Bending behavior of Orthotropic Skew Plate subjected to Point Load

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***Abstract*-Present paper deals with defection analysis of orthotropic skew plate using FSDT. A polynomial radial basis function base meshfree method is used to discretize the partial differential equations in displacement form. A MATLAB code is developed incorporating to obtain the solutions. Results related to flexure analysis of orthotropic skew plates are presented under point load. Effect of orthotropic ration, skew angle and span to thickness ratio is presented.**

***Index Terms*- Skew plate, Orthotropic, FSDT, Meshfree, Point Load**

I. INTRODUCTION

Plates are defined as plane structural elements with a small thickness. Plate deformation theories can be divided in to two groups: stress based and displacement based theories. Due to the presence of singularity at the obtuse corners skew plates are complicated then rectangular plates. Skew plates have several numbers of applications in various mechanical, civil and aero structures such as ship hulls, buildings, aircrafts etc. The present paper deals with the skew plates under point load. Analysis of skew plate using point load is a rare study in the field of research. Ferreira et al. [1] use the FSDT in the multiquadric radial basis function (MQRBF) procedure for predicting the free vibration behavior. Sengupta [2] has studied the performance of a simple finite element for the analysis of skew rhombic plates. Bending analysis of simply supported shear deformable Skew plates have been carried out by Liew and Han [3]. The spline-finite-strip/element method has also been applied to the bending analysis of skew plates (Tham et al. [4]; Li et al. [5]; Wang and Hsu [6]). Daripa and Singha [7] studied the influence of corner stresses on the stability behaviour of composite skew plates. The analysis of isotropic thick skew plates had been carried out by Muhammad and Singh, [8].Srinivasa et. al. [9] studies the buckling effect on skew plates using finite element.

II. MATHEMATICAL FORMULATION

The plate geometry of is shown in Fig. 1. Thickness h is along z axis whose mid plane is coinciding with x-y plane of the coordinate system is considered.
The displacement field at any point in the plate is expressed as ignoring initial displacements in X and Y direction:

$$u = -z\phi_x$$
$$v = -z\phi_y$$
$$w = w_0$$

(1)

The strain-displacement relations can be written as:

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} -z\dfrac{\partial \phi_x}{\partial x} \\ -z\dfrac{\partial \phi_y}{\partial y} \\ -z\dfrac{\partial \phi_x}{\partial y} - z\dfrac{\partial \phi_y}{\partial x} \end{Bmatrix}$$

(2)

$$\begin{Bmatrix} \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} = \begin{Bmatrix} -\phi_y + \dfrac{\partial w_0}{\partial y} \\ -\phi_x + \dfrac{\partial w_0}{\partial x} \end{Bmatrix}$$

(3)

The constitutive stress strain relation can be written as:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{zx} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & 0 & 0 & 0 \\ \bar{Q}_{12} & \bar{Q}_{22} & 0 & 0 & 0 \\ 0 & 0 & \bar{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \bar{Q}_{44} & 0 \\ 0 & 0 & 0 & 0 & \bar{Q}_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} \quad (4)$$

The governing differential equations of plate are obtained using Hamilton's principle and expressed as:

$$\frac{\partial M_{xx}}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x = 0$$

$$\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_{yy}}{\partial y} - Q_y = 0$$

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} - q_z = 0 \quad (5)$$

Where, $M_{xx} = D_{11}\frac{\partial \phi_x}{\partial x} + D_{12}\frac{\partial \phi_y}{\partial y}$

$$M_{yy} = D_{12}\frac{\partial \phi_x}{\partial x} + D_{22}\frac{\partial \phi_y}{\partial y}$$

$$M_{xy} = D_{16}\frac{\partial \phi_x}{\partial x} + D_{26}\frac{\partial \phi_y}{\partial y}$$

$$Q_x = kA_{55}\phi_x,\ Q_y = kA_{55}\phi_y$$

The boundary conditions for an arbitrary edge with simply supported conditions are as follows:

