© 2014 IJIRT | Volume 1 Issue 7 | ISSN: 2349-6002

INTEGRAL

Kunal Kumar, Himanshi Mann, Ashutosh Jha
Dronacharya college of engineering ece department

Abstract- The **integral** is an important concept in mathamatics. Integration is one of the two main operations in calculs with its inverse, differntition being the other. Given a function *f* of a real variable *x* and an interval[*a*, *b*] of the real line the **definite integral**

$$\int_a^b f(x)\,dx$$

is defined informally as the signed area of the region in the *xy*-plane that is bounded by the graph of *f*, the *x*-axis and the vertical lines *x* = *a* and *x* = *b*.

I. INTRODUCTION

Pre-calculus integration

The first documented systematic technique capable of determining integrals is the method of exhaustion of the anicent greek astronomer eudoxs(*ca.* 370 BC), which sought to find areas and volumes by breaking them up into an infinite number of divisions for which the area or volume was known. This method was further developed and employed by archimedas in the 3rd century BC and used to calculate areas for parabolas and an approximation to the area of a circle. Similar methods were independently developed in China around the 3rd century Adby lui hui, who used it to find the area of the circle. This method was later used in the 5th century by Chinese father-and-son mathematicians.

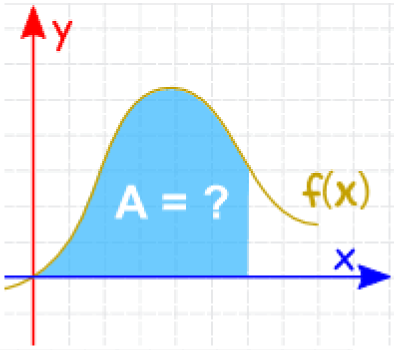
Newton and Leibniz

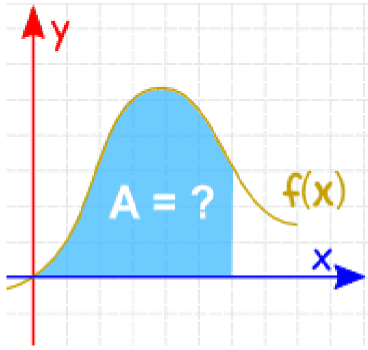
The major advance in integration came in the 17th century with the independent discovery of the fundamental theorm of integration by newton and leibniz. The theorem demonstrates a connection between integration and differentiation. This connection, combined with the comparative ease of differentiation, can be exploited to calculate integrals. In particular, the fundamental theorem of calculus allows one to solve a much broader class of problems. Equal in importance is the comprehensive mathematical framework that both Newton and Leibniz developed. Given the name infinitesimal calculus, it allowed for precise analysis of functions within continuous domains. This framework eventually became modern calculus, whose notation for integrals is drawn directly from the work of Leibniz.

Introduction to Integration

Integration is a way of adding slices to find the whole.

Integration can be used to find areas, volumes, central points and many useful things. But it is easiest to start with finding the **area under the curve of a function** like this:





What is the area under **y = f(x)** ?

That is a lot of adding up!

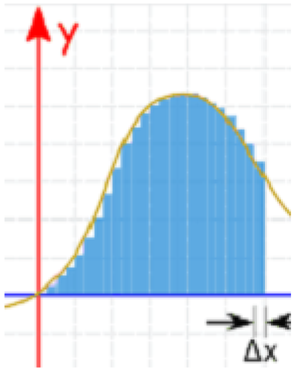
But we don't have to add them up, as there is a "shortcut". Because finding an Integral is the **reverse** of finding a Derivative.

(So you should really know abou derivative before reading more!)

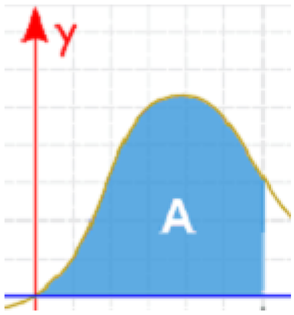
Like here: **Example: What is an integral of 2x?**

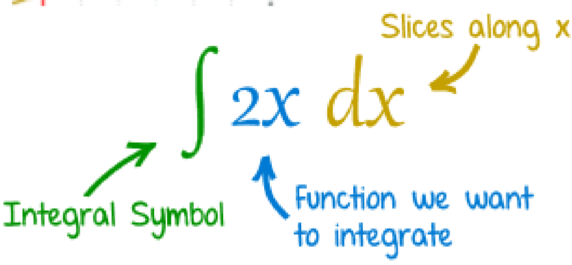
Slices

We can make Δx a lot smaller and **add up many small slices** (answer is getting better):



And as the slices **approach zero in width**, the answer approaches the **true answer**.





