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MATHEMATICAL MODELING AND PREDICTIVE CONTROLLER DESIGN FOR EXPERIMENTAL QUADRUPLE TANK SYSTEM

V.Jayashree, D.Angeline Vijula, M.E., Ph.D
Sri Ramakrishna Engineering College, Coimbatore.

***Abstract*--Many industrial processes such as distillation, manufacturing, refining etc. face interacting control loops, where the process is nonlinear and typically requires control over at least two variables. This could use at least two control loops and makes it Multi-Input Multi-Output (MIMO) or multivariable systems. It is common for models of industrial processes to have significant uncertainties, strong interactions and non-minimum phase behavior (i.e., right half plane transmission zeros). In recent process industries, PI and MPC are the two widely accepted control strategies, where PI is used at regulatory level control and MPC at supervisory level control. It was found that MPC controller provides better performance for many nonlinear processes. In this paper, a multivariable control called MPC is designed to control the process variables in Quadruple Tank Process to improve its performance.**

***Index Terms*- MPC, Decoupler, RGA analysis, Quadruple tank process, PI control.**

I. INTRODUCTION

Multivariable control system design is in great demand and need much attention in the process industries. When the interactions are not negligible, the plant must be considered as multiple inputs and multiple outputs system. Liquid level control is very important problem in chemical reactors, boilers etc.

The Quadruple tank process (QTP) is one of the multivariable laboratory processes with four interconnected water tanks. The quadruple tank apparatus has been developed for chemical engineering laboratories to illustrate the performance limitations for multivariable systems posed by ill-conditioning, right half plane transmission zeros and model uncertainties. The experiment is suitable for teaching how to select among multi-loop, decoupling, and fully multivariable control structures.

The salient feature of this system is an adjustable transmission zero which can be adjusted to operate in both minimum and non-minimum phase configuration, through the flow distribution to upper and lower tanks in quadruple tank system.

Non-linear equations are derived for the physical apparatus using various formulas and equations that describes the various components of the apparatus and the process in terms of tank height. These are used in the specification of the various apparatus components and secondly used as the equations that are linearized so that a linear representation of the plant in the form of a transfer function can be achieved and investigated. Stability and performance analysis has also been analysed for both in minimum and non-minimum phases.

The applications of multivariable process includes petrochemical industries, ceramic tile manufacturing industries, sugar industries, cement industries, paper making industries, dyeing industries.

II. QUADRUPLE TANK SYSTEM

The Quadruple tank process is a laboratory process that consists of four interconnected tank and two pumps. The setup of Quadruple tank process is shown in Fig.1 and the schematic diagram of the process is shown in Fig.2.

The process inputs are u_1 and u_2 (input voltages to pumps, (0-10 V) and the outputs are y_1 and y_2 (voltages from level measurement devices (0-10V). The target is to control the level of the lower two tanks with inlet flow rates.

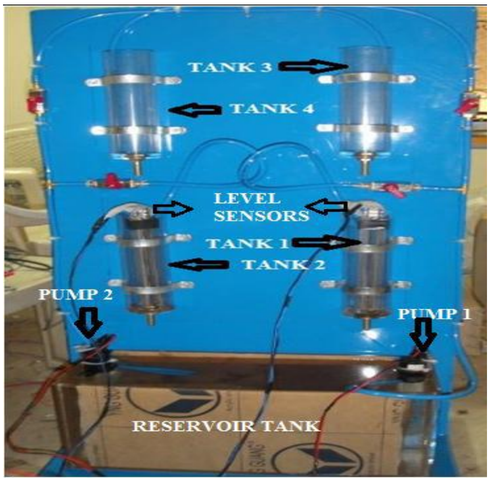

Fig.1. Setup of Quadruple Tank Process

The output of each pump is split into two using a three-way valve. Thus each pump output goes to two tanks, one lower and another upper, diagonally opposite and the ratio of the split up is controlled by the position of the valve. With the change in position of the two valves, the system can be appropriately placed either in the minimum phase or in the non-minimum phase. Let the parameter γ be determined by how the valves are set.