$$\phi^s, w, M_{nn} = 0 \quad (6)$$

Where,

$$\phi^s = -n_y.\phi^x + n_x.\phi^y$$

$$M_{nn} = n_x^2 M_{xx} + 2n_x n_y M_{xy} + n_y^2 M_{yy}$$

$$n_x = \cos(\theta),\quad n_y = \sin(\theta)$$

III. SOLUTION METHODOLOGY

The governing differential equations (5) are expressed in terms of displacement functions. Radial basis function based formulation works on the principle of interpolation of scattered data over entire domain. The variable w_0, ϕ_x *and* ϕ_y can be interpolated in form of radial distance between nodes. The solution of the linear governing differential equations is assumed in terms of polynomial radial basis function for nodes 1: N, as;

$$w_o, \phi_x, \phi_y = \sum_{j=1}^{N} (\alpha_j^w, \alpha_j^{\phi_x}, \alpha_j^{\phi_y})\ g\left(\left\| X - X_j \right\|, m\right)$$

Where, N is total numbers of nodes which is equal to summation of boundary nodes NB and domain interior nodes ND. $g\left(\left\| X - X_j \right\|, m\right)$ is polynomial radial basis function expressed as $g = r^m$, $\delta = \alpha_j^w, \alpha_j^{\phi_x}, \alpha_j^{\phi_y}$ are unknown coefficients. $\left\| X - X_j \right\|$ is the radial distance between two nodes.

Where, $r = \left\| X - X_j \right\| = \sqrt{\left(x - x_j\right)^2 + \left(y - y_j\right)^2}$ and m is shape parameter. The value of 'm' taken here is 5. Polynomial radial basis function becomes singular, when r = 0 i.e. for zero distance. In order to eliminate the singularity, an infinitesimally small value is added into the r^2 or zero distance. Mathematically it is explained as; $r^2 = r^2 + \mu^2$ when r = 0 or i = j; μ^2 is small numerical value of the order 10^{-10}.

The discretized governing equations for linear flexural analysis can be written as:

$$\begin{bmatrix} [K]_L \\ [K]_B \end{bmatrix}_{3N\times 3N} \{\delta\}_{3N\times 1} = \begin{Bmatrix} [F]_L \\ 0 \end{Bmatrix}_{3N\times 1} \quad (7)$$

The unknown coefficients $\{\delta\}$ are calculated from equation (7).

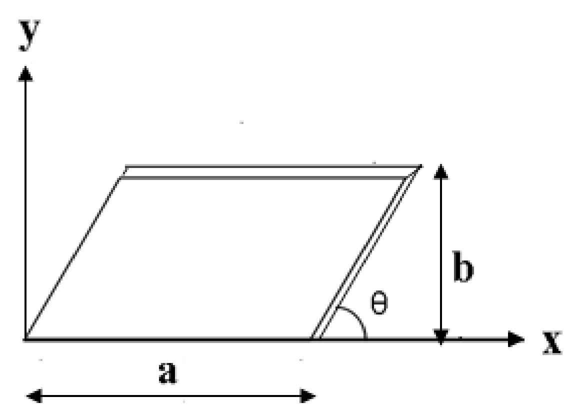

Fig. 1 Geometry of skew plate

IV. NUMERICAL RESULTS AND DISCUSSIONS

In order to demonstrate the accuracy and applicability of present formulation, a RBF based meshless code in MATLAB is developed following the analysis procedure as discussed above. Based on convergence study, a 13x13 node is used throughout the study.

The deflection and moments are normalized as:

$\bar{w} = w_{c\max}.100.h^3/(qa^4) \qquad \bar{M} = M_{c\max}.40/(qa^2)$

$\bar{\sigma}_{xx} = \sigma_{xx\max}.(qa^4/h^2) \qquad \bar{\sigma}_{yy} = \sigma_{yy\max}.(qa^4/h^2)$

$\bar{\sigma}_{xy} = \sigma_{xy\max}.(qa^4/h^2) \qquad \bar{\sigma}_{xz} = \sigma_{xz\max}.(qa^4/h^2)$

Unless until specified, the material properties are taken as: $E1 = 25, E2 = 1, \nu = 0.3, G1 = G2 = 0.5, G3 = 0.2$

