MATRICES AND ITS APPLICATIONS

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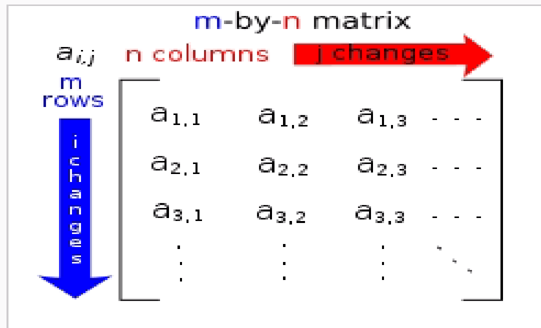
Abstract- **In mathematics, a matrix (plural matrices) is a rectangular array of numbers, symbols, or expressions, arranged in rows and columns. The individual items in a matrix are called its elements or entries. An example of a matrix with 2 rows and 3 columns is Matrices of the same size can be added or subtracted element by element. Applications of matrices are found in most scientific fields. In every branch of physics, including classical mechanics , optics ,electromagnetism, quantum mechanics, and quantum electrodynamics, they are used to study physical phenomena, such as the motion of rigid bodies. In computer graphics, they are used to project a 3-dimensional image onto a 2-dimensional screen. In probability theory and statistics, stochastic matrices are used to describe sets of probabilities. In this paper we have discussed about matrices and their applications .**

Index Terms- **mathematics , matrix , rows , columns , rigid bodies , 3-dimensional , stochastic matrices**

I. INTRODUCTION-

A matrix is a rectangular array of numbers or other mathematical objects, for which operations such as addition and multiplication are defined. Most commonly, a matrix over afield F is a rectangular array of scalars from F. Most of this article focuses on real and complex matrices, i.e., matrices whose elements are real numbers or complex numbers, respectively. More general types of entries are discussed below.



Each element of a matrix is often denoted by a variable with two subscripts. For instance, $a_{2,1}$ represents the element at the second row and first column of a matrix **A**.

The numbers, symbols or expressions in the matrix are called its entries or its elements. The horizontal and vertical lines of entries in a matrix are called rows and columns, respectively. Matrix calculus generalizes classical analytical notions such as derivatives and exponentials to higher dimensions. A major branch of numerical analysis is devoted to the development of efficient algorithms for matrix computations, a subject that is centuries old and is today an expanding area of research. Matrix decomposition methods simplify computations, both theoretically and practically. Algorithms that are tailored to particular matrix structures, such as sparse matrices and near-diagonal matrices, expedite computations in finite element method and other computations. Infinite matrices occur in planetary theory and in atomic theory. A simple example of an infinite matrix is the matrix representing the derivative operator, which acts on the Taylor series of a function.

II. VARIOUS OPERATIONS PERFORMED ON MATRICES

There are a number of basic operations that can be applied to modify matrices, called matrix addition, scalar multiplication, transposition,matrix multiplication, row operations, and submatrix

A table below is given for some basic operations of matrices

Operation	Definition	Example
Addition	The *sum* **A**+**B** of two *m*-by-*n* matrices **A** and **B** is calculated entrywise: $(\mathbf{A}+\mathbf{B})_{i,j} = \mathbf{A}_{i,j} + \mathbf{B}_{i,j}$, where $1 \le i \le m$ and $1 \le j \le n$.	$\begin{bmatrix}1&3&1\\1&0&0\end{bmatrix}+\begin{bmatrix}0&0&5\\7&5&0\end{bmatrix}=\begin{bmatrix}1+0&3+0&1+5\\1+7&0+5&0+0\end{bmatrix}=\begin{bmatrix}1&3&6\\8&5&0\end{bmatrix}$
Scalar multiplication	The *scalar multiplication* *c***A** of a matrix **A** and a number *c* (also called a scalar in the parlance of abstract algebra) is given by multiplying every entry of **A** by *c*: $(c\mathbf{A})_{i,j} = c \cdot \mathbf{A}_{i,j}$.	$2\cdot\begin{bmatrix}1&8&-3\\4&-2&5\end{bmatrix}=\begin{bmatrix}2\cdot1&2\cdot8&2\cdot-3\\2\cdot4&2\cdot-2&2\cdot5\end{bmatrix}=\begin{bmatrix}2&16&-6\\8&-4&10\end{bmatrix}$
Transpose	The *transpose* of an *m*-by-*n* matrix **A** is the *n*-by-*m* matrix $\mathbf{A}^T$ (also denoted $\mathbf{A}^{tr}$ or ${}^t\mathbf{A}$) formed by turning rows into columns and vice versa: $(\mathbf{A}^T)_{i,j} = \mathbf{A}_{j,i}$.	$\begin{bmatrix}1&2&3\\0&-6&7\end{bmatrix}^T=\begin{bmatrix}1&0\\2&-6\\3&7\end{bmatrix}$

III. MATRIX MULTIPLICATION

Multiplication of two matrices is defined if and only if the number of columns of the left matrix is the same as the number of rows of the right matrix. If **A** is an m-by-n matrix and **B** is an n-by-p matrix, then their matrix product **AB** is the m-by-p matrix whose entries are given by dot product of the corresponding row of **A** and the corresponding column of **B**:

$$[\mathbf{AB}]_{i,j} = A_{i,1}B_{1,j} + A_{i,2}B_{2,j} + \cdots + A_{i,n}B_{n,j} = \sum_{r=1}^{n} A_{i,r}B_{r,j}$$