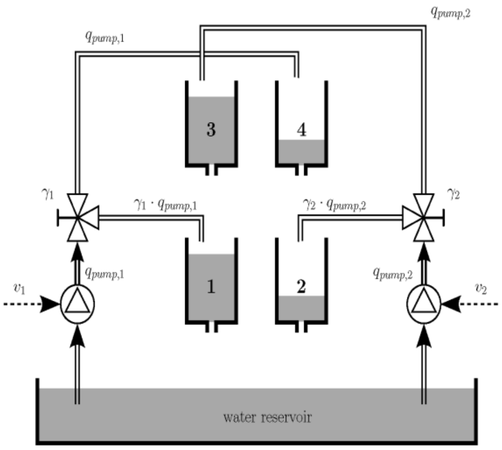

Fig.2. Schematic diagram of Quadruple tank process

Consider the Fig 1,if γ_1 is the ratio of the first tank, then $(1-\gamma_1)$ will be the flow to the fourth tank. The voltage applied to pump 1 is V_1 and the corresponding flow is k_1V_1. The parameters γ_1, γ_2 ε $(0, 1)$ are determined from how the valves are set prior to an experiment. The flow to tank 1 is $\gamma_1k_1V_1$ and the flow to flow tank 4 is $(1- \gamma_1)\ k_1V_1$ and similarly for tank 2 and tank 3. The acceleration due to gravity is denoted by g. The measured level signals are $y_1=kch1$ and $y_2=kch2$.

The pumps that are used in this system are identical. The error in mathematical model is significant if the pumps used in the experiment are not identical.

The parameter values are given by TABLE I.

TABLE I: QTS SPECIFICATION

Parameters	Values	Units
A_1,A_3	28	cm^2
A_2,A_4	32	cm^2
a_1,a_3	0.071	cm^2
a_2,a_4	0.051	cm^2
h_i	20	*Cm*
k_c	0.5	*V/cm*
G	981	cm/s^2

III. MATHEMATICAL MODEL

The mathematical model for this process using mass balance and Bernoulli's law is given by

$$\frac{dh_1}{dt} = -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1}\upsilon_1 \qquad (1)$$

$$\frac{dh_2}{dt} = -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2}\upsilon_2 \qquad (2)$$

$$\frac{dh_3}{dt} = -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_2)k_2}{A_3}\upsilon_2 \qquad (3)$$

$$\frac{dh_4}{dt} = -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_1)k_1}{A_4}\upsilon_1 \qquad (4)$$

Introducing the variables $x_i := h_i - h_i^{\ 0}$ and $u_i := \upsilon_i - \upsilon_i^{\ 0}$. The linearised state-space equation is then given by

$$\frac{dx}{dt} = \begin{bmatrix} -\frac{1}{T_1} & 0 & \frac{A_3}{A_1T_3} & 0 \\ 0 & -\frac{1}{T_2} & 0 & \frac{A_4}{A_2T_4} \\ 0 & 0 & -\frac{1}{T_3} & 0 \\ 0 & 0 & 0 & -\frac{1}{T_4} \end{bmatrix} x + \begin{bmatrix} \frac{\gamma_1k_1}{A_1} & 0 \\ 0 & \frac{\gamma_2k_2}{A_2} \\ 0 & \frac{(1-\gamma_2)k_2}{A_3} \\ \frac{(1-\gamma_1)k_1}{A_4} & 0 \end{bmatrix} u \qquad (5)$$

$$y = \begin{bmatrix} k_c & 0 & 0 & 0 \\ 0 & k_c & 0 & 0 \end{bmatrix} x \qquad (6)$$

where

$$T_i = \frac{A_i}{a_i}\sqrt{\frac{2h_i^{\ 0}}{g}}, i = 1,\ldots,4.$$

The corresponding transfer function matrix obtained is given by

$$G(s) = \begin{bmatrix} \frac{\gamma_1 c_1}{1+sT_1} & \frac{(1-\gamma_2)c_1}{(1+sT_3)(1+sT_1)} \\ \frac{(1-\gamma_1)c_2}{(1+sT_4)(1+sT_2)} & \frac{\gamma_2 c_2}{1+sT_2} \end{bmatrix} \qquad (7)$$

where $c_1 = \frac{T_1k_ck_1}{A_1}$ and $c_2 = \frac{T_2k_ck_2}{A_2}$

The operating points for minimum and non minimum phases are given below.