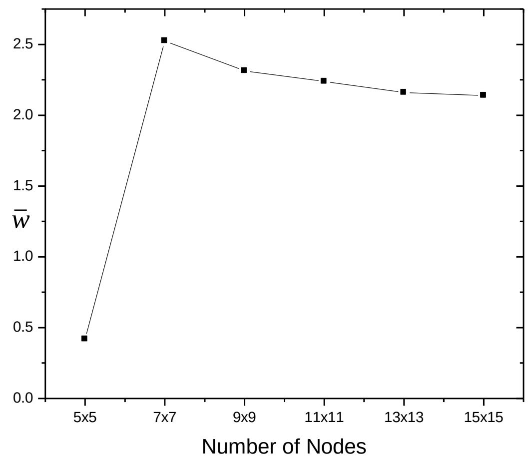

Fig. 2 Convergence study for deflection $\bar{w}$ of skew plate (a/h = 10)

From Fig.2 it can be seen that a good convergence is achieved for thick plate. The convergence is within 1% for nodes more than 9x9.

Table-1 Effect of span to thickness ratio on M_{xx} of a square orthotropic skew plate

	Skew angle				
a/h	90	75	60	45	30
5	20.9893	20.768	19.6	18.2046	15.5813
10	23.1592	22.1901	20.784	18.8794	15.4795
20	23.4296	22.3268	21.2779	19.0568	15.4538
30	23.6433	22.76	21.1556	18.7826	21.9364
40	23.6325	22.6562	20.8218	18.3357	14.8434
50	23.5647	22.4505	20.4015	17.8354	14.4386
100	23.2876	21.1278	18.18	15.5231	12.5853

Table-2 Effect of span to thickness ratio on Myy of a square orthotropic skew plate

	Skew angle				
a/h	90	75	60	45	30
5	4.3127	4.2427	4.1293	4.0409	3.5914
10	3.7336	3.6824	3.6327	3.5493	3.2592
20	3.2998	3.2408	3.1615	3.0369	2.7787
30	3.0875	3.0321	2.9255	2.7585	3.6991
40	2.9531	2.889	2.7585	2.5626	2.3077
50	2.8564	2.7787	2.6218	2.4032	2.1388
100	2.6406	2.4111	2.1173	1.8368	1.5645

Table-3 Effect of span to thickness ratio on Mxy of a square orthotropic skew plate

	Skew angle				
a/h	90	75	60	45	30
5	0.8154	0.742	0.8056	0.848	0.6972
10	0.5658	0.7331	0.895	0.9069	0.7568
20	0.5104	0.791	1.0039	1.0278	0.7732
30	0.548	0.8283	1.0284	1.0376	5.2862
40	0.5605	0.8424	1.0184	1.0038	0.7248
50	0.572	0.8505	0.9942	0.9547	0.676
100	0.6743	0.8638	0.8334	0.719	0.49

Table-4 Effect of span to thickness ratio on Mnn of a square orthotropic skew plate

	Skew angle				
a/h	90	75	60	45	30
5	4.3127	5.3402	7.9028	10.9711	12.4563
10	3.7336	4.8892	7.7813	10.9956	12.2184
20	3.2998	4.479	7.5359	10.8146	12.0601
30	3.0875	4.3074	7.3255	10.539	16.3455
40	2.9531	4.1637	7.1156	10.2211	11.4899
50	2.8564	4.0441	6.9072	9.8948	11.1456
100	2.6406	3.595	5.9712	8.4703	9.6238

Table-5 Effect of span to thickness ratio on Mns of a square orthotropic skew plate

Skew angle					
a/h	90	75	60	45	30
5	0.8154	4.1478	6.7534	7.0818	5.1181
10	0.5658	4.684	7.5071	7.665	5.1726
20	0.5104	4.8413	7.934	8.01	5.3586
30	0.548	5.0119	7.9848	8.0121	9.2275
40	0.5605	5.0273	7.9133	7.8866	5.3014
50	0.572	5.0087	7.7909	7.7161	5.2
100	0.6743	4.8001	7.0487	6.8432	4.6531

Table-6 Effect of span to thickness ratio on $\bar{\sigma}_{xx}$ of a square orthotropic skew plate

Skew angle					
a/h	90	75	60	45	30
5	14.7825	14.4985	14.3032	14.5186	14.0391
10	15.3368	15.2108	15.0446	15.3181	14.8214
20	16.1182	32.1906	15.8713	16.1865	15.8134
30	16.2707	16.113	16.0314	16.3705	110.7835
40	16.0447	15.9128	15.8618	16.1921	15.7976
50	15.6433	15.5511	15.5378	15.8367	15.4881
100	13.0423	13.3172	13.5223	13.5536	13.7869

