FOURIER SERIES

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***Abstract*- Through this paper we tend to propose the fourier series which contain the importance and the uses of fourier aeries in our dailty progress . Euler's formula , even and odd functions ,change of interval , real applications of forier series are also discussed in this paper. The best thing about fourier series is that first, almost any type of a wave can be approximated. Second, when fourier series converge, they converge very speedly. Periodic functions are also discussed in this paper. Periodic functions are of common occurrence in many physical and engineering problems; for example, in conductance of heat and mechanical vibrations. It is useful to express these functions in a series of sines and cosines.**

***Index Terms*- fourier series , periodic function , fourier coefficient, single valued function ,waveforms.**

I. INTRODUCTION

An infinite series whose terms are constants multiplied by sine and cosine functions and that can, if uniformly convergent, approximate a large variety of functions .

A Fourier series is a particular type of infinite mathematical series involving trigonometric functions. Fourier series are used in applied mathematics, and particularly in physics and electronics, to express periodic functions such as those that comprise communications signal waveform s.

Suppose { a , a_1 , a_2 , a_3 , ..., a_n , ...} and { b_1 , b_2 , b_3 , ..., b_n , ...} be infinite sets of constant s. These constants are known as the Fourier coefficient s. suppose x be a variable. The general Fourier series is given as:

$F(x) = a/2 + a_1 \cos x + b_1 \sin x + a_2 \cos 2x + b_2 \sin 2x + + a_n \cos nx + b_n \sin nx +$

Discussion

Periodic functions are of common occurrence in many physical and engineering problems; for example, in conductance of heat and mechanical vibrations. It is useful to express these functions in a series of sines and cosines. Most of the single valued functions which occure in applied mathematics can be expressed in the form

$F(x) = a/2 + a_1 \cos x + b_1 \sin x + a_2 \cos 2x + b_2 \sin 2x + ... + a_n \cos nx + b_n \sin nx +$ Within a desired range of values of the variables. Such a series is known as fourier series .

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L} \right)$$

Where a_0, a_n, b_n are constants , called the fourier coefficients.

Example1 The function $\sin x$ has periods 2π , 4π , 6π …, since $\sin(x+2\pi)$,$\sin(x+4\pi)$ …… all equal $\sin x$.

$$\boldsymbol{f(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L} \right)}$$

Where $\boldsymbol{a_0}$, $\boldsymbol{a_n}$, $\boldsymbol{b_n}$ are constants , called the fourier coefficients .

Basic Results

Most of the single valued functions which occure in applied mathematics can be expressed in the form $F(x) = a/2 + a_1 \cos x + b_1 \sin x + a_2 \cos 2x + b_2 \sin 2x + + a_n \cos nx + b_n \sin nx + ...$

Within a desired range of values of the variables. Such a series is known as fourier series .

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L} \right)$$

Where a_0, a_n, b_n are constants , called the fourier coefficients The Fourier polynomials are 2π-periodic functions. With the help of trigonometric identities

$$\sin(mx)\cos(nx) = \frac{1}{2}\left[\sin((m+n)x) + \sin((m-n)x)\right]$$

$$\cos(mx)\cos(nx) = \frac{1}{2}\left[\cos((m+n)x) + \cos((m-n)x)\right]$$

$$\sin(mx)\sin(nx) = \frac{1}{2}\left[\cos((m-n)x) - \cos((m+n)x)\right]$$

we can easily deduce the integral formulas

(1)for $n \geq 0$, we take

$$\int_{-\pi}^{\pi} \sin(nx)\,dx = 0,$$

for $n>0$,we take

$$\int_{-\pi}^{\pi} \cos(nx)\,dx = 0,$$

(2)for *m* and *n*, we take

$$\int_{-\pi}^{\pi} \sin(mx)\cos(nx)dx = 0,$$

(3) for n ≠ m, we take

$$\int_{-\pi}^{\pi} \cos(mx)\cos(nx)dx = 0, \text{ and } \int_{-\pi}^{\pi} \sin(mx)\sin(nx)dx = 0,$$

(4) For n ≥ 1 , we take

$$\int_{-\pi}^{\pi} \cos^2(nx)dx = \pi, \text{ and } \int_{-\pi}^{\pi} \sin^2(nx)dx = \pi.$$

With the help of above formulas, we can easily deduce the following result:

Theorem. Suppose

$$F_n(x) = a_0 + \sum_{k=1}^{k=n} \left(a_k \cos(kx) + b_k \sin(kx)\right).$$

We have

$$\begin{cases} a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} F_n(x)dx, \\ a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} F_n(x)\cos(kx)dx, \quad 1 \le k \le n \\ b_k = \frac{1}{\pi}\int_{-\pi}^{\pi} F_n(x)\sin(kx)dx, \quad 1 \le k \le n. \end{cases}$$

This theorem helps colleague a Fourier series to any **2π** -periodic function.