Table Ii. Operating Points

		P-	***P+***
(h_1^0,h_2^0)	(cm)	(12.4,12.7)	(12.6,13.0)
(h_3^0,h_4^0)	(cm)	(1.8,1.4)	(4.8,4.9)
(v_1^0,v_2^0)	(V)	(3.00,3.00)	(3.15,3.15)
(k_1, k_2)	(Cm^2/Vs)	(3.33,3.35)	(3.14,3.29)
(γ_1, γ_1)		(0.70,0.60)	(0.43,0.34)

For the two operating points P_- and P_+, we have the following time constants:

Table Iii. Time Constants

	P-	*P+*
(T_1,T_2)	(62,90)	(63,91)
(T_3,T_4)	(23,30)	(39,56)

The physical modeling gives the two transfer functions

$$G_{-}(s)=\begin{bmatrix}\dfrac{2.6}{62s+1} & \dfrac{1.5}{(23s+1)(62s+1)}\\ \dfrac{1.4}{(30s+1)(90s+1)} & \dfrac{2.5}{90s+1}\end{bmatrix} \tag{8}$$

and

$$G_{+}(s)=\begin{bmatrix}\dfrac{1.5}{63s+1} & \dfrac{2.5}{(23s+1)(62s+1)}\\ \dfrac{2.5}{(56s+1)(91s+1)} & \dfrac{2.5}{91s+1}\end{bmatrix} \tag{9}$$

IV. RELATIVE GAIN ARRAY

To fully appreciate the concept of relative gains, the RGA will be constructed for a system represented by a (2×2) P-Canonical structure as

$$\Lambda=\begin{bmatrix}\lambda_{11} & \lambda_{12}\\ \lambda_{21} & \lambda_{22}\end{bmatrix} \tag{10}$$

The suitable pairing is given by

- For **minimum** phase, $\boldsymbol{\lambda_{11}}$ is obtained as **0.63** which falls in the range 0:5 < λ_{11} < 1,so the pairing is determined as **y1 with u1** and **y2 with u2**.
- For **non minimum** phase, $\boldsymbol{\lambda_{11}}$ is obtained as **0.375** which falls in the range 0 < λ_{11} < 0:5, so the suitable pairing is found as **y1 with u2** and **y2 with u1**.

V. DESIGN OF DECOUPLER

The goal of decoupling control is to eliminate complicated loop interactions so that a change in one process variable will not cause corresponding changes in other process variables. To do this a non-interacting or decoupling control scheme is used. In this scheme, a compensation network called a decoupler is used right before the process. This decoupler is the inverse of the gain array and allows for all measurements to be passed through it in order to give full decoupling of all of the loops. This is shown Fig.3.

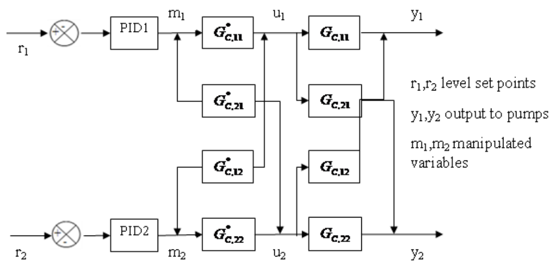

Fig.3. Design of decoupler

The decoupler design equations are

$$G^{*}_{C,12}=\frac{-G_{12}G^{*}_{C,22}}{G_{11}}$$ and

$$G^{*}_{C,21}=\frac{-G_{21}G^{*}_{C,11}}{G_{22}} \tag{11}$$

VI. MODEL PREDICTIVE CONTROL

Model Predictive Control (MPC) is an important advanced control technique for difficult multivariable control problems. The basic MPC concept can be summarized as follows. Suppose that we wish to control a multiple-input, multiple-output process while satisfying inequality constraints on the input and output variables. If a reasonably accurate dynamic model of the process is available, model and current measurements can be used to predict future values of the outputs. Then the appropriate changes in the input variables can be calculated based on both predictions and measurements. In essence, the changes in the individual input variables are coordinated after considering the input-output relationships represented by the process model.

