The Double Dispersion Effects on Unsteady Natural Convection Stratified Fluid past Vertical Plate with Flux

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Abstract- **The influence of double dispersion effects of unsteady natural convection along a vertical plate embedded in a Brinkman porous medium saturated with an electrically conducting fluid. In addition, the double stratification is considered and the variable surface heat and mass flux conditions are incorporated. The governing non-linear differential equations are converted into a set of non-dimensional non-linear coupled partial differential equations along with the suitable boundary conditions by using the non-dimensional transformations and then used implicit finite difference scheme named as Crank-Nicolson type to solve the reduced non-linear system of differential equations. The variations in velocity, temperature and concentration profiles are analyzed in-detail and depicted graphically for different physical parameters involved in the present investigation, after comparison of available results of the similar type in the literature with good agreement.**

Index Terms- **Brinkman porous medium, Crank-Nicolson method, Double stratification, Dufour and Soret effects, Termal and Solutal dispersion, Unsteady flow.**

I. INTRODUCTION

In a porous medium several researchers investigated on the natural convective heat and mass transfer flow due to the large amount of applications in science and technology such as heat exchangers, chemical reactors, cooling of electronic items, thermal engineering, geophysics, biology, ground water technology and geothermal systems etc. Furthermore, much of the work related to porous media transport phenomena has been presented in the handbook of porous media by Vafai [1] and a detailed discussion of these type applications is available in the monographs by Ingham and Pop [2]. Initially the study of porous media started with the simple Darcy model afterwards slowly extended to Darcy-Brinkman and Darcy-Brinkman–Forchheimer models too. The forced convection in a channel with a porous medium under the influence of variable porosity and Brinkman friction is investigated by Poulikakos and Renken [3]. And Nield *et al*. [4] used the Darcy-Brinkman–Forchheimer model to study the forced convection and fully developed flow in a channel filled with a porous medium. The effects of thermal and solutal stratifications on free convection in both Darcian and non-Darcian porous medium along with different geometry investigated by Murthy et al. [5], Lakshmi Narayana and Murthy [6] and Srinivasacharya and RamReddy [7].

The thermal stratification of the fluid arises due to temperature variations and in the same way solutal stratification of the fluid arises due to concentration differences. The case of the existence of different type fluids is also another cause for these double stratifications. And hence, it is favorable in specialized engineering areas such as thermal energy storage systems, heat transfer from thermal sources and heat rejection into the environment. With this motivation, most of the researchers have started their studies in the area of doubly stratified convective flows. By adding the thermal stratification in the energy equation same way solutal stratification in the concentration equation provides more realistic scenario than the conventional models as these additions produce a coupling effect on both the temperature concentration and velocity fields. Nakayama and Koyama [8] studied the thermal stratification effect, whereas an exact solution provided by Deka and Paul [9] for unsteady natural convection flow with thermal stratification effect over a vertical cylinder. An influence of electrophoresis and chemical reaction on unsteady convection doubly stratified flow past a vertical plate in the presence of a heat source/sink is analyzed and presented by Ganesan and Suganthi [10]. Soret and Dufour effects of free convection heat and mass transfer in the presence of doubly(thermal and solutal) stratified Darcy porous medium, analyzed and presented by Lakshmi Narayana and Murthy [11]. Recently Srinivasacharya and Upendar [12] analyzed and presented, mixed convection in MHD doubly (thermal and solutal) stratified micropolar fluid.

The concept of the thermal and solutal dispersion effects becoming prevalent in the porous media flow region where the inertia effect plays an important role (refer Nield and Bejan [13] and citations therein). When the fluid travels through convoluted ways in packed beds, mixing and recirculation of local fluid streams happens. This hydrodynamic mixing of fluid at pore level causes the double dispersion effects in porous media. These sorts of issues becomes more considerable for fast and moderate flows. Basically the dispersion theory related to miscible displacement and solute spreading on porous media. These areas have significant interest in secondary and tertiary oil recovery operations and contamination control of water resources engineering. With this consideration, El-Hakiem [14] analyzed the influence of thermal dispersion on Darcy free convection from a vertical plate in a non-Newtonian fluid saturated porous media. And the detailed literature survey and discussion of the thermal and solutal dispersion effects on different geometry are available in the Murthy [15] and Kairi and Murthy [16].

