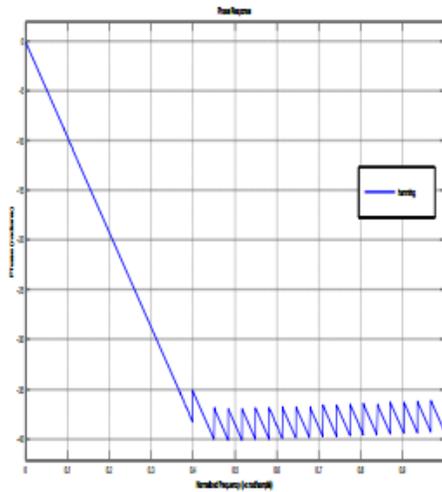


Fig:-6

3.7 Phase Response of Hanning window

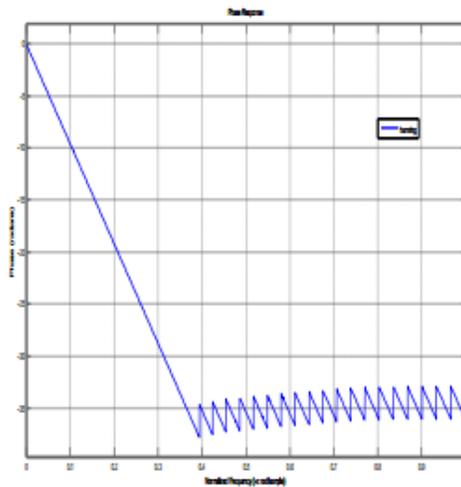


Fig:-7

III. CONCLUSION

FIR filter design by using hamming is stable as compare to rectangular and hanning windows techniques. Ripples in pass band are less in hamming as compare to other two techniques (as shown in fig 2, 3, 4). Hamming has linear phase as compare to rectangular and hanning windows.

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