The effects of cross-diffusion on unsteady natural convection stratified fluid past vertical plate with variable flux

Dr Madhava Reddy Ch

Department of Mathematics, Visvodaya Engineering College, Kavali, Andhra Pradesh, India

***Abstract*— In this paper, the influence over Dufour and Soret effects and viscous dissipation effects on unsteady natural convective doubly stratified flow of an electrically conducting fluid in a Brinkman porous medium has been analyzed along with vertical plate of variable surface heat and mass flux. By using the non-dimensional transformations, the governing differential equations are expressed into a set of non-linear coupled differential equations along with the suitable boundary conditions. The unconditionally stable implicit finite difference scheme of Crank-Nicolson type has been used to solve the reduced unsteady nonlinear boundary value problem. Numerical results for velocity, temperature and concentration profiles are analyzed in detail and depicted graphically for different physical parameter values and compared with available results in literature with good agreement.**

***Index Terms*—Brinkman porous medium, Crank-Nicolson method, Double stratification, Heat generation and Chemical reaction, Radiation, Unsteady.**

I. INTRODUCTION

In a porous medium, the natural convective transport has wide significance in heat and mass transfer problems due to their increasing applications in different areas such as scientific, biological, various engineering and industrial technology areas. In the monographs by Ingham and Pop [1], a detailed discussion of these applications are available. Much of the work related to porous media transport phenomena has been presented in the handbook of porous media by Vafai [2]. The solutal and thermal stratification of fluid arises due to concentration differences, temperature variations and in the form of the existence of different type fluids. Similarly, it is favorable in particular engineering areas such as thermal energy storage systems, heat rejection into the environment and heat transfer from thermal sources.

With this inspiration, most of the researchers have begun their studies in the area of doubly stratified convective flows. By Introducing the thermal stratification in the energy equation provides more realistic scenario than the conventional models as it produces a coupling effect on both the temperature and velocity fields. Nakayama and Koyama [3] discussed the thermal stratification effect, whereas Srinivasacharya and Upendar [4], Srinivasacharya and Ramreddy [5] and Murthy *et al.* [6] analysed the double stratification effects on natural convection flow.

An exact solution provided by Deka and Paul [7] for unsteady natural convection flow with thermal stratification effect over a vertical cylinder. An influence of electrophoresis and chemical reaction on unsteady convection doubly stratified flow past a vertical plate in the presence of a heat source/sink is presented by Ganesan and Suganthi [8]. Soret and Dufour effects of free convection heat and mass transfer in the presence of doubly stratified Darcy porous medium, analyzed and presented by Lakshmi Narayana and Murthy [9]. Recently Srinivasacharya and Upendar [4] analyzed and presented, mixed convection in MHD doubly stratified micropolar fluid.

The mass fluxes caused by temperature gradients and energy flux caused by a concentration gradient are popularly known as thermal-diffusion (Soret) and diffusion-thermo (Dufour) effects respectively. A broad investigation of these effects is considered theoretically and experimentally in both gas and fluids. These are found to be very important in different areas like hydrology, petrology and Geosciences (eg., see Eckert and Drake [10] and references cited therein). Dursunkaya and Worek [11] studied about cross-diffusion effects in unsteady and steady free convection, whereas Kafoussias and Williams [12], Ahmed [13] and Srinivasacharya and RamReddy [14] discussed the steady convective flows in Newtonain and non-Newtoanian fluids with cross diffusion effects. Nield and Kuznetsov [15] investigated and discussed the cross-diffusion in nanofluids, with the aim of making a detailed comparison with regular cross diffusion effects. The cross-diffusion impacts are peculiar to nanofluids, and at the same time investigating the interaction between these effects when the base fluid of the nanofluid is itself a binary fluid like salty water. The Hall current, thermal diffusion, and heat sources are considered and discussed by Ahmed and Barua[16] to investigate an unsteady natural convective flow in a porous medium. Recently, Loganathan *et al.* [17] observed that the local heat transfer rate enhances with an enhancement in Dufour and Soret numbers for both air and water by analyzing the problem of the cross-diffusion effects on the unsteady natural convection and Srinivasacharya and Surender [18] studied and presented the effect of double stratification along with Dufour and Soret type diffusivities on mixed convection boundary layer flow of a nanofluid past a vertical plate in a porous medium.