The variable surface heat and mass flux in an unsteady natural convection fluid past vertical plate along with the effects of double dispersion and with double stratification in porous media is one interesting area. Ganesan and Rani [17] presented the investigation of unsteady free convection on a vertical cylinder along with variable heat and mass flux in detail. Using an implicit finite difference scheme, the problem of transient free convection along with mass transfer with constant heat flux on a vertical plate solved and presented by Soundalgekar and Ganesan [18]. The analysis of the numerical solution of transient free convection flow past a semi-infinite inclined plate along with variable surface heat flux and mass flux has been done by Ganesan and Palani [19]. Both Kabeir *et al* [20] and Sacheti *et al*. [21] has studied and analyzed an unsteady MHD convection in a fluid saturated porous medium over a vertical plate with constant heat flux only. Recently S. Nadeem *et al* [22] studied in detail, the numerical study of MHD boundary layer flow of a Maxwell fluid past a stretching sheet along with variable surface heat flux and mass flux in the presence of nanoparticles

The main aim of the present investigation is to analyze the effects of combined thermal and solutal dispersion, thermal and solutal stratification parameters on unsteady natural convection flow in a MHD doubly stratified fluid saturated Brinkman porous medium along with variable surface heat and mass flux using the Crank-Nicolson type scheme. The velocity, temperature and concentration are illustrated and graphically analyzed under the effects of governing parameters.

II. MATHEMATICAL FORMULATION

In this section, a problem of two-dimensional, unsteady, laminar free convection flow past a vertical plate in a viscous incompressible doubly stratified fluid saturated porous medium with variable surface heat flux and mass flux under the influence of thermal and solutal (double) dispersion and a magnetic field is formulated mathematically. The coordinate system is chosen to represent the *x*-axis along the vertical plate and the *y*-axis as upward normal to the plate. The fluid and the plate are assumed to be at the constant temperature and constant concentration initially at $t'=0$, whereas the surface heat flux and mass flux are supplied to the fluid from the plate at a rate of $q_w(x) = x^m$ and $q_w^*(x) = x^n$ respectively, where m is a heat flux exponent and n is the mass flux exponent in the power variations and both heat and mass flux are maintained at the same level for all the time $t'>0$. The ambient medium is assumed to be vertically linearly stratified with respect to both temperature and concentration in the form $T_\infty(x) = T_{\infty,0}+Ax$ and $C_\infty(x) = C_{\infty,0}+Bx$ respectively. Then, under these assumptions, the set of governing boundary layer equations of dimensionless mass, momentum, energy and species concentration for the free convection flows with Boussinesq's and Brinkman porous medium approximation are as follows

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\frac{1}{\varepsilon}\frac{\partial u}{\partial t'} + \frac{u}{\varepsilon^2}\frac{\partial u}{\partial x} + \frac{v}{\varepsilon^2}\frac{\partial u}{\partial y} = \frac{\upsilon}{\varepsilon}\frac{\partial^2 u}{\partial y^2} - \frac{\mu}{k}u - \frac{\sigma B_0^{\ 2}}{\rho}\frac{u}{\varepsilon} + g[\beta_T(T'-T_\infty(x)) + \beta_C(C'-C_\infty(x))] \quad (2)$$

$$\frac{\partial T'}{\partial t'}+u\frac{\partial T'}{\partial x}+v\frac{\partial T'}{\partial y}=\frac{\partial}{\partial y}[\alpha_e\frac{\partial T'}{\partial y}] \quad (3)$$

$$\frac{\partial C'}{\partial t'}+u\frac{\partial C'}{\partial x}+v\frac{\partial C'}{\partial y}=\frac{\partial}{\partial y}[D_e\frac{\partial C'}{\partial y}] \quad (4)$$