We know that the derivative of x^2 is 2x so an integral of 2x is x^2. You will see more examples later.

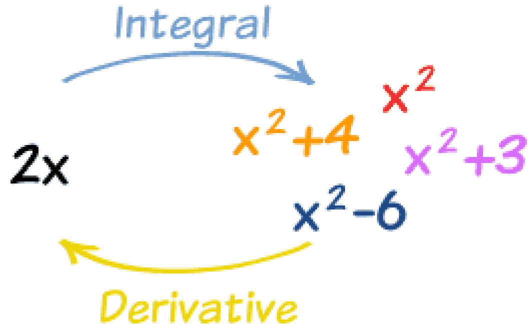
Notation

The symbol for "Integral" is a stylish "S" (for "Sum", the idea of summing slices).After the Integral Symbol we put the function we want to find the integral of (called the Integrand), and then finish with **dx** to mean the slices go in the x direction (and approach zero in width) And here is how we write the answer:

$$\int 2x \, dx = x^2 + C$$

Plus C

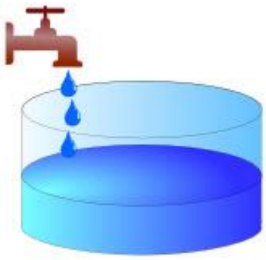
We wrote the answer as x^2 but why + C ? It is the "Constant of Integration". It is there because of **all the functions whose derivative is 2x**:



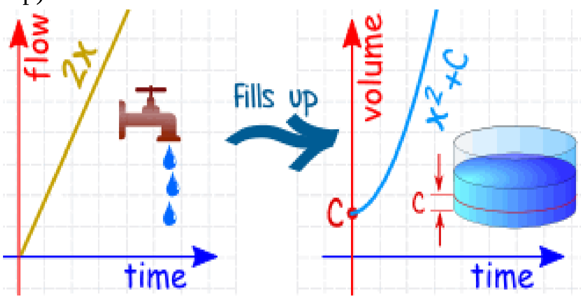
The derivative of $\mathbf{x^2+4}$ is **2x**, and the derivative of $\mathbf{x^2+99}$ is also **2x**, and so on! Because the derivative of a constant is zero.

So when we **reverse** the operation (to find the integral) we only know **2x**, but there could have been a constant of any value. So we wrap up the idea by just writing + C at the end.

Tap and Tank



Integration is like filling a tank from a tap. The input (before integration) is the **flow rate** from the tap.Integrating the flow (adding up all the little bits of water) gives us the **volume of water** in the tank.Imagine the flow starts at 0 and gradually increases (maybe a motor is slowly opening the tap).

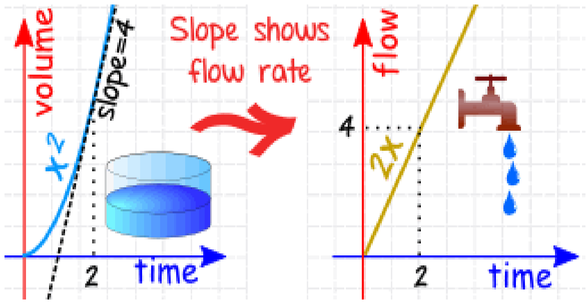


As the flow rate increases, the tank fills up faster and faster.With a flow rate of **2x**, the tank fills up at $\mathbf{x^2}$.We have integrated the flow to get the volume.

Example: (assuming the flow is in liters per minute) after 3 minutes (**x=3**):

- the flow rate has reached **2x = 2×3 = 6** liters/min,
- and the volume has reached $\mathbf{x^2 = 3^2 = 9}$ liters.

We can do the reverse, too:



Imagine you don't know the flow rate. You only know the volume is increasing by $\mathbf{x^2}$. We can go in reverse (using the derivative, which gives us the slope) and find that the flow rate is **2x**.

Other functions

Well, we have played with **y=2x** enough now, so how do we integrate other functions?If we are lucky enough to find the function on the **result** side of a derivative, then (knowing that derivatives and integrals are opposites) we have an answer. But remember to add C

$\int \cos(x)\, dx = \sin(x) + C$

But a lot of this "reversing" has already been done

Example: What is $\int x^3\, dx$?

On the rules of integration there is a "Power Rule" that says:

$\int x^n\, dx = x^{n+1}/(n+1) + C$

We can use that rule with n=3:

$\int x^3\, dx = x^4/4 + C$

Knowing how to use those rules is the key to being good at Integration.So get to know those rules and **get lots of practice**.

Definite vs Indefinite Integrals

We have been doing **Indefinite Integrals** so far.A **Definite Integral** has actual values to calculate between (they are put at the bottom and top of the "S"):