Table-7 Effect of span to thickness ratio on $\bar{\sigma}_{yy}$ of a square orthotropic skew plate

Skew angle					
a/h	90	75	60	45	30
5	6.4202	6.1999	5.9407	6.534	6.6295
10	5.9777	5.7749	5.5194	6.0496	6.1597
20	5.2674	5.1002	4.8675	5.2635	5.3268
30	4.7318	4.601	4.4041	4.6958	6.1042
40	4.3371	4.2235	4.0413	4.2618	4.4132
50	4.0262	3.9188	3.7395	3.9072	4.2249
100	3.118	2.9156	2.693	2.7156	3.1726

Table-8 Effect of span to thickness ratio on $\bar{\sigma}_{xy}$ of a square orthotropic skew plate

Skew angle					
a/h	90	75	60	45	30
5	1.1426	1.1377	1.1271	1.1325	1.1195
10	0.9467	0.9512	0.9411	0.9319	0.9339
20	0.7002	1.1697	0.7421	1.0618	0.8976
30	0.5666	0.5693	0.8252	1.225	1.5342
40	0.4864	0.4887	0.8501	1.2673	1.1572
50	0.4573	0.4839	0.8481	1.254	1.1485
100	0.4722	0.4884	0.7413	1.0219	0.9141

Table-9 Effect of span to thickness ratio on $\bar{\sigma}_{xz}$ of a square orthotropic skew plate

Skew angle					
a/h	90	75	60	45	30
5	0.7208	0.7104	0.6998	0.707	0.6821
10	0.4472	0.4392	0.4288	0.4255	0.3994
20	0.5817	0.5216	0.459	0.3689	0.3022
30	0.8452	0.7716	0.6375	0.488	0.3306
40	1.1232	1.0137	0.8098	0.603	0.3959
50	1.4169	1.258	0.9714	0.7088	0.4644
100	3.1799	2.4849	1.6599	1.1689	0.7572

Table-10 Effect of orthotropic ratio on Mxx of a skew plate

Skew angle				
E1/E2	**90**	**75**	**60**	**45**
3	12.8986	10.3245	8.702	7.2069
5	15.5884	12.6542	10.5493	8.8052
15	20.871	18.2913	15.4492	13.0965
20	22.2114	19.87	16.9523	14.4276
25	23.2876	21.1278	18.18	15.5231
30	24.2049	22.18	19.2158	16.4613
40	25.7406	23.8902	20.6581	18.0201

Table-11 Effect of orthotropic ratio on M_{yy} of a skew plate

	Skew angle			
E1/E2	**90**	**75**	**60**	**45**
3	5.8982	5.0389	4.3181	3.7164
5	4.8873	4.2742	3.6697	3.173
15	3.1757	2.8999	2.5331	2.1987
20	2.8576	2.6136	2.2907	1.9879
25	2.6406	2.4111	2.1173	1.8368
30	2.4791	2.2571	1.9853	1.721
40	2.247	2.0326	1.8265	1.5519

Table-12 Effect of orthotropic ratio on M_{xy} of a skew plate

	Skew angle			
E1/E2	**90**	**75**	**60**	**45**
3	2.5744	2.4651	2.0368	1.4811
5	1.8099	2.0076	1.6799	1.2445
15	0.8571	1.1556	1.0565	0.8707
20	0.748	0.9828	0.9261	0.7831
25	0.6743	0.8638	0.8334	0.719
30	0.62	0.776	0.7626	0.6689
40	0.5434	0.653	0.6616	0.5944

Table-13 Effect of orthotropic ratio on M_{nn} of a skew plate

	Skew angle			
E1/E2	**90**	**75**	**60**	**45**
3	5.8982	5.1678	5.0078	4.949
5	4.8873	4.6514	5.0474	5.5618
15	3.1757	3.8329	5.5513	7.3794
20	2.8576	3.6885	5.7741	7.9738
25	2.6406	3.595	5.9712	8.4703
30	2.4791	3.5298	6.1467	8.8997
40	2.247	3.4456	6.4148	9.6209