where u and v are Darcy velocity components along with the x-axis and y-axis directions respectively and μ is the viscosity coefficient, ε is the porosity, g is the acceleration due to gravity, ρ is the density, k is the permeability, σ is the electrical conductivity, T' is the dimensional temperature, C' is the dimensional concentration, β_T and β_C are the coefficients of thermal and solutal expansions, α is the thermal diffusivity. The effective thermal diffusivity (α_e) and can be written as $\alpha_e=\alpha_0+\alpha_d$, where $\alpha_0= k/\rho C_P$, where C_p is the specific heat and $\alpha_d=\gamma^* du$. The effective solutal diffusivity (De) and can be written as $De=D+D_d$ where $D_d=\xi^* du$ and D is the mass. And γ^* is dimensional thermal dispersion coefficient ξ^* is dimensional solutal dispersion coefficient.

The boundary conditions are

$$t'\le 0: u(x,y,t')=0,\ v(x,y,t')=0,$$
$$T'(x,y,t')=T_\infty(x),\ C'(x,y,t')=C_\infty(x)$$
$$t'>0: u(x,0,t')=0,\ v(x,0,t')=0,$$
$$\frac{\partial T'(x,y,t')}{\partial y}_{at\, y=0}=-\frac{q_w(x)}{k},$$
$$\frac{\partial C'(x,y,t')}{\partial y}_{at\, y=0}=-\frac{q_w^*(x)}{D}$$
$$t'>0: u(0,y,t')=0,\quad v(0,y,t')=0,$$
$$T'(0,y,t')=T_{\infty,0},\quad C'(0,y,t')=C_{\infty,0}$$
$$t'>0: u(x,\infty,t')\to 0,\ T'(x,\infty,t')\to T_\infty(x),$$
$$C'(x,\infty,t')\to C_\infty(x) \quad (5)$$

The physical parameters local skin friction, Nusselt number and Sherwood number are obtained as

$$\tau_x=-\mu\left(\frac{\partial u}{\partial y}\right)_{y=0},$$
$$Nu_x=-\frac{x\left(\frac{\partial T'}{\partial y}\right)_{y=0}}{T'_{y=0}}, \quad (6)$$
$$Sh_x=-\frac{x\left(\frac{\partial C'}{\partial y}\right)_{y=0}}{C'_{y=0}}$$

and average skin friction, Nusselt number and Sherwood number are obtained as

$$\overline{\tau_L}=-\frac{1}{L}\int_0^L\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}dx,$$
$$\overline{Nu_L}=-\int_0^L\frac{\left(\frac{\partial T'}{\partial y}\right)_{y=0}}{T'_{y=0}}dx, \quad (7)$$
$$\overline{Sh_L}=-\int_0^L\frac{\left(\frac{\partial C'}{\partial y}\right)_{y=0}}{C'_{y=0}}dx$$

Using the following non-dimensional variables

$$X=\frac{x}{L},\ Y=\frac{y}{L}Gr^{1/4},\ U=\frac{uL}{\upsilon}Gr^{-1/2},$$
$$V=\frac{vL}{\upsilon}Gr^{-1/4},$$
$$t=\frac{t'\upsilon}{L^2}Gr^{1/2},\ T=\frac{(T'-T_\infty(x))Gr^{1/4}}{q_w(L)L/k},$$
$$C=\frac{(C'-C_\infty(x))Gr^{1/4}}{q_w^*(L)L/D} \quad (8)$$

in to the Eqs. (1) to (4), we obtain the following system of coupled non-dimensional partial differential equations

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \quad (9)$$

$$\frac{1}{\varepsilon}\frac{\partial U}{\partial t}+\frac{U}{\varepsilon^2}\frac{\partial U}{\partial X}+\frac{V}{\varepsilon^2}\frac{\partial U}{\partial Y}=\frac{1}{\varepsilon}\frac{\partial^2 U}{\partial Y^2}-\frac{1}{DaGr^{1/2}}U-\frac{1}{\varepsilon}\frac{M}{Gr^{1/2}}U+Gr^{-1/4}T+Gr^{-1/4}NC \quad (10)$$

$$\frac{\partial T}{\partial t}+U\frac{\partial T}{\partial X}+(V-\gamma\frac{\partial U}{\partial Y})\frac{\partial T}{\partial Y}=$$
$$(\frac{1}{Pr}+\gamma U)\frac{\partial^2 T}{\partial Y^2}-Gr^{1/4}\varepsilon_1 U \quad (11)$$