Studying the effects of cross-diffusion along with double stratification in porous medium on unsteady natural convection fluid past vertical plate with variable surface heat and mass flux is one interesting area. Ganesan and Rani [19] presented the study of the unsteady free convection on a vertical cylinder with variable heat and mass flux in detail. The effects of mass transfer on the transient free convection flow of a dissipative fluid along a semi infinite vertical plate with constant heat flux presented by Gokhale and Samman [20]. The problem for transient free convection with mass transfer on a vertical plate with constant heat flux solved by Soundalgekar and Ganesan [21] using an implicit finite difference scheme. Ganesan and Palani [22] have studied and analyzed numerically for transient free convection flow past a semi-infinite inclined plate with variable surface heat and mass flux. Both Kabeir *et al* [23] and Sacheti *et al*. [24] has analyzed unsteady MHD convection over a vertical plate in a fluid saturated porous medium with constant heat flux. Shanker and Kishan [25] presented the effect of mass transfer on the MHD flow past an impulsively started vertical plate with variable temperature but constant heat flux. Recently S. Nadeem *et al* [26] analyzed numerical study of MHD boundary layer flow of a Maxwell fluid past a stretching sheet in the presence of nanoparticles with variable surface heat and mass flux.

The aim of the present study is to investigate the effects of cross-diffusion in the Brinkman porous medium on unsteady natural convection doubly stratified fluid past vertical plate with variable surface heat and mass flux using an implicit finite difference method of Crank-Nicolson type. The influence of velocity, temperature and concentration are analyzed and exhibited graphically for variations in governing parameters in detail.

II. MATHEMATICAL FORMULATION

Consider a problem of two-dimensional, unsteady, laminar free convection flow past a vertical plate of a viscous incompressible doubly stratified fluid saturated porous medium in an electrically conducting fluid with variable surface heat and mass flux under the influence of magnetic field is formulated mathematically in this section by taking into account the effect of viscous dissipation. The coordinate system is chosen to represent the *x*-axis along the vertical plate and the *y*-axis as upward normal to the plate. The fluid and the plate are assumed to be at the constant temperature and constant concentration initially at $t'=0$, whereas the surface heat and mass flux aer supplied to the fluid from the plate at a rate of $q_w(x) = x^m$ and $q_w^*(x) = x^n$ respectively and both are maintained at the same level for all time $t'>0$. The ambient medium is assumed to be vertically linearly stratified with respect to both temperature and concentration in the form $T_\infty(x) = T_{\infty,0}+Ax$ and $C_\infty(x) = C_{\infty,0}+Bx$ respectively. Then, under these assumptions, the governing boundary layer equations of mass, momentum, energy and species concentration for free convection flows with Boussinesq's and

Brinkman porous medium approximation are as follows

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{1}{\varepsilon}\frac{\partial u}{\partial t'}+\frac{u}{\varepsilon^2}\frac{\partial u}{\partial x}+\frac{v}{\varepsilon^2}\frac{\partial u}{\partial y}= \frac{\upsilon}{\varepsilon}\frac{\partial^2 u}{\partial y^2}-\frac{\mu}{k}u-\frac{\sigma B_0^{\,2}}{\rho}\frac{u}{\varepsilon}+ g[\beta_T(T'-T_\infty(x))+\beta_C(C'-C_\infty(x))] \tag{2}$$

$$\frac{\partial T'}{\partial t'}+u\frac{\partial T'}{\partial x}+v\frac{\partial T'}{\partial y}=\alpha\frac{\partial^2 T'}{\partial y^2}+ \frac{\mu}{\rho Cp}\left(\frac{\partial u}{\partial y}\right)^2+\frac{DK_T}{CsCp}\frac{\partial^2 C'}{\partial y^2} \tag{3}$$

$$\frac{\partial C'}{\partial t'}+u\frac{\partial C'}{\partial x}+v\frac{\partial C'}{\partial y}=D\frac{\partial^2 C'}{\partial y^2}+\frac{DK_T}{T_m}\frac{\partial^2 T'}{\partial y^2} \tag{4}$$

Where u and v are Darcy velocity components along the x and y directions respectively, ρ is the density, g is the acceleration due to gravity, C_p is the specific heat, μ is the coefficient of viscosity, σ is the electrical conductivity, k is the permeability, ε is the porosity, T' is the temperature, C' is the concentration, β_T and β_C are the coefficients of thermal and solutal expansions, α is the thermal diffusivity, D is the mass diffusivity and T_m is the mean fluid temperature.

