

GRAPH THEORY

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Abstract- we know that an electric circuit or a network is an interconnection of passive elements (R,L,C) and active elements (voltage source,current source). An electric circuit is a closed energized network,i.e., all electric circuits are networks , but a network is not necessarily a circuit , e.g. T-network The analysis of an electrical circuit should result in knowing the voltages across and currents through all the circuit elements.We have seen that KCL, KVL and the V-I relations of the elements suffice to analyse a circuit . however , when the circuits become more elaborate , with more nodes and more loops , a systematic method for formulating the network equilibrium. Equations is needed . there are two such methods : mesh analysis and nodal analysis

A network comprised of B branches involves 2B unknowns, i.e., each of the branch voltages and currents.

However the branch volt-ampere relations of the network, presumed to be known, relate the current and the voltage of each branch,. Hence a calculation of either B currents or B voltages (or some combination of B voltages and currents), and then substitution in the B branch volt-ampere relations, provides all the voltages and currents.

In general however neither the B branch voltages nor the B branch currents are independent, i.e., some of the B voltage variables for example can be expressed as a combination of other voltages using KVL, and some of the branch currents can be related using KCL. Hence there generally are fewer than B independent unknowns. In this we determine the minimum number of independent variables for a network analysis, the relationship between the independent and dependent variables, and efficient methods of obtaining independent equations to determine the variables. In doing so we make use of the mathematics of Graph Theory.

Graph Theory

A circuit graph is a description of the just the topology of the circuit, with details of the circuit elements suppressed. The graph contains branches and nodes. A branch is a curve drawn between two nodes to indicate an electrical connection between the nodes.

A directed graph is one for which a polarity marking is assigned to all branches (usually an arrow) to distinguish between movement from node A to B and the converse movement from B to A.

A connected graph is one in which there is a continuous path through all the branches (any of which may be traversed more than once) which touches all the nodes. A graph that is not connected in effect has completely separate parts, and for our purposes is more conveniently considered to be two (or more) independent graphs.

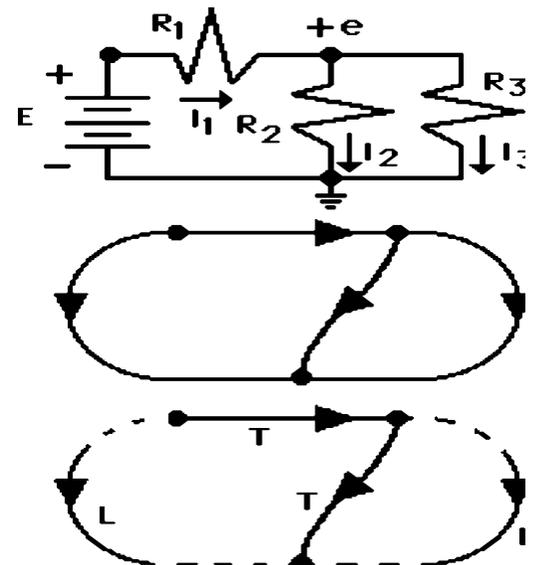
We consider a directed, connected graph. An electrical circuit and its graph are illustrated to the right. The graph retains only topological information about the circuit.

For reasons to appear branches are divided into two topological groups, tree branches and links. A tree of the graph is a subset of the branches such that all graph nodes are connected by branches but without forming a closed path. These branches then are the tree branches. The remaining branches (collectively called a co-tree) are the links. A tree and its links for the illustrative circuit are shown in the figure (labeled T and L respectively).