Definition. Suppose $f(x)$ be a 2π -periodic function which is integrable on **[-π,π]** . Set

$$\begin{cases} a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx, \\ a_n = = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)dx, \quad 1 \le n \\ b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)dx, \quad 1 \le n. \end{cases}$$

The trigonometric series

$$a_0 + \sum \left(a_n \cos(nx) + b_n \sin(nx)\right)$$

is known as the **Fourier series** associated to the function $f(x)$. We will use the notation

$$f(x) \sim a_0 + \sum_{n=1}^{\infty} \left(a_n \cos(nx) + b_n \sin(nx)\right).$$

Example. Find the Fourier series of the function

$$f(x) = x, \quad -\pi \le x \le \pi.$$

Answer. Since $f(x)$ is odd, then $a_n = 0$, for n ≥ 0. We pay attention to the coefficients b_n. For any n ≥1, we consider

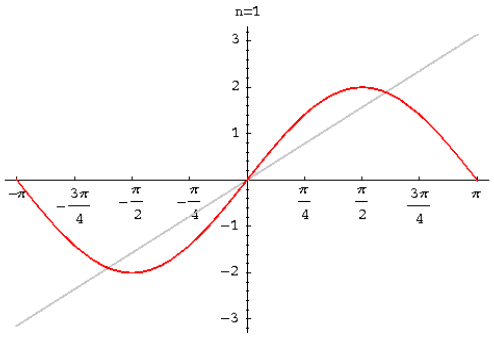
$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x\sin(nx)dx = \frac{1}{\pi}\left[-\frac{x\cos(nx)}{n} + \frac{\sin(nx)}{n^2}\right]_{-\pi}^{\pi}.$$

We figure out

$$b_n = -\frac{2}{n}\cos(n\pi) = \frac{2}{n}(-1)^{n+1}.$$

thus

$$f(x) \sim 2\left(\sin(x) - \frac{\sin(2x)}{2} + \frac{\sin(3x)}{3} \ldots.\right).$$



Change of interval In many engineering problems , it is desired to expand a function in a fourier series over an interval of length 2L. in order to apply foregoing theory , this interval must be transformed into an interval of length 2π . this can be achieved by a transformation of variable. suppose that $f(x)$ is defined and integrable on the interval [-*L*,*L*]. Set

$$F(x) = f\left(\frac{Lx}{\pi}\right).$$

suppose the Fourier series of $F(x)$

$$F(x) = f\left(\frac{Lx}{\pi}\right) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos(nx) + b_n \sin(nx)\right).$$

Replacing the value of t by given value t= Lx ÷ π , we find the following definition:

Definition. Suppose $f(x)$ be a function defined and integrable on [-*L*,*L*]. The Fourier series of $f(x)$ function is

$$f(t) \sim a_0 + \sum_{n=1}^{\infty} \left(a_n \cos\left(n\frac{\pi t}{L}\right) + b_n \sin\left(n\frac{\pi t}{L}\right)\right)$$

Whereas

$$\begin{cases} a_0 &= \frac{1}{2\pi}\int_{-\pi}^{\pi} f\left(\frac{Lx}{\pi}\right)\cos(nx)dx = \frac{1}{2L}\int_{-L}^{L} f(x)\cos\left(n\frac{\pi x}{L}\right)dx, \\ a_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} f\left(\frac{Lx}{\pi}\right)\cos(nx)dx = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(n\frac{\pi x}{L}\right)dx, \\ b_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} f\left(\frac{Lx}{\pi}\right)\sin(nx)dx = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(n\frac{\pi x}{L}\right)dx, \end{cases}$$

For $1 \leq n$.

Example. Find the Fourier series of

$$f(x) = \begin{cases} 0, & -2 \leq x < 0 \\ x, & 0 \leq x \leq 2 \end{cases}$$

Answer. Since $L = 2$, we find

$$a_0 = \frac{1}{4}\int_0^2 x\,dx = \frac{1}{2},$$

$$a_n = \frac{1}{2}\int_0^2 x\cos\left(n\frac{\pi x}{2}\right)dx = \frac{1}{2}\left(\frac{2}{n\pi}\right)^2\left(\cos(n\pi)-1\right) = \frac{1}{2}\left(\frac{2}{n\pi}\right)^2\left((-1)^n-1\right)$$

$$b_n = \frac{1}{2}\int_0^2 x\sin\left(n\frac{\pi x}{2}\right)dx = -\frac{2\cos(n\pi)}{n\pi} = \frac{2}{n\pi}(-1)^{n+1}$$

For $n \geq 1$. Thus, we take

$$f(x) \sim \frac{1}{2} + \sum_{n=1}^{\infty}\left[\frac{2}{n^2\pi^2}\left((-1)^n-1\right)\cos\left(n\frac{\pi x}{2}\right) + \frac{2}{n\pi}(-1)^{n+1}\sin\left(n\frac{\pi x}{2}\right)\right].$$

3. Applications

some real world function of Fourier series: advanced noise cancellation and cell phone network technology uses Fourier series where digital filtering is used to reduce noise and bandwidth demands respectively.It bring out that (almost) any kind of a wave can be written as a sum of sines and cosines. So for example, assuming that I was to record your voice for one second saying something, I can find its fourier series which may look entity like this for example voice=sin(x)+110sin(2x)+1100sin(3x)+ and this interactive module demonstrates you how when you add sines and/or cosines the graph of cosines and sines becomes closer and closer to the original graph we are trying to approximate.The really cool thing about fourier series is that first, almost any type of a wave can be approximated. Second, when fourier series converge, they converge very speedly.one of many applications is compression. Everyone's favorite MP3 format uses this for audio squeezing. You take a sound, extend its fourier series. It'll most same to be an infinite series but it converges so fast that taking the first few terms is enough to reproduce the original sound. The rest of the terms can be avoided because they add so little that a human ear can likely tell no difference.

Signal Processing: It may be the best utilization of Fourier analysis.

1. Approximation Theory: We use Fourier series to note down a function as a trigonometric polynomial.

Control Theory: The Fourier series of functions in the differential equation generally gives some prediction about the behavior of the solution of differential equation. They are important to find out the dynamics of the solution.

II. CONCLUSION

This paper concludes that as the number of calculated terms in a Fourier series increases, the series more and more closely approximates the exact function that explains a complex signal waveform. Computers can manupulate Fourier series out to hundreds, thousands, or millions of terms.

REFERENCES

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