Table-14 Effect of orthotropic ratio on M_{ns} of a skew plate

	Skew angle			
E1/E2	**90**	**75**	**60**	**45**
3	2.5744	1.8917	2.1328	1.7467
5	1.8099	2.414	3.1765	2.8161
15	0.8571	4.0177	5.7145	5.4489
20	0.748	4.4545	6.4538	6.2199
25	0.6743	4.8001	7.0487	6.8432
30	0.62	5.0879	7.5455	7.3701
40	0.5434	5.5531	8.2234	8.2341

Table-15 Effect of orthotropic ratio on $\bar{\sigma}_{xx}$ of a skew plate

	Skew angle			
E1/E2	**90**	**75**	**60**	**45**
3	6.3801	6.5417	6.7813	6.497
5	7.8849	8.0694	8.304	8.0166
15	11.368	11.6492	11.8632	11.7269
20	12.3105	12.5939	12.8034	12.7577
25	13.0423	13.3172	13.5223	13.5536
30	13.6374	13.8997	14.0995	14.1947
40	14.5646	14.7991	15.3627	15.1787

Table-16 Effect of orthotropic ratio on $\bar{\sigma}_{yy}$ of a skew plate

	Skew angle			
E1/E2	**90**	**75**	**60**	**45**
3	3.8303	3.9576	3.8448	5.1127
5	3.6747	3.7089	3.5353	4.0589
15	3.314	3.1702	2.9375	3.0259
20	3.2069	3.0275	2.8012	2.8506
25	3.118	2.9156	2.693	2.7156
30	3.0411	2.8233	2.604	2.6066
40	2.9116	2.6755	2.481	2.4378

Table-17 Effect of orthotropic ratio on $\bar{\sigma}_{xy}$ of a skew plate

Skew angle				
E1/E2	**90**	**75**	**60**	**45**
3	0.9289	1.2979	2.0776	2.6183
5	0.7868	1.056	1.6404	2.0823
15	0.5488	0.6301	0.9607	1.2876
20	0.5034	0.5461	0.8309	1.1316
25	0.4722	0.4884	0.7413	1.0219
30	0.4487	0.4457	0.6746	0.9401
40	0.4142	0.389	0.5822	0.8329

Table-18 Effect of orthotropic ratio on $\overline{\sigma}_{xz}$ of a skew plate

Skew angle				
E1/E2	**90**	**75**	**60**	**45**
3	13.8162	8.043	4.8038	3.3454
5	10.4717	6.4033	3.773	2.6299
15	4.7581	3.4504	2.187	1.5378
20	3.7847	2.8741	1.8757	1.3194
25	3.1799	2.4849	1.6599	1.1689
30	2.7623	2.2021	1.498	1.0574
40	2.2169	1.8147	1.2388	0.9009

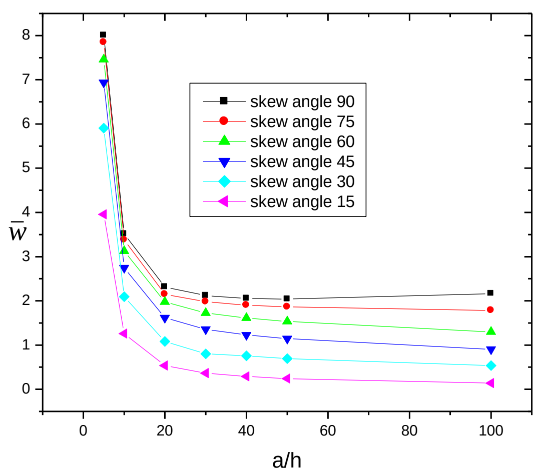

Fig. 3 Effect of span to thickness ratio for deflection $\overline{w}$ of a skew plate

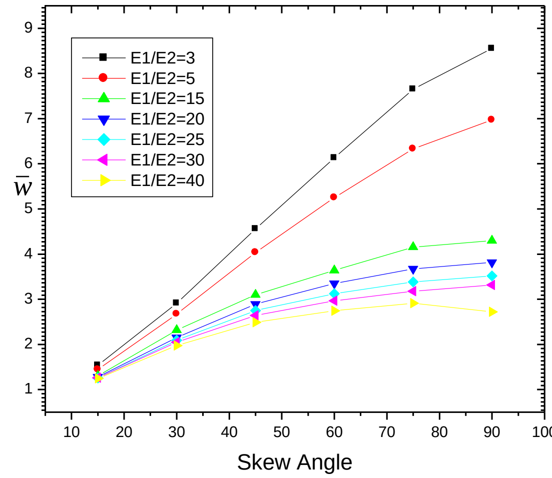

Fig. 4 Effect of skew angle with variation of orthotropic ratio for deflection $\overline{w}$ of a skew plate (a/h=1/10)