The boundary conditions are

$$t'\le 0: u(x,y,t')=0,\ v(x,y,t')=0,$$
$$T'(x,y,t')=T_\infty(x),\ C'(x,y,t')=C_\infty(x)$$
$$t'>0: u(x,0,t')=0,\ v(x,0,t')=0,$$
$$\frac{\partial T'(x,y,t')}{\partial y}_{at\, y=0}=-\frac{q_w(x)}{k},$$
$$\frac{\partial C'(x,y,t')}{\partial y}_{at\, y=0}=-\frac{q_w^*(x)}{D}$$
$$t'>0: u(0,y,t')=0,\quad v(0,y,t')=0,$$
$$T'(0,y,t')=T_{\infty,0},\quad C'(0,y,t')=C_{\infty,0}$$
$$t'>0: u(x,\infty,t')\to 0,$$
$$T'(x,\infty,t')\to T_\infty(x),$$
$$C'(x,\infty,t')\to C_\infty(x) \tag{5}$$

The physical parameters local skin friction, Nusselt number and Sherwood number are obtained as

$$\tau_x=-\mu\left(\frac{\partial u}{\partial y}\right)_{y=0},$$
$$Nu_x=-\frac{x\left(\frac{\partial T'}{\partial y}\right)_{y=0}}{T'_{y=0}}, \tag{6}$$
$$Sh_x=-\frac{x\left(\frac{\partial C'}{\partial y}\right)_{y=0}}{C'_{y=0}}$$

and average skin friction, Nusselt number and Sherwood number are obtained as

$$\overline{\tau}_L=-\frac{1}{L}\int_0^L\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}dx,$$
$$\overline{Nu_L}=-\int_0^L\frac{\left(\frac{\partial T'}{\partial y}\right)_{y=0}}{T'_{y=0}}dx, \tag{7}$$
$$\overline{Sh_L}=-\int_0^L\frac{\left(\frac{\partial C'}{\partial y}\right)_{y=0}}{C'_{y=0}}dx$$

Using the following non-dimensional variables

$$X=\frac{x}{L},\ Y=\frac{y}{L}Gr^{1/4},\ U=\frac{uL}{\upsilon}Gr^{-1/2},$$
$$V=\frac{vL}{\upsilon}Gr^{-1/4},\ t=\frac{t'\upsilon}{L^2}Gr^{1/2},$$
$$T=\frac{(T'-T_{\infty,0}-Ax)Gr^{1/4}}{q_w(L)L/k}, \tag{8}$$
$$C=\frac{(C'-C_{\infty,0}-Bx)Gr^{1/4}}{q_w^*(L)L/D}$$

in to the Eqs. (1) to (4), we obtain the following system of non-dimensional partial differential equations

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{9}$$

$$\frac{1}{\varepsilon}\frac{\partial U}{\partial t}+\frac{U}{\varepsilon^2}\frac{\partial U}{\partial X}+\frac{V}{\varepsilon^2}\frac{\partial U}{\partial Y}=\frac{1}{\varepsilon}\frac{\partial^2 U}{\partial Y^2}-$$

$$\frac{1}{DaGr^{1/2}}U-\frac{1}{\varepsilon}\frac{M}{Gr^{1/2}}U+Gr^{-1/4}T+Gr^{-1/4}NC \qquad (10)$$

$$\frac{\partial T}{\partial t}+U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y}=\frac{1}{\Pr}\frac{\partial^2 T}{\partial Y^2}-$$

$$Gr^{1/4}\varepsilon_1 U+Gr^{1/4}Ec\left(\frac{\partial U}{\partial Y}\right)^2+Df\frac{\partial^2 C}{\partial Y^2} \qquad (11)$$

$$\frac{\partial C}{\partial t}+U\frac{\partial C}{\partial X}+V\frac{\partial C}{\partial Y}=\frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2}-$$

