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OBSERVATIONS ON THE HYPERBOLA

$$y^2 = 60x^2 + 4$$

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***Abstract*- The binary quadratic equation $y^2 = 60x^2 + 4$ is considered and a few interesting properties among the solutions are presented. Employing the integer solutions of the equation under consideration, a special Pythagorean triangle is obtained.**

***Index Terms*- binary quadratic, integer solutions**

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I. INTRODUCTION

The binary quadratic equation of the form $y^2 = Dx^2 + 1$ where D is non-square positive integer has been studied by various mathematicians for its non-trivial integral solutions when D takes different integral values[1-4]. For an extensive review of various problems, one may refer [5-23]. In this communication, yet another interesting hyperbola given by $y^2 = 60x^2 + 4$ is considered and infinitely many integer solutions are obtained.

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II. METHOD OF ANALYSIS

Consider the binary quadratic equation

$$y^2 = 60x^2 + 4 \tag{1}$$

with the least positive integer solutions $x_0 = 1, y_0 = 8$

To obtain the other solutions of (1), consider the Pellian equation

$$y^2 = 60x^2 + 4$$

whose general solution $(\tilde{x}_n, \tilde{y}_n)$ is given by,

$$\tilde{x}_n = \frac{g}{4\sqrt{15}} \quad \text{and} \quad \tilde{y}_n = \frac{f}{2}$$

in which,

$$f = \left(31 + 4\sqrt{60}\right)^{n+1} + \left(31 - 4\sqrt{60}\right)^{n+1}$$

$$g = \left(31 + 4\sqrt{60}\right)^{n+1} - \left(31 - 4\sqrt{60}\right)^{n+1},$$

where n = -1,0,1,2,…

Applying Brahmagupta lemma between the solutions of (x_0, y_0) and $(\tilde{x}_n, \tilde{y}_n)$, the general solution of (1) is found to be

$$x_{n+1} = \frac{f}{2} + \frac{2g}{\sqrt{15}} \tag{2}$$

$$y_{n+1} = 4f + \sqrt{15}g \tag{3}$$

where n = -1,0,1,2,….

Thus (2) and (3) represent non-zero distinct integral solutions of (1) which represents a hyperbola. The recurrence relations satisfied by the values of x and y are respectively

$$x_{n+3} - 62x_{n+2} + x_{n+1} = 0$$

$y_{n+3} - 62y_{n+2} + y_{n+1} = 0$

A few numerical examples are presented in the table below.

n	x_{n+1}	y_{n+1}
-1	1	8
0	63	848
1	3905	30248
2	242047	1874888
3	46845617	116212808
4	929944511	7203319208

A few interesting relations among the solutions are presented below.

1. x_{n+1} is always odd and y_{n+1} is always even.
2. $x_{n+1} \equiv 1(\bmod 2)$
3. $y_{n+1} \equiv 0(\bmod 8)$
4. $x_{3n+1} \equiv 0(\bmod 3)$
5. $24y_{2n+2} - 180x_{2n+2} + 12$ is a Nasty number.
6. $4y_{2n+2} - 30x_{2n+2} + 2$ is a quadratic number.
7. $4y_{3n+3} - 30x_{3n+3} + 3(4y_{n+1} - 30x_{n+1})$ is a Cubic integer.
8. $$60(4y_{3n+3} - 30x_{3n+3} + 12y_{n+1} - 90x_{n+1}) - 900(8x_{n+1} - y_{n+1})^2(4y_{n+1} - 30x_{n+1}) = 240(4y_{n+1} - 30x_{n+1})$$
9. $x_{n+2} = 4y_{n+1} + 31x_{n+1}$.
10. $x_{n+3} = 248y_{n+1} + 1921x_{n+1}$.
11. $y_{n+2} = 31y_{n+1} + 240x_{n+1}$.
12. $y_{n+3} = 1921y_{n+1} + 14880x_{n+1}$.
13. $4y_{2n+2} - 30x_{2n+2} + 2 = (4y_{n+1} - 30x_{n+1})^2$.
14. $$4y_{3n+3} - 30x_{3n+3} + 3(4y_{n+1} - 30x_{n+1}) = (4y_{n+1} - 30x_{n+1})^3.$$
15. $x_{n+3}y_{n+1} - x_{n+1}y_{n+3} = 992$.
16. $60x_{n+1}x_{n+3} - y_{n+1}y_{n+3} = -7684$
17. $x_{n+2}y_{n+1} - x_{n+1}y_{n+2} = 16$.
18. $60x_{n+2}x_{n+1} - y_{n+1}y_{n+2} = -124$)

III. REMARKABLE OBSERVATIONS

1. Define $X = 4y_{n+1} - 30x_{n+1}$ and $Y = 8x_{n+1} - y_{n+1}$, then the pair (X, Y) satisfies the hyperbola $X^2 = 15Y^2 + 4$
2. Define $X = 4y_{2n+2} - 30x_{2n+2} + 2$ and $Y = 8x_{n+1} - y_{n+1}$, then the pair (X, Y) satisfiesthe parabola $15Y^2 = X - 4$
3. Define $X = x_{n+2} - 61x_{n+1}$ and $Y = 8x_{n+1} - y_{n+1}$, then the pair (X, Y) satisfies the hyperbola $X^2 = 15Y^2 + 4$
4. Let $p = (x_{n+1} + y_{n+1})$, $q (= x_{n+1})$ be any two non-zero distinct positive integers ,note that $p > q$.

Treat p,q as the generaters of the Pythagorean triangle $T(\alpha, \beta, \gamma)$,where $\alpha = 2pq$

$\beta = p^2 - q^2, \gamma = p^2 + q^2$.

Let A,P are triangle.In the above A,P represent the area and perimeter of the Pythagorean triangle T,

then the following relations are observed:

(i) $\alpha - 30\beta + 29\gamma = -4$

(ii) $\beta - \frac{4A}{P} - 30(\gamma - \beta) = 4$

(iii) $31\alpha - \gamma + 4 = \frac{120A}{P}$

IV. CONCLUSION

In this paper, infinitely many non-zero distinct integer solutions for the hyperpola $y^2 = 60x^2 + 4$ are obtained. As binary quadratic diophantine equations are rich in variety, one may search for integer solutions and the corresponding properties for other choices of binary quadratic diophantine equations.

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