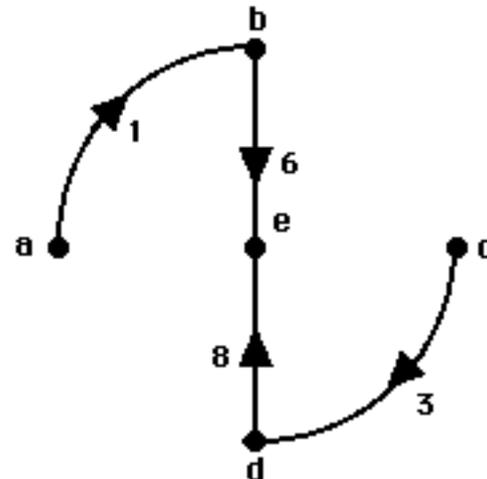
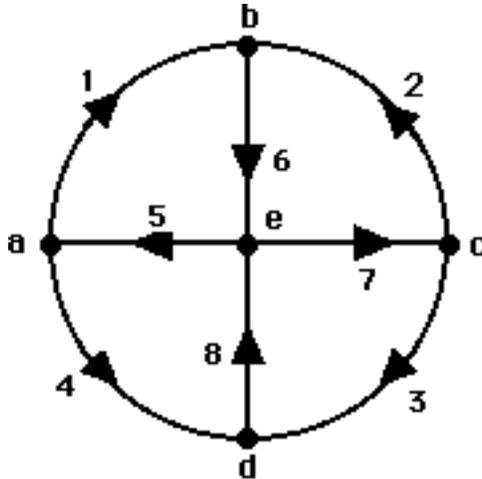
For a network with B branches and N nodes the number of tree branches is $N-1$; this is the minimum number of branches needed to connect N nodes, and connecting another branch will form a loop. In general a graph contains many different trees, but for each of these trees there are $N-1$ tree branches. It follows then that there are $B-N+1$ links.

Choosing Independent Current Variables:

Given a network graph with B branches and N nodes select



a tree, any one will do for the present purpose. Remove all the link branches so that, by definition, there are no loops formed by the remaining tree branches. It follows from the absence of any closed paths that all the branch currents become zero. Hence by 'controlling' just the link currents all the branch currents



can be controlled. This control would not exist in general using fewer than all the link branches because a loop would be left over; depending on the nature of the circuit elements branches making up the loop current could circulate around the loop. Using more than the link branches is not necessary. Hence it should be possible to express all the branch currents in terms of just the link currents, i.e., there are $B - N + 1$ independent current variables, and link currents provide one such set of independent variables. A straightforward formal procedure for describing tree branch currents in terms of link currents is illustrated as follows; the graph on the left side of the figure below is used for illustration.

The first step is selection of a tree, e.g., the tree on the right of the figure above. The tree makes clear that a set of independent current variables is formed using link branches 2, 4, 5, and 7. (Branch numbering and specification of the branch polarity arrows is more or less arbitrarily.)

Select a link; start with any one since each link is used in turn. Remove any other link used previously and then insert the single selected link to the tree to form a single loop. For example inserting link branch 2 defines a loop formed by branches 2, 3, 6, and 8. Write a KVK equation for this loop.

As an aid for keeping track of information, particularly for larger graphs, we can use a matrix such as that shown below. The rows of the table define the $N - B + 1$ loops, four for the illustration, which will be formed by the links. Rows are labeled by the ID for the link branch used to form the loop, since this branch is unique for the loop so

designated. The columns of the matrix correspond to the full set of graph branches. Since it is generally helpful to be systematic in these matters we choose to write all the KVL loop equations circulating about a loop in the direction of the link arrow. The loop equation will consist of a sum of loop branch voltage drops; the coefficient of a branch voltage drop will be +1 if circulation around the loop is in the same direction as the branch arrow, and -1 otherwise. Enter the coefficients into the table, row by row. Branches that are not in the

| | | Branch # | | | | | | | |
|--------|--|----------|---|----|---|---|----|---|----|
| Loop # | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 2 | | | 1 | -1 | | | 1 | | -1 |
| 4 | | -1 | | | 1 | | -1 | | 1 |
| 5 | | 1 | | | | 1 | 1 | | |
| 7 | | | | 1 | | | | 1 | 1 |