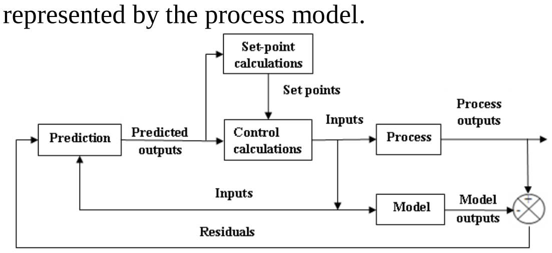

Fig.4. Block Diagram of MPC controller

A block diagram of a model predictive control system is shown in Fig 4. A process model is used to predict the current values of the output variables. The residuals, the differences between the actual and predicted outputs, serve as the feedback signal to a *Prediction* block. The predictions are used in two types of MPC calculations that are performed at each sampling instant set-point calculations and control calculations. Inequality constraints on the input and output variables, such as upper and lower limits, can be included in either type of calculation.

The set points for the control calculations, also called *targets*, are calculated from an economic optimization based on a steady-state model of the process, traditionally, a linear model. Typical optimization objectives include maximizing a profit function, minimizing a cost function, or maximizing a production rate. The optimum values of set points change frequently due to varying process conditions, especially changes in the inequality constraints. The constraint changes are due to variations in process conditions, equipment, and instrumentation, as well as economic data such as prices and costs. In MPC the set points are typically calculated each time the control calculations are performed.

The MPC calculations are based on current measurements and predictions of the future values of the outputs. The objective of the MPC control calculations is to determine a sequence of *control moves* (that is, manipulated input changes) so that the predicted response moves to the set point in an optimal manner. The actual output *y*, predicted output and manipulated input *u* for SISO control are shown in Fig 5. At the current sampling instant, denoted by *k*, the MPC strategy calculates a set of *M* values of the input $\{u(k + i - 1), i = 1, 2, . . , M\}$.

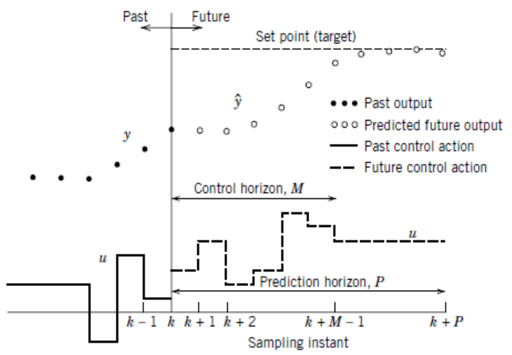

Fig.5. Basic concept for model predictive control

The set consists of the current input $u(k)$ and M _ 1 future inputs. The input is held constant after the *M* control moves. The inputs are calculated so that a set of *P* predicted outputs $(k + i)$, $i = 1, 2,...,P\}$ reaches the set point in an optimal manner. The control calculations are based on optimizing an objective function. The number of predictions *P* is referred to as the *prediction horizon* while the number of control moves *M* is called the *control horizon*.

A distinguishing feature of MPC is its *receding horizon approach*. Although a sequence of *M* control moves is calculated at each sampling instant, only the first move is actually implemented. Then a new sequence is calculated at the next sampling instant, after new measurements become available; again only the first input move is implemented. This procedure is repeated at each sampling instant.

Selection Of Design And Tuning Parameters

A number of design parameters must be specified in order to design an MPC system.

Sampling period Δt and model horizon N

The sampling period Δt and model horizon *N* should be chosen so that $N\Delta t = t_s$ where t_s is the settling time for the open-loop response.

Control M and prediction P horizons

As control horizon *M* increases, the MPC controller tends to become more aggressive and the required computational effort increases. Decreasing the value of *P* tends to make the controller more aggressive. The prediction horizon *P* is often selected to be $P = N + M$ so that the full effect of the last MV move is taken into account.

Weighting Matrices, Q and R

The output weighting matrix ***Q*** allows the output variables to be weighted according to their relative importance. ***R*** allows input MVs to be weighted according to their relative importance.