$$Gr^{1/4}\varepsilon_2 U+Sr\frac{\partial^2 T}{\partial Y^2} \qquad (12)$$

along with the corresponding initial and boundary conditions in a non-dimensional form are

$$t\le 0: U(X,Y,t)=0,\ V(X,Y,t)=0,$$
$$T(X,Y,t)=0,\ C(X,Y,t)=0$$
$$t>0: U(X,0,t)=0,\ V(X,0,t)=0,$$
$$\frac{\partial T(X,Y,t)}{\partial Y}_{at\,Y=0}=-X^m,$$
$$\frac{\partial C(X,Y,t)}{\partial Y}_{at\,Y=0}=-X^n$$
$$t>0: U(0,Y,t)=0,\ V(0,Y,t)=0,$$
$$T(0,Y,t)=0,\ C(0,Y,t)=0$$
$$t>0: U(X,\infty,t)\to 0,$$
$$T(X,\infty,t)\to 0,\ C(X,\infty,t)\to 0 \qquad (13)$$

where $Gr=g\beta_T L^4 q_w(L)/k\upsilon^2$ is the thermal Grashof number, $Gc=g\beta_C L^4 q_w^*(L)/D\upsilon^2$ is the solutal Grashof number, $N=Gc/Gr$ is the buoyancy ratio, $Da=k\upsilon/(\mu L^2)$ is the Darcy number, $M=\sigma B_0^2 L^2/(\rho\upsilon)$ is the magnetic parameter, $EC=n_0^2/(Cpq_w(L))$ is the Eckert number, $n_0=(k\mu\upsilon/(\rho L^3))^{1/2}Gr^{1/2}$, $\alpha=k/(\rho Cp)$ is the thermal diffusitivity, $Pr=\upsilon/\alpha$ and $Sc=\upsilon/D$ are the Prandtl and Schmidt numbers, $Sr=D^2K_T q_w(L)/(T_m\,\upsilon k q_w^*(L))$ and $Df=kK_T q_w^*(L)/CsCp\upsilon q_w(L)$ are the Soret and Dufour numbers, $\varepsilon_1=Ak/q_w(L)$ and $\varepsilon_2=BD/q_w^*(L)$ are the thermal and solutal stratification parameters.

The non-dimensional forms of physical parameters of interest local skin friction, Nusselt number and Sherwood number are obtained as

$$\tau_X=Gr^{3/4}\left(\frac{\partial U}{\partial Y}\right)_{Y=0},$$
$$Nu_X=-\frac{X\left(\frac{\partial T}{\partial Y}\right)_{Y=0}}{T_{Y=0}}, \qquad (14)$$
$$Sh_X=-\frac{X\left(\frac{\partial C}{\partial Y}\right)_{Y=0}}{C_{Y=0}}$$

The non-dimensional forms of average skin friction, Nusselt number and Sherwood number are obtained as

$$\bar{\tau}=Gr^{3/4}\int_0^1\left(\frac{\partial U}{\partial Y}\right)_{Y=0}dX,$$
$$\overline{Nu}=-\int_0^1\frac{\left(\frac{\partial T}{\partial Y}\right)_{Y=0}}{T_{Y=0}}dX, \qquad (15)$$
$$\overline{Sh}=-\int_0^1\frac{\left(\frac{\partial C}{\partial Y}\right)_{Y=0}}{C_{Y=0}}dX$$

III. NUMERICAL TECHNIQUE

In order to solve the unsteady, non-linear, coupled partial differential equations (9) - (12), under the conditions (13), an implicit finite difference scheme of Crank-Nicolson type has been employed.

The mesh sizes have been fixed as ΔX = 0.05, ΔY = 0.25 with time step Δt = 0.01. The computations are done first by reducing the spatial mesh sizes by 50% in one direction, and later in both directions by 50% and then the results are compared. It is observed in both the cases, that the results differ just in the fifth decimal place. Thus, the choice of the mesh sizes is by all accounts suitable. The scheme Crank-Nicolson type is unconditionally stable with the local truncation error $O(\Delta t^2+\Delta Y^2+\Delta X)$ and this error tends to zero as Δt, ΔY and ΔX tend to zero. Subsequently the scheme is compatible, stable and ensures the convergence.