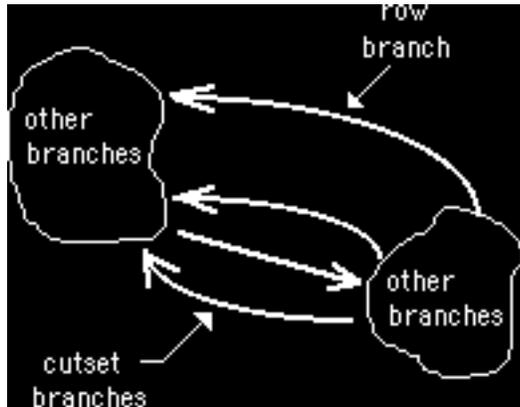
loop for a given row will have no entry (or enter 0), indicating their lack of involvement in a particular loop.

Choosing Independent Voltage Variables:

Voltages and current

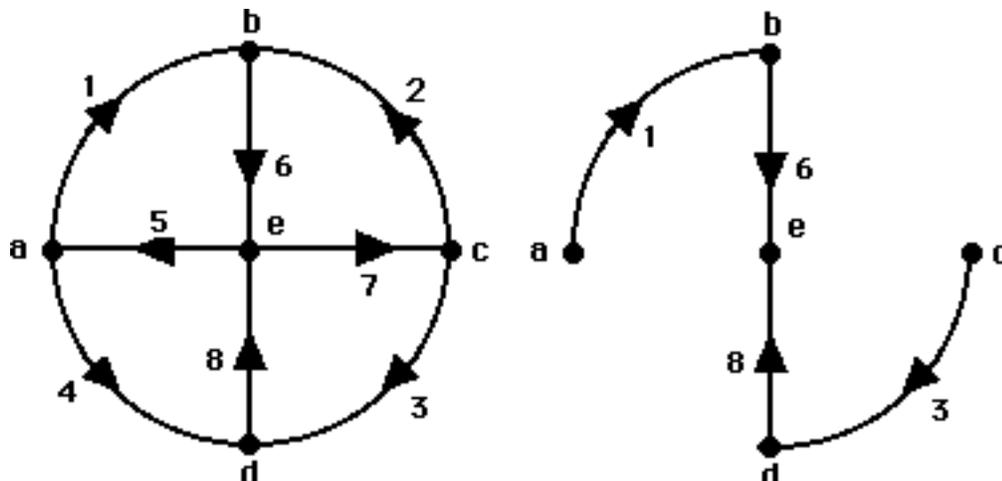
appear in the KCL and KVL in a formally symmetric fashion, and so it should not be surprising to find a procedure employing voltage variables dual to the tieset procedure for current variables. Thus consider again a network with B branches and N nodes, and select a tree for the graph. For an illustration we use the same graph as before and, although it is not necessary, for comparison to the preceding discussion we use the same tree as before.

Whereas the link currents provide a set of independent current variable the N-1 tree branch voltages provide a set of independent voltage variables; setting these branch voltages to zero forces all branch voltages to zero. Remember that each link is a connection between nodes, and hence KVL enables each link voltage to be expressed in terms of only tree branch voltages. It is convenient to record these expressions in a ('cutset') matrix similarly to what was



done for the current variables with a 'tieset' matrix. The rows of the matrix are associated with the voltage drop across tree branches (in place of link currents for the tieset matrix) and the columns are the graph branches.

A column entry in a given row is '1, or -1, determined as follows. Insert each branch into the tree in turn;



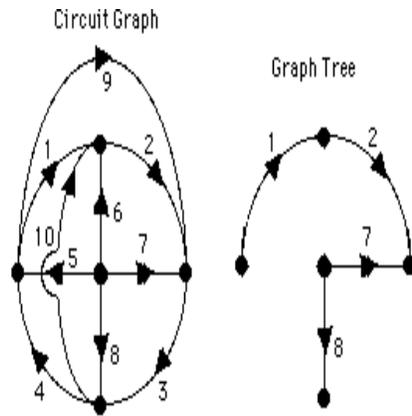
remove branches inserted earlier. Express the voltage drop across the inserted branch in terms of tree branch voltages. This is always possible since the inserted branch either is a tree branch, or it is a link and forms a loop with tree branches. The table for the illustrative network and the tree selected is shown to the left; the coefficients of the voltage expressions are entered into the table. The expressions for branch voltage drop in terms of tree branch voltages can be read back from the columns, e.g.,