$$\frac{\partial C}{\partial t}+U\frac{\partial C}{\partial X}+(V-\xi\frac{\partial U}{\partial Y})\frac{\partial C}{\partial Y}=$$
$$(\frac{1}{Sc}+\xi U)\frac{\partial^2 C}{\partial Y^2}-Gr^{1/4}\varepsilon_2 U \quad (12)$$

along with the corresponding non-dimensional initial and boundary conditions are

$$t \le 0: U(X,Y,t)=0,\ V(X,Y,t)=0,$$
$$T(X,Y,t)=0,\ C(X,Y,t)=0$$
$$t>0: U(X,0,t)=0,\ V(X,0,t)=0,$$
$$\left.\frac{\partial T(X,Y,t)}{\partial Y}\right|_{at\,Y=0} = -X^m,$$
$$\left.\frac{\partial C(X,Y,t)}{\partial Y}\right|_{at\,Y=0} = -X^n$$
$$t>0: U(0,Y,t)=0,\ \ V(0,Y,t)=0,$$
$$T(0,Y,t)=0,\ \ C(0,Y,t)=0$$
$$t>0: U(X,\infty,t)\to 0,$$
$$T(X,\infty,t)\to 0,\ C(X,\infty,t)\to 0 \quad (13)$$

where $Gr = g\beta_T L^4 q_w(L)/k\upsilon^2$, $Gc = g\beta_C L^4 q_w^*(L)/D\upsilon^2$ are the thermal Grashof number and the solutal Grashof number, $N = Gc/Gr$ is the buoyancy ratio, $M = \sigma B_0^2 L^2/(\rho\upsilon)$ is the magnetic parameter, $Da = k\upsilon/(\mu L^2)$ is the Darcy number, $\alpha = k/(\rho Cp)$ is the thermal diffusitivity, $Pr = \upsilon/\alpha$ is the Prandtl numbers and $Sc = \upsilon/D$ is the Schmidt numbers, $\varepsilon_1 = Ak/q_w(L)$ and $\varepsilon_2 = BD/q_w^*(L)$ are the thermal and solutal stratification parameters, $\gamma = \gamma^* Gr^{1/2}$ d/L is the thermal dispersion coefficient and $\xi = \xi^* Gr^{1/2}$ d/L is the solutal dispersion coefficient.

III. RESULTS AND DISCUSSION

To verify the accuracy of the present numerical results, we noticed that the steady state obtained at X = 1.0 and the present numerical solutions compared with available similar solutions of Kumar *et al*. [24], Kairi *et al*. [25] and Ganesan *et al*. [26] in the literature and confirmed that, the present results following the trend observed in the literature with great agreement. The effects of various parameters on nondimensional local and average skin friction, Nusselt number and Sherwood numbers are analyzed and plotted with fixed values $Pr=1.0$, $Gr=5.0$, $Sc=0.96$, $N=1.0$, $M=1.0$, $Da=1.0$, $\varepsilon=0.6$, $\varepsilon_1=0.1$, $\varepsilon_2=0.5$, $\gamma=0.3$, $\xi=0.3$, $m=0.5$, $n=0.5$ unless particularly otherwise specified and all the graphs are drawn at steady state time *t*.

Consideration of the energy equations in conjunction with the thermal dispersion is favorable for the heat conduction over the heat convection. This means that the energy equation gives thermal conduction more dominance in the supply of dispersion. With this idea, Fig 1 is prepared to represent the influence of thermal stratification parameter along with thermal dispersion coefficient on non-dimensional velocity, temperature and concentration for fixed values of other parameters. From Fig. 1(a) and Fig. 1(b), it has been observed that, the velocity and temperature decrease with an increase in the value of thermal stratification parameter and increase with an increase in the value of thermal dispersion. That is, thermal dispersion enhances the transport of heat along the normal direction to the wall as compared with the case where dispersion is neglected. In general, this work may be useful in showing that the use of porous media with better heat dispersion properties may result in better heat transfer rate characteristics that may be required in many industrial applications (like those concerned with packed bed reactors, nuclear waste disposal, etc.). From Fig. 1 (c), it can be found that, the concentration of the fluid increases by increasing the values of the thermal stratification parameter and decreases slightly with an increase in the value of thermal dispersion coefficient.