IV. RESULTS AND DISCUSSION

In order to verify the accuracy of the present numerical results, the steady state obtained at X = 1.0 and the present numerical solutions compared with available similar solutions of Ganesan *et al*. [28] and Vasu *et al.* [29] in the literature and confirmed that, the present results following the trend observed in the literature with great agreement. The effects of various parameters on non-dimensional velocity, temperature and concentration along with the physical quantities are analyzed and plotted with fixed values $Pr=0.71$, $Gr=5.0$, $Ec=1.0$, $Sc=0.22$, $N=1.0$.

Figs.1. (a) to 1. (c) represent profiles of non-dimensional velocity, temperature and concentration for various estimations of cross-diffusion parameters for $Da=0.1$, $\varepsilon=0.6$, $\varepsilon_1=0.1$, $\varepsilon_2=0.5$, $M=1.0$, $m=0.5$, $n=0.5$.

Figs.1. (a) to 1. (c) show velocity, temperature and concentration profiles with an upgrade in Dufour number and simultaneous decrease in Soret number for both the cases $t=1.5$ and steady state. Graphs for all local physical parameters are drawn for both the cases $t=1.5$ and t at steady state.

The variations in non-dimensional velocity, temperature and concentration for various values of thermal and solutal stratification parameters are investigated and plotted with $Da=0.1$, $\varepsilon=0.6$, $Sr=2.0$, $Df=0.03$, $M=1.0$, $m=0.5$, $n=0$ in Figs. 2. (a) to 2. (c).

From Fig. 2. (a), it has been observed that, the velocity decreases with an increase in the value of both thermal and solutal stratification parameters. It can be observed from Fig. 2. (b) that, the temperature of the fluid in the medium has increased with an increase in the value of the solutal stratification parameter and decreases with an increase in the value of thermal stratification parameter. From Fig. 2. (c), it can be found that, the concentration of the fluid is expanded by increasing the estimations of the both thermal and solutal stratification parameters. Furthermore, it is also observed that, the value of time t at steady state increases with an increase in the value of thermal stratification parameter.

From Fig. 3. (a) to Fig. 3. (c) demonstrated the impacts of exponents in the power law variation of the heat mass flux m and n values on non-dimensional velocity, temperature and concentration profiles with $\varepsilon=0.6$, $\varepsilon_1=0.1$, $\varepsilon_2=0.5$, $Sr=2.0$, $Df=0.03$, $Da=0.1$, $M=1.0$.

Fig. 3. (a) present the velocity profile and it is observed that, the velocity of the fluid is diminishing as the exponents either m or n values increasing. From Fig. 3. (b), it can be observed that, the temperature of the fluid in the medium is diminished with an increase in the value of exponent m and expands with an increase in the value of the exponent n. From Fig. 3. (c), it can be found that, the concentration of the fluid is slightly increased by increasing the estimations of the exponent m and decreasing with an increase in the value of exponent n. Further, to achieve the steady state values, required more time in both the cases of more values of exponents m and n.

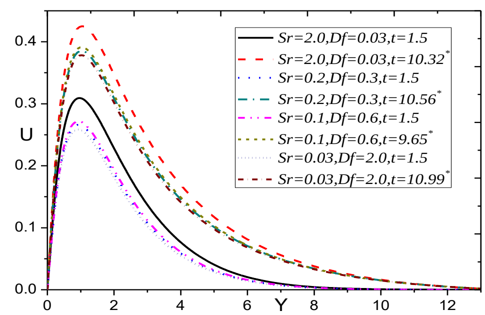
V. CONCLUSION

This paper analyses the problem of unsteady MHD free convective flow past a vertical plate with variable surface heat flux and mass flux embedded with doubly stratified porous medium in the presence of porosity, Soret and Dufour effects. The resulting non-dimensional governing coupled partial differential equations are solved numerically by using the unconditionally stable implicit finite difference method of Crank-Nicolson type. However, the comparison of present results with Ganesan *et al*. [28] and Vasu *et al.* [29] in the literature is performed and observed excellent agreement. The important findings of this study are listed as follows:

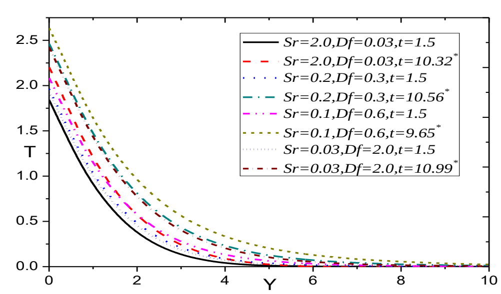
a) The decrease in the velocity and concentration profiles and an increase in the temperature profile with an increase in the value of the Dufour number (or simultaneous decrease in the Soret number) except at $Sr=0.1$ and $Df=0.6$.With the influence of Soret and Dufour parameters,

b) For increasing value of the thermal stratification parameter, velocity, temperature, depict the reverse trend while the depicts the same trend. In case of a rise in value of the solutal stratification parameter, velocity, concentration, depict the reverse trend while temperature, depicts the same trend.

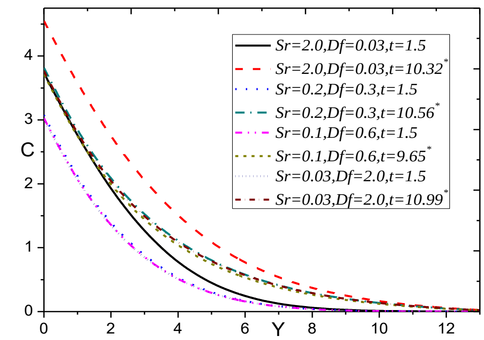
c) The velocity, temperature decreases and concentration, increase with an increase in the exponent *m*. But except temperature, velocity, and concentration, decrease with the rise in the exponent *n*. Further, to reach the steady state values, the required time is more when the exponents *m*, *n* are more.



(a) Velocity profiles at $X=1$.

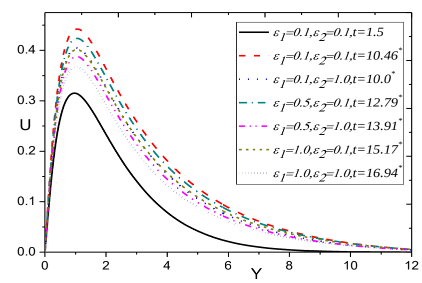


(b) Temperature profiles at $X=1$.

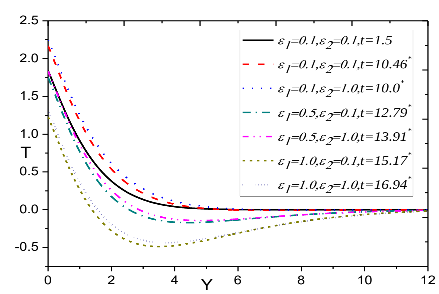


(c) Concentration profiles at $X=1$.

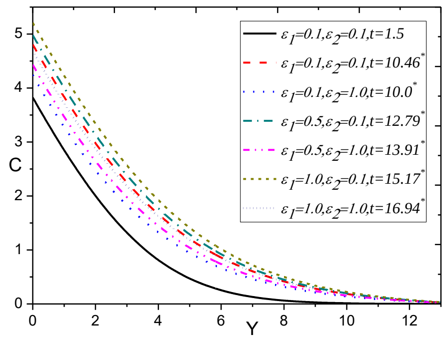
Fig. 1. Profiles with the effect of cross-diffusion for $Da=0.1$, $\varepsilon=0.6$, $\varepsilon_1=0.1$, $\varepsilon_2=0.5$, $M=1.0$, $m=0.5$, $n=0.5$. (*-steady state)



(a) Velocity profiles at $X=1$.

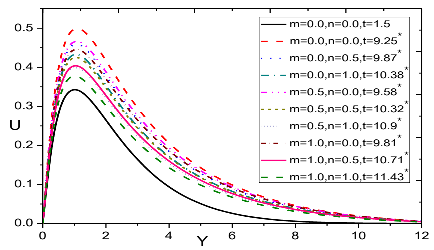


(b) Temperature profiles at $X=1$.