$$v_4 = v_1 + v_6 - v_8$$

The rows of the matrix provide very useful information also; the group of branches corresponding to nonzero entries in a row is called a cutset. The voltage expression for each of the cutset branches requires use of the tree branch voltage corresponding to the row. Suppose all the tree branches shrink to zero length except the one corresponding to the row. This divide the graph nodes into two group such that a path from a node in one group to a node in the other group must pass through a cutset branch (see figure to right). Hence the cutset branches can

| Tree Branch | Branch # | | | | | | | |
|-------------|----------|----|---|----|----|---|----|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 1 | 1 | | | 1 | -1 | | | |
| 3 | | 1 | 1 | | | | -1 | |
| 6 | | -1 | | 1 | -1 | 1 | | |
| 8 | | 1 | | -1 | | | -1 | 1 |

be used to write KCL equations.



CUTSET EXAMPLE

This is the same graph and tree as before; however now a cutset analysis (tree voltages as voltage variables) follows. As before the graph has 10 branches and 5 nodes. There are therefore 4 tree branches and 6 links. The number of independent voltage variables, equal to the number of tree branches, is 4.

In general the cutset matrix is filled in easiest by columns. Add a link to the tree, each link separately so that the loop formed is clearly evident. Express the link voltage in terms of the tree branch voltages, using KVL, i.e., the link voltage difference in the direction of the link arrow is the same as that calculated circulating on tree branches. Enter the tree voltage coefficients, + 1 if the circulation is in the direction of the branch arrow and -1 otherwise. Note that the link branch is not included in the column directly, only as a column header. Tree branch columns also are shown but the entry is trivial

CutSet matrix

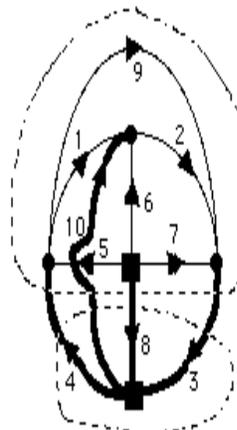
| tree branch | circuit branch | | | | | | | | | |
|-------------|----------------|---|---|----|----|----|----|---|---|----|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 1 | 1 | | | | -1 | -1 | | | | 1 |
| 2 | | 1 | | | -1 | -1 | -1 | | | 1 |
| 7 | | | | -1 | 1 | 1 | 1 | | | 1 |
| 8 | | | | 1 | -1 | | | | 1 | - |

$$\begin{aligned}
 v_3 &= -v_7 + v_8 \\
 v_4 &= -v_1 - v_2 + v_7 - v_8 \\
 v_5 &= -v_1 - v_2 + v_7 \\
 v_6 &= -v_2 + v_7 \\
 v_9 &= v_1 + v_2 \\
 v_{10} &= -v_2 + v_7 - v_8
 \end{aligned}$$

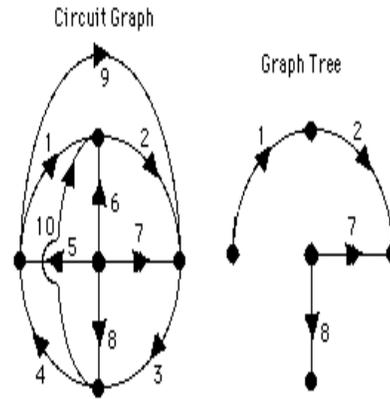
The tree branch voltages provide a set of voltage variables in terms of which all the branch voltages can be expressed. Note that the KVL equations are independent of one another since each involves a branch voltage, the link voltage, which is not involved in any other equation. The voltage equations, shown to the left, can be read back directly from the matrix