Matrix multiplication satisfies the rules (**AB**)**C** = **A**(**BC**) (associativity), and (**A**+**B**)**C** = **AC**+**BC** as well as **C**(**A**+**B**) = **CA**+**CB** (left and right distributivity), whenever the size of the matrices is such that the various products are defined. The product **AB** may be defined without **BA** being defined, namely if **A** and **B** arem-by-n and n-by-k matrices, respectively, and m ≠ k. Even if both products are defined, they need not be equal, i.e., generally

AB ≠ **BA**,

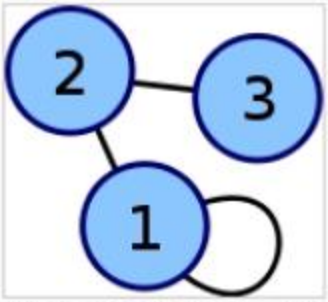
i.e., matrix multiplication is not commutative

APPLICATIONS OF MATRICES

There are numerous applications of matrices, both in mathematics and other sciences. Some of them merely take advantage of the compact representation of a set of numbers in a matrix. For example, in game theory and economics, the payoff matrix encodes the payoff for two players, depending on which out of a given (finite) set of alternatives the players choose. Text mining and automated thesaurus compilation makes use of document-term matrices such as tf-idf to track frequencies of certain words in several documents.

Graph theory

The adjacency matrix of a finite graph is a basic notion of graph theory. It records which vertices of the graph are connected by an edge. Matrices containing just two different values (1 and 0 meaning for example "yes" and "no", respectively) are called logical matrices. The distance (or cost) matrix contains information about distances of the edges. These concepts can be applied to websites connected hyperlinks or cities connected by roads etc., in which case (unless the road network is extremely dense) the matrices tend to be sparse, i.e., contain few nonzero entries. Therefore, specifically tailored matrix algorithms can be used in network theory



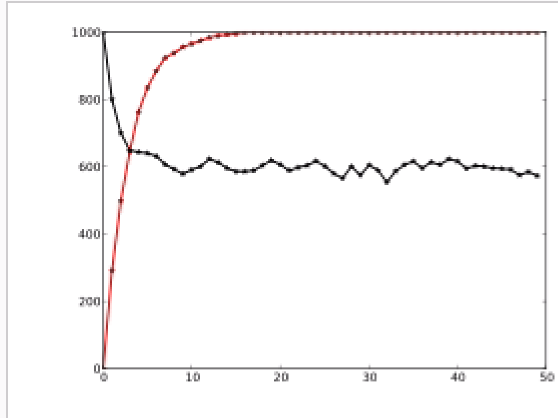
An undirected graph with adjacency matrix

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}.$$

Probability theory and statistics

Stochastic matrices are square matrices whose rows are probability vectors, i.e., whose entries are non-negative and sum up to one. Stochastic matrices are used to define Markov chains with finitely many states. A row of the stochastic matrix gives the probability distribution for the next position of some particle currently in the state that corresponds to the row. Properties of the Markov chain like absorbing states, i.e., states that any particle attains eventually, can be read off the eigenvectors of the transition matrices.

Statistics also makes use of matrices in many different forms. Descriptive statistics is concerned with describing data sets, which can often be represented as data matrices, which may then be subjected to dimensionality reduction techniques. The covariance matrix encodes the mutual variance of several random variables.



Two different Markov chains. The chart depicts the number of particles (of a total of 1000) in state "2". Both limiting values can be determined from the transition matrices, which are given by $\begin{bmatrix} .7 & 0 \\ .3 & 1 \end{bmatrix}$ (red) and $\begin{bmatrix} .7 & .2 \\ .3 & .8 \end{bmatrix}$ (black).

Symmetries and transformations in physics

Linear transformations and the associated symmetries play a key role in modern physics. For example, elementary particles in quantum field theory are classified as representations of the Lorentz group of special relativity and, more specifically, by their behavior under the spin group. Concrete representations involving the Pauli matrices and more general gamma matrices are an integral part of the physical description of fermions, which behave as spinors. For the three lightest quarks, there is a group-theoretical representation involving the special unitary group SU(3); for their calculations, physicists use a convenient matrix representation known as the Gell-Mann matrices, which are also used for the SU(3) gauge group that forms the basis of the modern description of strong nuclear interactions, quantum chromodynamics. The Cabibbo–Kobayashi–Maskawa matrix, in turn, expresses the fact that the basic quark states that are important for weak interactions are not the same as, but linearly related to the basic quark states that define particles with specific and distinct masses

Electronics

Traditional mesh analysis in electronics leads to a system of linear equations that can be described with a matrix.The behaviour of many electronic components can be described using matrices. Let A be a 2-dimensional vector with the component's input voltage v_1 and input current i_1 as its elements, and let B be a 2-dimensional vector with the component's output voltage v_2 and output current i_2 as its elements. Then the behaviour of the electronic component can be described by $B = H \cdot A$, where H is a 2 x 2 matrix containing one impedance element (h_{12}), one admittance element (h_{21}) and two dimensionless elements (h_{11} and h_{22}). Calculating a circuit now reduces to multiplying matrices.

Geometrical optics

Geometrical optics provides further matrix applications. In this approximative theory, the wave nature of light is neglected. The result is a model in which light rays are indeedgeometrical rays. If the deflection of light rays by optical elements is small, the action of a lens or reflective element on a given light ray can be expressed as multiplication of a two-component vector with a two-by-two matrix called ray transfer matrix: the vector's components are the light ray's slope and its distance from the optical axis, while the matrix encodes the properties of the optical element. Actually, there are two kinds of matrices, viz. a refraction matrix describing the refraction at a lens surface, and a translation matrix, describing the translation of the plane of reference to the next refracting surface, where another refraction matrix applies. The optical system, consisting of a combination of lenses and/or reflective elements, is simply described by the matrix resulting from the product of the components' matrices.

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