Figure 2 represent the influence of a solutal stratification parameter along with a solutal dispersion coefficient on non-dimensional velocity, temperature and concentration for fixed values of other parameters. Fig. 2 (a) and Fig. 2 (c) present the velocity and concentration profiles, and it is observed that, the velocity and concentration of the fluid is decreasing with increase in solutal stratification parameter and ae slightly increasing with a rise in the solutal dispersion coefficient. From Fig. 2 (b), it can be observed that, the temperature of the fluid in the medium is increasing with an increase in the value of solutal stratification parameter while decreasing slightly with an increase in the value of solutal dispersion coefficient.

Fig. 3 demonstrate the impact of exponents in the power law variation of the heat mass flux *m* and *n* values on non-dimensional velocity, temperature and concentration profiles for fixed values of other parameters. It is observed that the velocity decrease with increasing values of both heat and mass flux exponents *m* and *n* as shown in Fig. 3 (a). Figure 3

(c) display that, the concentration increase with increasing values of heat flux exponent *m*, and decrease with increasing value of mass flux exponent *n*. From Fig. 3 (b), it is observed that the temperature, decrease as heat flux exponent *m* increases while they increase with increasing mass flux exponent *n*. Further, to achieve the steady state values, it is required more time for higher values of both heat and mass flux exponents *m* and *n* when the values of both m and n are non zeros.

IV. CONCLUSION

This present study analyzed numerically the double dispersion unsteady MHD free convective flow of doubly stratified fluid past a vertical plate with variable surface heat flux and mass flux in a Brinkman porous medium. First the governing nonlinear partial differential equations subjected to their boundary conditions are reduced to non dimensional form under suitable non dimensional transformations and then solved numerically using unconditionally stable Crank-Nicolson type of implicit finite difference scheme. However, the comparison of present results with study of Kumar *et al*. [23], Kairi *et al*. [24] and Ganesan *et al*. [25] in the literature is performed and confirmed the excellent agreement. The main consolidate conclusions of the present study are listed as follows:

(a) The velocity increases slightly with an increase in both thermal and solutal (double) dispersion coefficients while decreases with an increase in both thermal, solutal (double) stratification parameters and heat, mass flux exponents.

(b) The concentration increases with an increase in the thermal stratification parameter, solutal dispersion coefficient and heat flux exponent, but decreases with a rise in the solutal stratification parameter, thermal dispersion coefficient and mass flux exponent.

(c) The temperature increases with an increase in the solutal stratification parameter, thermal dispersion coefficient and mass flux exponent while decreases with a rise in the thermal stratification parameter, solutal dispersion coefficient and heat flux exponent.

(d) The time taken to reach steady state increases with a rise in both thermal, solutal (double) stratification parameters and heat, mass flux exponents, while decreases with a rise in the solutal dispersion coefficient when these quantities are non zeros.