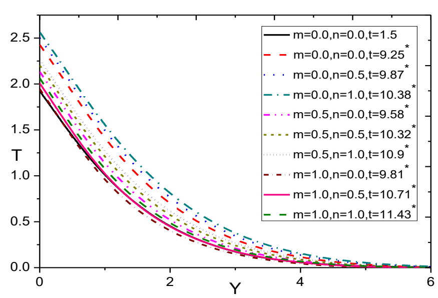


(c) Concentration profiles at $X=1$.

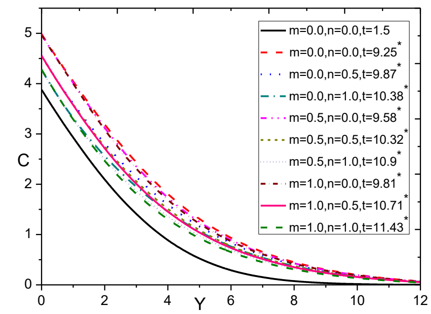
Fig. 2.Profiles with the effect of double stratification for $Da=0.1$, $\varepsilon=0.6$, $Sr=2.0$, $Df=0.03$, $M=1.0$, $m=0.5$, $n=0.5$. (*-steady state)



(a) Velocity profiles at $X=1$.



(b) Temperature profiles at $X=1$.



(c) Concentration profiles at $X=1$.

Fig. 3. Profiles with the effect of exponents m, n for $\varepsilon=0.6$, $\varepsilon_1=0.1$, $\varepsilon_2=0.5$, $Sr=2.0$, $Df=0.03$, $Da=0.1$, $M=1.0$. (*-steady state)

REFERENCES

[1] Ingham, D. B., Pop, I., Transport Phenomena in Porous Media II, *Pergamon, Oxford*, 2002.

[2] Vafai, K. (Ed.).: Handbook of Porous Media, 2nd edition, *CRC Press, Boca Raton*, 2005.

[3] A. Nakayama and H. Koyama, Effect of thermal stratification on free convection within a porous medium. *J. Thermo physics and Heat Transfer*, 1(3), 282-285, 1987.

[4] D. Srinivasacharya and Upendar Mendu, Mixed convection in MHD doubly stratified micropolar fluid, *J Braz. Soc. Mech. Sci. Eng.*, 37, 431-440, 2015.

[5] D. Srinivasacharya and Ch. RamReddy, Free convective heat and mass transfer in a doubly stratified non-Darcy micropolar fluid. *Korean J. Chem. Eng.*, 28(9), 1824-1832, 2011.

[6] P.V.S.N. Murthy, D. Srinivasacharya and P.V.S.S.S.R. Krishna, Effect of double stratification on free convection in a Darcian porous medium. *J. Heat Transfer* 126(2), 297-300, 2004.

[7] R.K. Deka and A.Paul, Transient free convection flow past infinite vertical cylinder with thermal stratification, *J. of Mechanical Science and Engineering*, 26(8), 2229-2237, 2012.

[8] P. Ganesan and R.K. Suganthi, Free convective flow over a vertical plate in a doubly stratified medium with electrophoresis, heat source/sink and chemical reaction effects. *Korean J. Chem. Eng.*, 30(4), 813-822, 2013.

[9] Lakshmi Narayana PA, Murthy PVSN, Soret and Dufour effects on free convection heat and mass transfer in a doubly stratified Darcy porous medium, *ASME J Heat Transf*, 128, 1204–1212, 2006.

[10] E.R.G. Eckeret and R.M. Drake, Analysis of heat and mass transfer. *McGraw Hill, Newyork*, 1972.

[11] Z. Dursunkaya and W.M. Worek, Diffusion-thermo and thermal-diffusion effects in transient and steady natural convection from a vertical surface. *Int. J. Heat Mass Transfer*. 35, 2060–2065, 1992.

[12] N.G. Kafoussias and N.G. Williams, Thermal-diffusion and diffusion-thermo effects on mixed free-forced convective and mass transfer boundary layer flow with temperature dependent viscosity. *Int.J. Engng. Sci.* 33, 1369–1384, 1995.