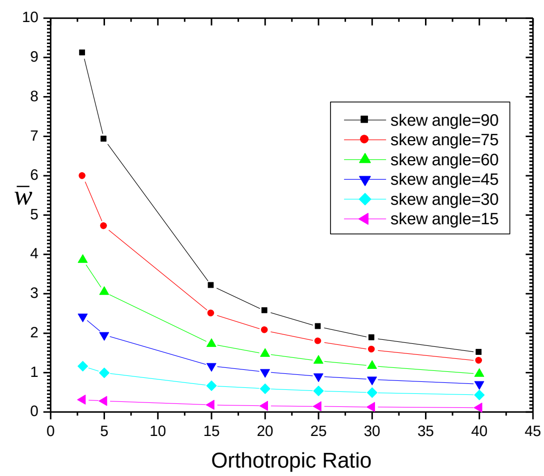

Fig. 5 Effect of orthotropic ratio for deflection $\overline{w}$ of a skew plate (a/h=1/100)

Other numerical examples have been also considered and the results obtained for different values of span to thickness ratio is shown in Table-1 to Table 9 and for different orthotropic ratio is shown in Table-10 to Table-18.

Fig. 3 to Fig. 7 shows the effect of skew angle on deflection. It is observed that as skew angle increases, the deflection decreases. The effect of span to thickness ratio seems to be negligible after a/h=40. The effect of orthotropy ratio seems to be negligible after E1/E2=30.

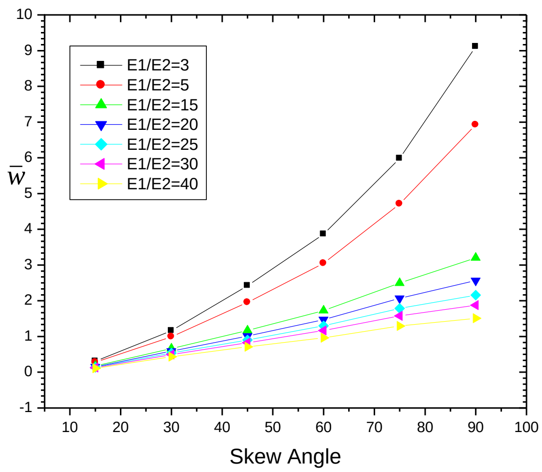

Fig. 6 Effect of skew angle with variation of orthotropic ratio for deflection $\bar{w}$ of a skew plate (a/h=1/100)

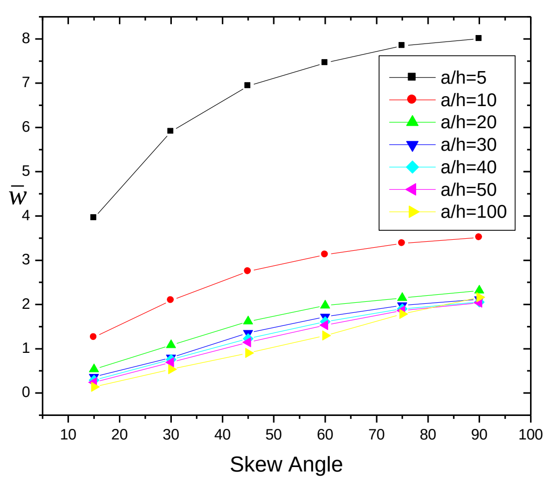

Fig. 7 Effect of thickness along skew angle for deflection $\bar{w}$ of skew plate

V. CONCLUSION

The present study shows that the proposed RBFs are capable to accurately predict the flexure behavior of skew plates subjected to concentrated load. Effect of skewness on deflection, moments and stresses is obtained. It is found that all the parameters decrease as skewness increases. Effect is more prominent for thick plates as compared to thick plate.

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