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The effects double stratification on unsteady natural convection past a vertical flat plate

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***Abstract*—In this paper, the influence of viscous dissipation and cross-diffusion effects on unsteady natural convective flow of an electrically conducting doubly stratified fluid in a Brinkman porous medium has been analyzed. By using the similarity transformations, the governing differential equations are expressed into a set of non-linear coupled ordinary differential equations along with the suitable boundary conditions. The Crank-Nicolson type scheme developed based on implicit finite difference method has been used to solve the reduced nonlinear boundary value problem. Numerical results for velocity, temperature and concentration profiles are depicted graphically and analyzed in detail for different physical parameters.**

***Index Terms*— Brinkman porous medium, Crank-Nicolson method, Double stratification, Dufour and Soret effects, Unsteady.**

I. INTRODUCTION

Natural convective transport in a porous medium has wide significance in heat and mass transfer problems because of their increasing applications in diverse areas such as scientific, various engineering and industrial areas (e.g., see Nield and Bejan[1]). The thermal and solutal stratifications of fluid arise due to temperature variations, concentration differences, or the existence of different fluids. Also, it is favourable in distinct engineering applications such as heat rejection into the environment, thermal energy storage systems and heat transfer from thermal sources.

With this motivation, several researchers have started their study in the area of doubly stratified convective flows. Nakayama and Koyama [2] discussed the thermal stratification effect, whereas Murthy *et al.* [3] and Srinivasacharya and Ramreddy [4] analysed the double stratification effects on natural convection flow in a porous medium. Deka and Paul[5] provided an exact solution for unsteady natural convection flow over a vertical cylinder with thermal stratification effect. An influence of electrophoresis and chemical reaction on unsteady convection doubly stratified flow past a semi-infinite vertical plate in the presence of heat source/sink is presented by Ganesan and Suganthi [6].

The mass fluxes created by temperature gradients and energy flux caused by a concentration gradient are termed as thermal-diffusion (Soret) and diffusion-thermo (Dufour) effects respectively. An extensive study of these effects are considered theoretically and experimentally in both gases and liquids. These are found to be important in different areas like hydrology, petrology and Geosciences (eg., see Eckert and Drake [7] and references cited therein). Dursunkaya and Worek [8] analyzed cross-diffusion effects in unsteady and steady free convection, whereas Kafoussias and Williams [9], Ahmed [10] and Srinivasacharya and RamReddy[11] discussed the steady convective flows in Newtonain and non-Newtoanian fluids with Soret and Dufour effects. The Hall current, thermal diffusion, and heat source are considered by Ahmed and Barua[12] to investigate an unsteady natural convective flow in a porous medium. Recently, Loganathan *et al.* [13] observed that the local heat transfer rate enhances with an enhancement in Dufour and Soret numbers for both air and water, while analyzing the problem of the cross-diffusion effects on the unsteady natural convection (see the citations therein).

The objective of the present study is to investigate the effects of cross-diffusion on unsteady MHD free convection flow in a doubly stratified fluid saturated Brinkman porous medium using the Crank-Nicolson type scheme. The influence of various physical parameters for velocity, temperature and concentration are exhibited graphically and analyzed in detail.

II. MATHEMATICAL FORMULATION

Consider a two-dimensional laminar incompressible unsteady natural convective flow past a vertical plate in an electrically conducting and doubly stratified fluid saturated porous medium. The coordinate system is chosen to represent the *x*-axis along the vertical plate and the *y*-axis as normal to the plate. The fluid and the plate are assumed to be at the constant temperature and constant concentration initially at $t'=0$ whereas the temperature and concentration of the plate are changed to T_w and C_w respectively for the time $t'>0$. The ambient medium is assumed to be vertically linearly stratified with respect to both temperature and concentration in the form $T_\infty(x) = T_{\infty,0}+Ax$ and $C_\infty(x) = C_{\infty,0}+Bx$ respectively. A magnetic field of uniform strength B_0 is introduced normal to the direction of the flow.

Under the above assumptions, the governing boundary layer equations for the flow with usual Boussinesq's and Brinkman porous medium approximation are as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{1}{\varepsilon}\frac{\partial u}{\partial t'}+\frac{u}{\varepsilon^2}\frac{\partial u}{\partial x}+\frac{v}{\varepsilon^2}\frac{\partial u}{\partial y}+\frac{\mu}{k}u+\frac{\sigma B_0^{\,2}}{\rho}\frac{u}{\varepsilon}=\frac{\nu}{\varepsilon}\frac{\partial^2 u}{\partial y^2}+g\left[\beta_T(T'-T_\infty(x))+\beta_C(C'-C_\infty(x))\right] \tag{2}$$

$$\frac{\partial T'}{\partial t'}+u\frac{\partial T'}{\partial x}+v\frac{\partial T'}{\partial y}=\alpha\frac{\partial^2 T'}{\partial y^2}+\frac{\mu}{\rho Cp}\left(\frac{\partial u}{\partial y}\right)^2+\frac{DK_T}{CsCp}\frac{\partial^2 C'}{\partial y^2} \tag{3}$$

$$\frac{\partial C'}{\partial t'}+u\frac{\partial C'}{\partial x}+v\frac{\partial C'}{\partial y}=D\frac{\partial^2 C'}{\partial y^2}+\frac{DK_T}{T_m}\frac{\partial^2 T'}{\partial y^2} \tag{4}$$

Where u and v are Darcy velocity components along the x and y directions respectively, ρ is the density, g is the acceleration due to gravity, C_p is the specific heat, μ is the coefficient of viscosity, σ is the electrical conductivity, k is the permeability, ε is the porosity, T' is the temperature, C' is the concentration, β_T and β_C are the coefficients of thermal and solutal expansions, α is the thermal diffusivity, D is the mass diffusivity and T_m is the mean fluid temperature.