Reference Trajectory α_i

The reference trajectory has a tuning factor that can be used to adjust the desired speed of response for each output.

Predictions For Mimo Models

The previous analysis for SISO systems can be generalized to MIMO systems by using the Principle of Superposition. For simplicity, we first consider a process control problem with two outputs, *y*1 and *y*2, and two inputs, *u*1 and *u*2. The predictive model consists of two equations and four individual step-response models; one for each input-output pair is given as

$$\hat{y}_1(k+1)=\sum_{i=1}^{N-1}S_{11.i}\Delta u_1(k-i+1)+S_{11,N}u_1(k-N+1)+\sum_{i=1}^{N-1}S_{12,i}\Delta u_2(k-i+1)+S_{12,N}u_2(k-N+1) \tag{12}$$

$$\hat{y}_1(k+1)=\sum_{i=1}^{N-1}S_{21.i}\Delta u_1(k-i+1)+S_{21,N}u_1(k-N+1)+\sum_{i=1}^{N-1}S_{22,i}\Delta u_2(k-i+1)+S_{22,N}u_2(k-N+1) \tag{13}$$

where $S_{12,i}$ denotes the *i*th step-response coefficient for the model that relates y_1 and u_2. The other step-response coefficients are defined in an analogous manner. The MIMO model for the corrected predictions can be expressed in dynamic matrix form is

$$\tilde{Y}(k+1)=S\Delta U(k)+\hat{Y}^0(k+1)+[y(k)-\hat{y}(k)] \tag{14}$$

where

$\tilde{Y}(k+1)$ is the mP-dimensional vector of corrected predictions over the prediction horizon P

$\hat{Y}^0(k+1)$ is the mP-dimensional vector of predicted unforced responses

$\Delta U(k)$ is the rM-dimensional vector of the next M control moves

The mP x m matrix is defined as

$$S \cong [I_m I_m .....I_m]^T \tag{15}$$

where I_m denotes the m x m identity matrix.

The dynamic matrix $\boldsymbol{S}$ is defined by

$$s \equiv \begin{bmatrix} S_1 & 0 & \dots & 0 \\ S_2 & S_1 & 0 & \dots \\ . & . & . & . \\ S_M & S_{M-1} & \dots & S_1 \\ S_{M+1} & S_M & \dots & S_2 \\ . & . & \dots & . \\ S_P & S_{P-1} & \dots & S_{P-M+1} \end{bmatrix} \tag{16}$$

where $\boldsymbol{Si}$ is the m x r matrix of step-response coefficients for the ith time step.

VII SIMULATIONS AND RESULTS

The closed loop responses for minimum phase and non minimum phase are obtained with servo and regulator operation. The servo operation is for set point tracking and the regulator operation is for disturbance rejection.

The output response using PI controllers for minimum and non minimum phases are shown below.

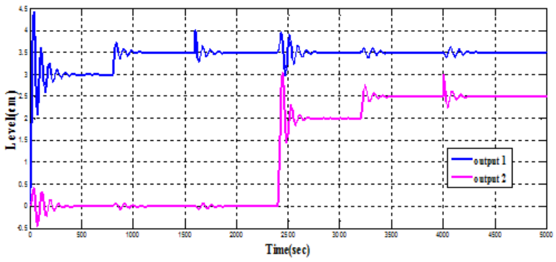

Fig.6. Response using PI controller without decoupler – minimum phase

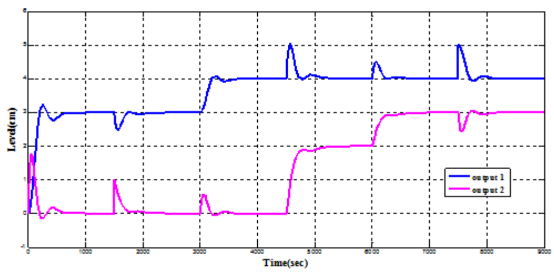

Fig.7. Response using PI controller without decoupler – non minimum phase

From the above responses, it is seen that there are interaction present between the two loops. Hence to reduce the interactions, decouplers are used. The output response using decoupled PI controller is shown below.