[13] A.A. Ahmed, Similarity solution in MHD effects of thermal diffusion and diffusion thermo on free convective heat and mass transfer over a stretching surface considering suction or injection. *Commun Nonlinear Sci. Numer. Simulat*, 14, 2202–2214, 2009.

[14] D. Srinivasacharya and Ch. RamReddy, Mixed convection heat and mass transfer in a non-Darcy micropolar fluid with Soret and Dufour effects. *Nonlinear Analysis: Modelling and Control*, 16(1), 100–115, 2011.

[15] D. A. Nield, and A. V. Kuznetsov, The Cheng–Minkowycz problem for the double-diffusive natural convective boundary layer flow in a porous medium saturated by a nanofluid, *International Journal of Heat and Mass Transfer*, 54, 374-378, 2011.

[16] N.K. Ahmed and H.D.P. Barua, Unsteady MHD free convective flow past a vertical porous plate immersed in a porous medium with Hall current, thermal diffusion and heat source. *Int. J. of Engineering, Science and Technology*, 2(6), 59–74, 2010.

[17] P. Loganathan, D. Iranian and P. Ganesan, Dufour and Soret effects on unsteady free convective flow past a semi-infinite vertical plate with variable viscousity and thermal conductivity. *Int. J. Engineering and Technology*, 7(1), 303-316, 2015.

[18] D. Srinivasacharya and Ontela Surender, Effect of double stratification on mixed convection boundary layer flow of a nanofluid past a vertical plate in a porous medium, Applied Nanoscience, 5, 29-38, 2015.

[19] Ganesan, P. and H.P. Rani, Unsteady free convection MHD flow past a vertical cylinder with mass transfer. *Int. J. Ther. Sci.*, 39, 265-272, 2000.

[20] Gokhale, M.Y. and F.M. Al Samman, Effects of mass transfer on the transient free convection flow of a dissipative fluid along a semi-infinite vertical plate with constant heat flux. *Int. J. Heat and MassTransfer*, 46, 6, 999-1011, 2003.

[21] V.M. Soundalgekar, P. Ganesan, Finite difference analysis of transient free convection with mass transfer on an isothermal vertical flat plate, *Int. J. Eng. Sci*. 19, 757–770, 1981.

[22] P. Ganesan, G. Palani, Convective flow over an inclined plate with variable heat and mass flux, *in Proceedings of the Fourth ISHMT/ASME Heat and Mass transfer Conference and Fifteenth National Heat and Mass Transfer Conference, Pune, India*, 323–329, January 12–14, 2000.

[23] S.M.M. EL-Kabeir, A.M. Rashad, R.S.R. Gorla, Unsteady MHD combined convection over a moving vertical sheet in a fluid saturated porous medium with uniform surface heat flux, *Math.Comput. Mod.*, 46, 384–397, 2007.

[24] N.C. Sacheti, P. Chandran, A.K. Singh, An exact solution for unsteady magnetohydrodynamics free convection flow with constant heat flux, *Int. Commun. Heat Mass Transfer* , 21, 131–142, 1994.

[25] B. Shanker, N. Kishan, The effects of mass transfer on the MHD flow past an impulsively started infinite vertical plate with variable temperature or constant heat flux, *J. Energy, Heat Mass Transfer*, 19, 273–278, 1997.

[26] S. Nadeem, Rizwan Ul Haq, Z.H. Khan, Numerical study of MHD boundary layer flow of a Maxwell fluid past a stretching sheet in the presence of nanoparticles, *Journal of the Taiwan Institute of Chemical Engineers,* 45(1), 121–126, 2014.

[27] B. Carnahan, H.A. Luther, J.O. Wilkes, Applied Numerical Methods, *John Wiley and Sons,New York*, 1969.

[28] P. Ganesan, G. Palani, Finite difference analysis of unsteady natural convection MHD flow past an inclined plate with variable surface heat and mass flux. *International Journal of Heat and Mass Transfer,* 47, 4449–4457, 2004.

[29] B.Vasu, V. Ramachandra Prasad and N. Bhaskar Reddy, Radiation and Mass Transfer Effects on Transient Free Convection Flow of a Dissipative Fluid past Semi-Infinite Vertical Plate with Uniform Heat and Mass Flux. *Journal of Applied Fluid Mechanics,* 4(1), 15-26, 2011.