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HURWITZ POLYNOMIAL

Gajender, Gaurav, Himanshu Sharma

Abstract- **In mathematics, a Hurwitz polynomial, named after Adolf Hurwitz, is a polynomial whose coefficients are positive real numbers and whose roots (zeros) are located in the left half-plane of the complex plane or on the jω axis, that is, the real part of every root is zero or negative. The term is sometimes restricted to polynomials whose roots have real parts that are strictly negative, excluding the axis (i.e., a Hurwitz stable polynomial) We present a new criterion to determine the**

stability of polynomial with real coefficients

For a polynomial to be Hurwitz, it is necessary but not sufficient that all of its coefficients be positive. A necessary and sufficient condition that a polynomial is Hurwitz is that it passes the Routh-Hurwitz stability criterion.

I. INTRODUCTION

In mathematics, a **Hurwitz polynomial**, named after Adolf Hurwitz, is a polynomial whose coefficients are positive real numbers and whose roots (zeros) are located in the left half-plane of the complex plane or on the jω axis, that is, the real part of every root is zero or negative. The term is sometimes restricted to polynomials whose roots have real parts that are strictly negative, excluding the axis (i.e., a Hurwitz stable polynomial).

A polynomial function P(s) of a complex variable s is said to be Hurwitz if the following conditions are satisfied:

1. P(s) is real when s is real.
2. The roots of P(s) have real parts which are zero or negative.

Hurwitz polynomials are important in control systems theory, because they represent the characteristic equations of stable linear systems. Whether a polynomial is Hurwitz can be determined by solving the equation to find the roots, or from the coefficients without solving the equation by the Routh-Hurwitz stability criterion.

II. PROPERTIES

For a polynomial to be Hurwitz, it is necessary but not sufficient that all of its coefficients be positive. A necessary and sufficient condition that a polynomial is Hurwitz is that it passes the Routh-Hurwitz stability criterion. A given polynomial can be efficiently tested to be Hurwitz or not by using the Routh continued fraction expansion technique.

The properties of Hurwitz polynomials are:

1. All the poles and zeros are in the left half plane or on its boundary, the imaginary axis.
2. Any poles and zeros on the imaginary axis are simple (have a multiplicity of one).
3. Any poles on the imaginary axis have real strictly positive residues, and similarly at any zeros on the imaginary axis, the function has a real strictly positive derivative.
4. Over the right half plane, the minimum value of the real part of a PR function occurs on the imaginary axis (because the real part of an analytic function constitutes a harmonic function over the plane, and therefore satisfies the maximum principle).
5. The polynomial should not have missing powers of s.

$$p(s) = k(s)\,[\hat{M}(s) + \hat{N}(s)] = k(s)\hat{p}(s)$$

where $M(s) = k(s)\hat{M}(s)$, $N(s) = k(s)\hat{N}(s)$, and $\hat{p}(s) = \hat{M}(s) + \hat{N}(s)$.

CHECKING WEATHER A POLYNOMISL IS HURWITZ OR NOT

Let p(s) be the polynomial in question. Assume first that p(s) is neither an even nor an odd polynomial. To test whether such a polynomial p(s) is indeed a Hurwitz polynomial, we may use the Hurwitz test.

 First decompose p(s) into its even and odd parts, M(s) and N(s), respectively, as p(s) = M(s) + N(s).

$$T(s) = \frac{N(s)}{M(s)} \quad \text{if } d \text{ is an odd integer}$$

$$T(s) = \frac{M(s)}{N(s)} \quad \text{if } d \text{ is an even integer}$$

 Using M(s) and N(s) we form the test ratio T(s), whose numerator has a higher degree than that of its denominator. Suppose that p(s) is a polynomial of degree d. Then

 Next, we perform the continued fraction expansion about infinity on the test ratio T(s), removing one pole at a time in the form of a quotient qs, resulting in:

$$T(s) = q_1 s + \cfrac{1}{q_2 s + \cfrac{1}{q_3 s + \cfrac{1}{\ddots + \cfrac{1}{q_d s}}}}$$

where qis is the ith quotient, and qi , is the associated coefficient.

 If there is one or more quotients with negative coefficients, then p(s) is neither a Hurwitz nor a modified Hurwitz polynomial.

 On the other hand, if there are d quotients (d = d^) and every quotient has a positive coefficient, then p(s) is a Hurwitz polynomial.

 Finally, if the number of quotient d^ is less than d but every quotient has a positive coefficient, this means that there is a common factor k(s) between M(s) and N(s). Hence, we can write p(s) as:

 Because all the d^ quotients of T(s) have positive coefficients, the polynomial p(s) in (4-10) is Hurwitz. Thus, if k(s) is a modified Hurwitz polynomial [i.e., if all the roots of k(s) are simple and purely imaginary], then p(s) is a modified Hurwitz polynomial.

 A procedure to determine if k(s) is a modified Hurwitz polynomial is described in the following in conjunction with the case when p(s) is either an even or an odd polynomial.

 Suppose now that p(s) is either an even or an odd polynomial of degree d. is a modified Hurwitz polynomial if and only if p(s) has only simple and imaginary axis roots (including the origin).

 To determine if p(s) is a modified Hurwitz polynomial, we form a test ratioT^(s):

$$\hat{T}(s) = \frac{p(s)}{(d/ds)\,p(s)} = \frac{p(s)}{p'(s)}$$

and perform the continued fraction expansion about infinity on T^(s), as in (4-9). Then p(s) is a modified Hurwitz polynomial if and only if there are d quotients in the expansion and each quotient has a positive coefficient.

 In the case when p(s) is either an even or an odd polynomial, if there is one or more negative coefficient in the continued fraction expansion of T^(s), then p(s) has a RH s-plane root; and if all coefficients are positive but thereare only d^ < d quotients, then all roots of p(s) are on the imaginary axis ofthe s-plane, but p(s) has non-simple or multiple roots. Either situation implies that p(s) is not a modified Hurwitz polynomial.

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