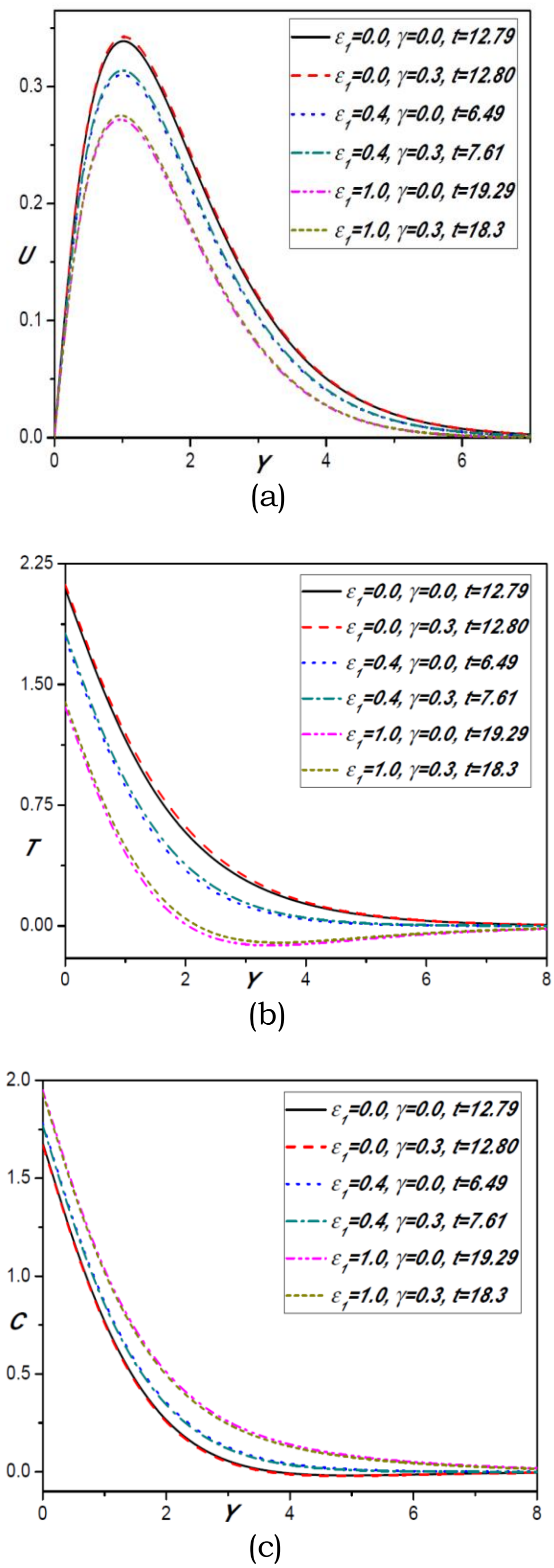

Fig 1: Effect of thermal stratification with thermal dispersion coefficients on (a) Velocity, (b) Temperature and (c) Concentration profile at X=1

