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NETWORK SYNTHESIS USING HURWITZ POLYNOMIAL AND POSITIVE REAL FUNCTION

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***Abstract-* In this paper we will be studying about Network synthesis the derivation of the components, their values, and their arrangement within an electric circuit, so as to synthesize a circuit which provides a given output signal in response to a given input signal using hurwitz polynomial and positive real function.**

***Index Terms-* Synthesis, network, polynomial, inductors, resistors, capacitors, circuit, output.**

I. INTRODUCTION

As the name suggests, in theory of network synthesis we are going to study about the synthesis of various networks which consists of both the active (resistors) and passive elements (inductors and capacitors).Let us know what is a network function? In the frequency domain, network functions are defined as the quotient obtained by dividing the phasor corresponding to the circuit output by the phasor corresponding to the circuit input.In simple words, network functions are the ratio of output phasor to the input phasor when phasors exists in frequency domain. The general form of network functions are given below:

$$F(s) = \frac{a_n s^n + a_{n-1} s^{n-1} + \cdots\cdots\cdots\cdots + a_1 s^1 + a_0}{b_m s^m + b_{m-1} s^{m-1} + \cdots\cdots\cdots\cdots + b_1 s^1 + b_0}$$

II. HURWITZ POLYNOMIAL

In Mathematics, a Hurwitz polynomial, named after Adolf Hurwitz, is a polynomial whose coefficients are positive real numbers and whose roots (zeros) are located in the left half-plane of the complex plane or on the jω axis, that is, the real part of every root is zero or negative.The term is sometimes restricted to polynomials whose roots have real parts that are strictly negative, excluding the axis (i.e., a Hurwitz stable polynomial).

A polynomial function P(s) of a complex variable s is said to be Hurwitz if the following conditions are satisfied:

1. P(s) is real when s is real.
2. The roots of P(s) have real parts which are zero or negative.

Hurwitz polynomials are important in control systems theory, because they represent the characteristic equations of stable linear systems. Whether a polynomial is Hurwitz can be determined by solving the equation to find the roots, or from the coefficients without solving the equation by the Routh-Hurwitz stability criterion.

III. PROPERTIES

For a polynomial to be Hurwitz, it is necessary but not sufficient that all of its coefficients be positive. A necessary and sufficient condition that a polynomial is Hurwitz is that it passes the Routh-Hurwitz stability criterion. A given polynomial can be efficiently tested to be Hurwitz or not by using the Routh continued fraction expansion technique.

The properties of Hurwitz polynomials are:

All the poles and zeros are in the left half plane or on its boundary, the imaginary axis.

- Any poles and zeros on the imaginary axis are simple (have a multiplicity of one).
- Any poles on the imaginary axis have real strictly positive residues, and similarly at any zeros on the imaginary axis, the function has a real strictly positive derivative.
- Over the right half plane, the minimum value of the real part of a PR function occurs on the imaginary axis (because the real part of an analytic function constitutes a harmonic function over the plane, and therefore satisfies the maximum principle).

- The polynomial should not have missing powers of s.

Example 7 - Check whether the given polynomial $P(s) = s^4 + s^3 + 5s^2 + 3s + 4$ is Hurwitz or not

Condition 1 - Since all coefficients of P(s) are positive so P(s) is real for s real

Condition (2). The even & odd parts of P(s) are

$M(s) = s^4 + 5s^2 + 4$

$N(s) = s^3 + 3s$

$s^3+3s \,\big)\, s^4+5s^2+4 \,\big(\, s$

s^4+3s^2

$2s^2+4 \,\big)\, s^3+3s \,\big(\, \frac{1}{2}s$

s^3+2s

$s \,\big)\, 2s^2+4 \,\big(\, 2s$

$2s^2$

$4 \,\big)\, s \,\big(\, \frac{s}{4}$

s

$\times$

$$\psi(s) = \frac{M(s)}{N(s)} = s + \cfrac{1}{\frac{s}{2} + \cfrac{1}{2s + \cfrac{1}{s/4}}}$$

it is an Hurwitz

IV. POSTIVE REAL FUNCTION

Any function which is in the form of F(s) will be called as a **positive real function** if fulfill these four important conditions:

- F(s) should give real values for all real values of s.
- P(s) should be a Hurwitz polynomial.
- If we substitute s = jω then on separating the real and imaginary parts, the real part of the function should be greater than or equal to zero, means it should be non negative. This most important condition and we will frequently use this condition in order to find out the whether the function is positive real or not.
- On substituting s = jω, F(s) should posses simple poles and the residues should be real and positive.

V. PROPERTIES OF POSITIVE REAL FUNCTION

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- On substituting s = jω, F(s) should posses simple poles and the residues should be real and positive.

Example Show that the function $F(s) = \frac{(s+2)(s+4)}{(s+1)(s+3)}$ is p.r

Solution: $F(s) = \frac{s^2+6s+8}{s^2+4s+3}$

Condition (1) We see that all roots of D(s) lie on the negative real axis or left half of s-plane Therefore, D(s) is Hurwitz polynomial.

Condition (2) Does not exist

Condition (3) $M_1 = s^2+8$ $N_1 = 6s$ $M_2 = s^2+3$ $N_2 = 4s$

$M_1M_2 - N_1N_2 \geq 0$

$(s^2+8)\cdot(s^2+3) - (6s)\cdot(4s) \geq 0$

$s^4 - 13s^2 + 24 \geq 0$

$A(\omega^2) = \omega^4 + 13\omega^2 + 24 \geq 0$ for all ω

Therefore, F(s) is p.r.f.

Example $Z(s) = \frac{2s^2+5}{s(s^2+1)}$ is p.r or not

Solution

Condition (1). $D(s) = s^3 + s$, then $D'(s) = 3s^2+1$

$3s^2+1 \,\big)\, s^3+s \,\big(\, \frac{1}{3}s$

$s^3 + \frac{1}{3}s$

$\frac{2s}{3} \,\big)\, 3s^2+1 \,\big(\, \frac{3}{2}\cdot 3s = \frac{9}{2}s$

$3s^2$

$1 \,\big)\, \frac{2}{3}s \,\big(\, \frac{2}{3}s$

$\frac{2}{3}s$

$\times$

Therefore, D(s) is Hurwitz polynomial

Condition (2) Z(s) has pair of poles $s = \pm j1$

The partial fraction Expansion of Z(s) is

$$Z(s) = \frac{-3s}{s^2+1} + \frac{5}{s}$$

which shows that the residue of the poles at $s = \pm j1$ is negative.

Therefore, Z(s) is not p.r.f.

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