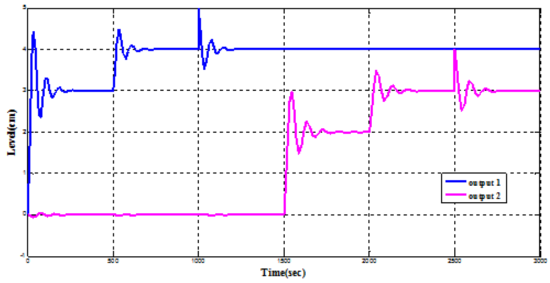

Fig.8. Response using PI controller with decoupler – minimum phase

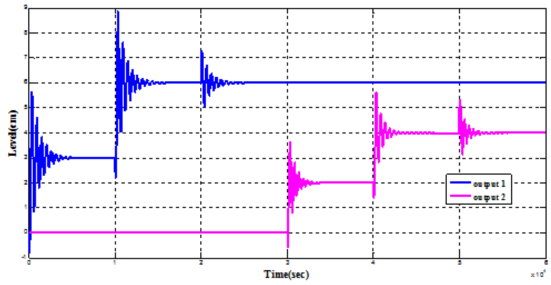

Fig.9. Response using PI controller with decoupler – non minimum phase

From the responses of decoupled PI controller, it is seen that the interactions are reduced. But it has large settling time, larger tuning parameters. So to reduce these parameter ranges, Model Predictive Control can be implemented. The output response using MPC is shown below.

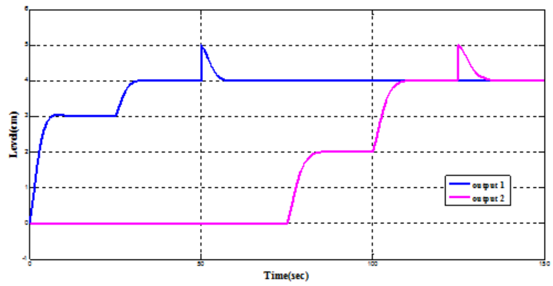

Fig.10. Response using MPC – minimum phase

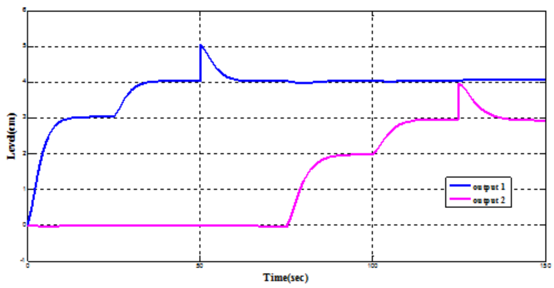

Fig.11. Response using MPC – non minimum phase

It is seen that the MPC provides better response than PI and decoupled PI controllers with smaller settling time, time constants etc.

Table Iv: Quantitative Comparison Of The Performances Of Pi, Decoupled Pi And Mpc For Non Minimum Phase System

S.No	Controller	Parameter	Minimum phase		Non minimum phase	
			Tank 1	Tank 2	Tank 1	Tank 2
1	PI controller	Settling time(sec)	450	490	1000	1200
		Peak overshoot(%)	4.4	1.0	0.2	0.0
2	Decoupled PI controller	Settling time(sec)	270	350	6000	2000
		Peak overshoot(%)	1.4	1.0	2.6	1.5
3	Model Predictive Control	Settling time(sec)	10	12	14	19
		Peak overshoot(%)	0.0	0.0	0.0	0.0

VIII CONCLUSION

The Quadruple Tank Process is modeled and simulation is done with conventional PI controller, Decoupled PI controller and MPC controller. The transfer function matrix is obtained for the minimum phase and non-minimum phase using the corresponding operating conditions. PI controller, Decoupled PI controller and MPC controller are simulated and tested for both minimum phase and non-minimum phase conditions with step input. The Decoupled PI controller minimized the interaction between the two loops but due to large settling time, parameters the performance of the system is not improved. MPC was able to control both minimum phase and non-minimum phase modes of behavior with smaller settling time and negligible interactions. MPC performs better than PI and Decoupled PI in terms of interaction between the two loops and overshoot.

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