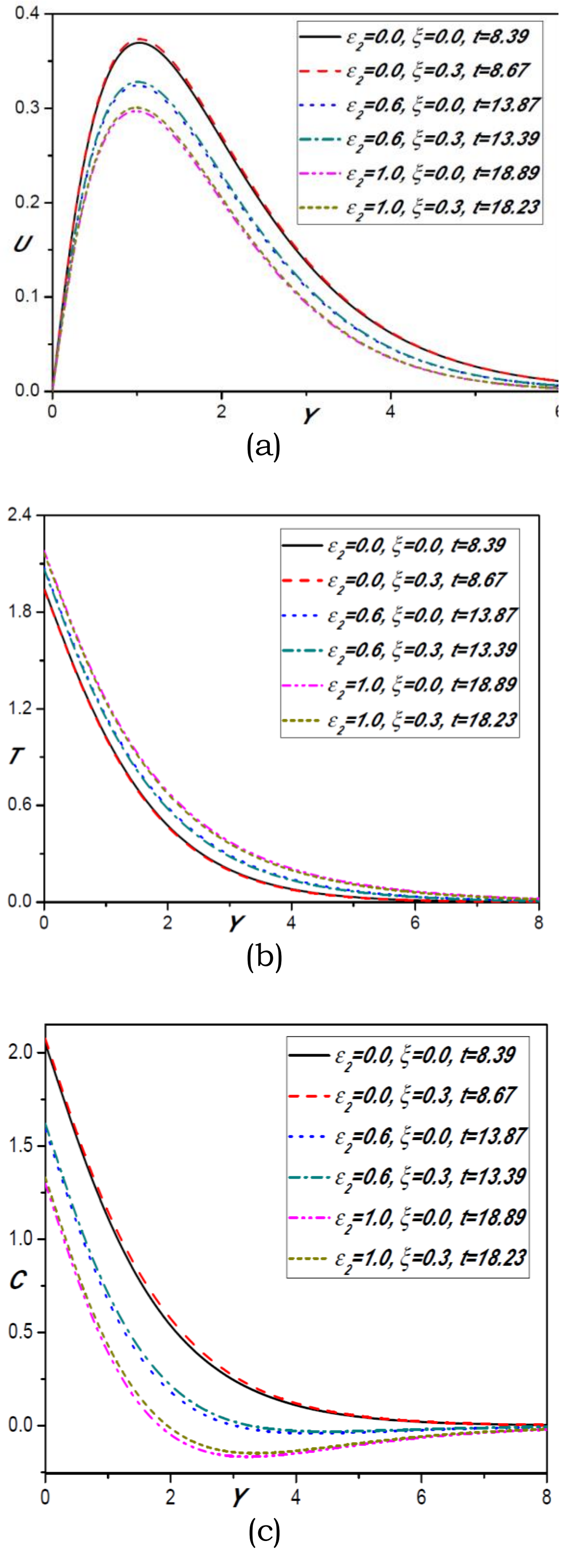

Fig 2: Effect of solutal stratification with solutal dispersion coefficients on (a) Velocity, (b) Temperature and (c) Concentration profile at X=1

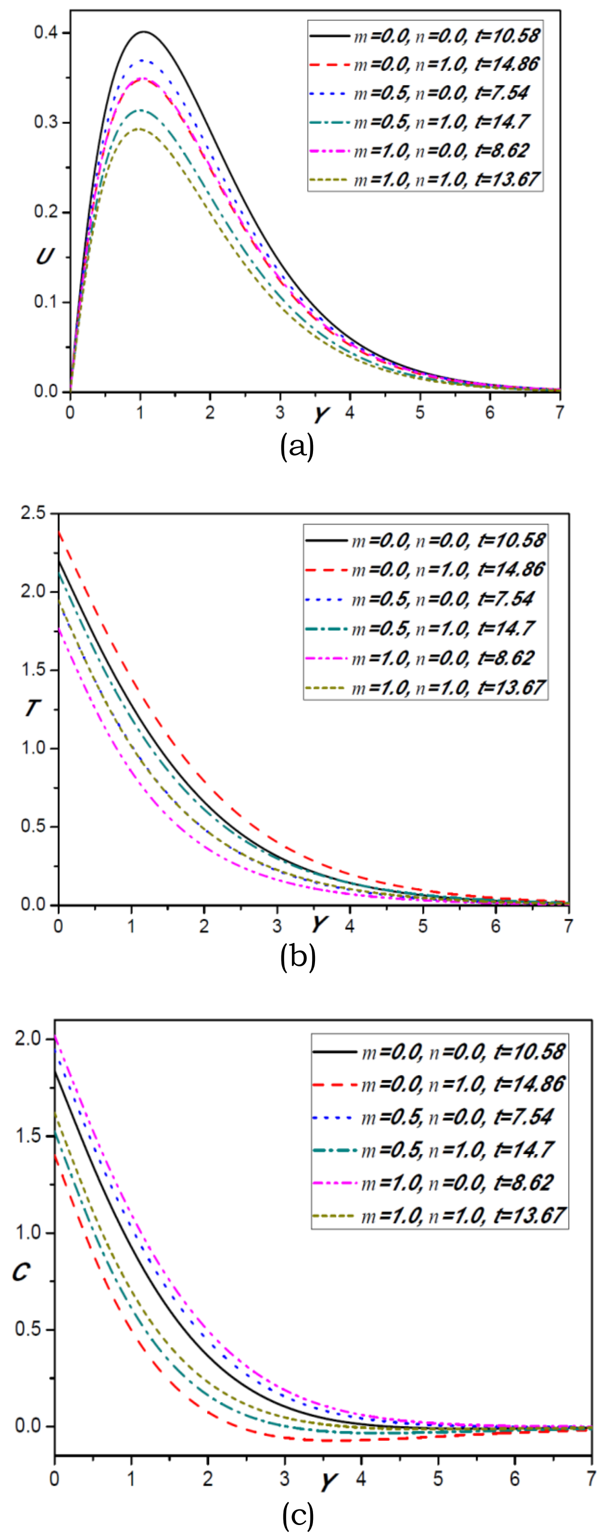

Fig 3: Effect of exponents m, n on (a) Velocity, (b) Temperature and (c) Concentration profile at X=1

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