Each tree branch separates the graph nodes into two groups which are connected through links. The sketch to the right indicates the separation for tree branch #8; the nodes are grouped by imagining the tree branches to shrink to zero length. The links connecting the node groups for a given tree branch (and the tree branch also) are indicated by each row entry, the matrix entries also indicate the appropriate coefficients. The KCL equations corresponding to the rows are shown below.

$$\begin{aligned}
 i_1 - i_4 - i_5 + i_9 &= 0 \\
 i_2 - i_4 - i_5 - i_6 + i_9 - i_{10} &= 0 \\
 -i_3 + i_4 + i_5 + i_6 + i_7 + i_{10} &= 0 \\
 i_3 - i_4 + i_8 - i_{10} &= 0
 \end{aligned}$$



TIESET EXAMPLE



An illustrative circuit graph (directed, connected) and one of its trees is drawn to the left. A tieset analysis (link currents as current variables) follows. The graph has 10 branches and 5 nodes. There are therefore 4 tree branches and 6 links. The number of independent current variables, equal to the number of links, is 6.

In general the tieset matrix is filled in easiest by rows. Add a link to the tree, one link at a time, and write a KVL equation for the loop so formed. For simplicity and clarity the matrix to the right is filled in always circulating in the direction of the link direction arrow. Only the coefficients of the branch voltages are entered; +1 if the circulation is in the direction of the branch arrow, -1 if it is opposite. Note that although the circuit graph has a modest topological complexity the loop formed by each link is very evident.

TieSet Matrix

| link branch | circuit branch | | | | | | | | | |
|-------------|----------------|----|---|---|---|---|----|----|---|----|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 3 | | | 1 | | | | 1 | -1 | | |
| 4 | 1 | 1 | | 1 | | | -1 | 1 | | |
| 5 | 1 | 1 | | | 1 | | -1 | | | |
| 6 | | 1 | | | | 1 | | -1 | | |
| 9 | -1 | -1 | | | | | | | 1 | |
| 10 | | 1 | | | | | -1 | 1 | | 1 |

$$\begin{aligned}
 v_3 + v_7 - v_8 &= 0 \\
 v_1 + v_2 + v_4 - v_7 + v_8 &= 0 \\
 v_1 + v_2 + v_5 - v_7 &= 0 \\
 v_2 + v_6 - v_8 &= 0 \\
 -v_1 - v_2 + v_9 &= 0 \\
 v_2 - v_7 + v_8 + v_{10} &= 0
 \end{aligned}$$

The KVL equations are shown to the left; these can easily be read back from the matrix. Note that the equations are independent of one another since each involves a voltage, a link voltage, not involved in any other equation.

A tree branch divides the graph nodes into two groups, and the matrix column corresponding to that tree branch reveals the links which connect one group to the other. On applying KCL the tree branch current can be expressed in terms of those link currents. The appropriate coefficients are indicated by the matrix entries. The current expressions for the illustration are shown to the right. Note that the equations are independent of one another; each equation involves a branch current not included in any other equation.

$$\begin{aligned}
 i_1 &= i_4 + i_5 - i_9 \\
 i_2 &= i_4 + i_5 + i_6 - i_9 + i_{10} \\
 i_7 &= i_3 - i_4 - i_5 - i_{10} \\
 i_8 &= -i_3 + i_4 - i_6 + i_9 + i_{10}
 \end{aligned}$$

In general the branch volt-ampere relations would be known. These can be used to substitute branch currents for branch voltages in the KVL equations. The KCL equations then can be substituted to eliminate the tree branch currents, leaving 6 equations involving the 6 link currents. Solve this set of linear simultaneous equations for the link currents and back-substitute to determine the remaining branch currents and voltages.

I. CONCLUSION

The graph of a planar network is drawn by keeping all the points of intersection of two or more branches known as nodes, and representing the network elements by lines, ideal

voltage source and ideal current source are replaced by short circuit and open circuit respectively.

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