The boundary conditions are

$$u(x,y,t')=0,\ v(x,y,t')=0,$$
$$T'(x,y,t')=T_\infty(x),\ C'(x,y,t')=C_\infty(x)$$
for $t'\le 0$
$$u(x,0,t')=0,\ v(x,0,t')=0,$$
$$T'(x,0,t')=T_w,\ C'(x,0,t')=C_w$$
for $t'>0$
$$u(0,y,t')=0,\ v(0,y,t')=0,$$
$$T'(0,y,t')=T_{\infty,0},\ C'(0,y,t')=C_{\infty,0}$$
for $t'>0$
$$u(x,y,t')\to 0,\ T'(x,y,t')\to T_\infty(x),$$
$$C'(x,y,t')\to C_\infty(x)$$
as $y\to\infty$ and for $t'>0$ (5)

Introducing the following non-dimensional variables

$$X=\frac{x}{L},\ Y=\frac{y}{L}Gr^{1/4},\ U=\frac{uL}{\upsilon}Gr^{-1/2},$$
$$V=\frac{vL}{\upsilon}Gr^{-1/4},\ t=\frac{t'\upsilon}{L^2}Gr^{1/2},$$
$$T=\frac{T'-T_\infty(x)}{T_w-T_{\infty,0}},\ C=\frac{C'-C_\infty(x)}{C_w-C_{\infty,0}}$$

into Eqs. (1)-(4), we obtain

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{6}$$

$$\frac{1}{\varepsilon}\frac{\partial U}{\partial t}+\frac{U}{\varepsilon^2}\frac{\partial U}{\partial X}+\frac{V}{\varepsilon^2}\frac{\partial U}{\partial Y}+\frac{1}{DaGr^{1/2}}U+\frac{1}{\varepsilon}\frac{M}{Gr^{1/2}}U=\frac{1}{\varepsilon}\frac{\partial^2 U}{\partial Y^2}+T+NC \tag{7}$$

$$\frac{\partial T}{\partial t}+U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y}+\varepsilon_1 U=\frac{1}{\Pr}\frac{\partial^2 T}{\partial Y^2}+Ec\left(\frac{\partial U}{\partial Y}\right)^2+Df\frac{\partial^2 C}{\partial Y^2} \tag{8}$$

$$\frac{\partial C}{\partial t}+U\frac{\partial C}{\partial X}+V\frac{\partial C}{\partial Y}+\varepsilon_2 U=\frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2}+Sr\frac{\partial^2 T}{\partial Y^2} \tag{9}$$

where $Gr = g\beta_T L^3 (T_w - T_{\infty,0}) / \upsilon^2$ is the Grashof number, $N = \beta_C (C_w - C_{\infty,0}) / (\beta_T (T_w - T_{\infty,0}))$ is the buoyancy ratio, $Da = k\upsilon / (\mu L^2)$ is the Darcy number, $M = \sigma B_0^2 L^2 / (\rho\upsilon)$ is the magnetic parameter, $EC = n_0^2 / (C_p (T_w - T_{\infty,0}))$ is the Eckert number, $n_0 = (\mu / (\rho L)) Gr^{1/2}$, $\alpha = k / (\rho C_P)$ is the thermal diffusivity, $Pr = \upsilon / \alpha$ and $Sc = \upsilon / D$ are the Prandtl and Schmidt numbers, $Sr = DK_T (T_w - T_{\infty,0}) / (T_m \upsilon (C_w - C_{\infty,0}))$ and $Df = DK_T (C_w - C_{\infty,0}) / (C_S C_p \upsilon (T_w - T_{\infty,0}))$ are the Soret and Dufour numbers, $\varepsilon_1 = AL / (T_w - T_{\infty,0})$ and $\varepsilon_2 = BL / (C_w - C_{\infty,0})$ are the thermal and solutal stratification parameters.

The non-dimensional conditions associated with the reduced equations are

$$U(X,Y,t) = 0,\ V(X,Y,t) = 0,$$
$$T(X,Y,t) = 0,\ C(X,Y,t) = 0$$

for $t \le 0$

$$U(X,\ 0,\ t) = 0,\ V(X,\ 0,\ t) = 0,$$
$$T(X,\ 0,\ t) = 1 - \varepsilon_1 \mathrm{X},\quad C(X,\ 0,\ t) = 1 - \varepsilon_2 \mathrm{X}$$

for $t > 0$

$$U(0,\ Y,t) = 0,\quad V(0,\ Y,t) = 0,$$
$$T(0,\ Y,t) = 0,\quad C(0,\ Y,t) = 0$$

for $t > 0$

$$U(X,Y,t) \to 0,\ T(X,Y,t) \to 0,$$
$$C(X,Y,t) \to 0$$

as $Y \to \infty$ and for $t > 0$ (10)

The non-dimensional forms of physical parameters of interest such as local skin friction, heat and mass transfer rates are obtained as

$$\tau_X = Gr^{3/4} \left(\frac{\partial U}{\partial Y} \right)_{Y=0},$$

$$Nu_X = -Gr^{1/4} \frac{X \left(\frac{\partial T}{\partial Y} \right)_{Y=0}}{1 - \varepsilon_1 X} \quad \text{and}$$

$$Sh_X = -Gr^{1/4} \frac{X \left(\frac{\partial C}{\partial Y} \right)_{Y=0}}{1 - \varepsilon_2 X} \tag{11}$$

The non-dimensional forms of average skin friction, heat and mass transfer rates are obtained as

$$\bar{\tau} = Gr^{3/4} \int_0^1 \left(\frac{\partial U}{\partial Y} \right)_{Y=0} dX,$$

$$\overline{Nu} = -Gr^{1/4} \int_0^1 \frac{\left(\frac{\partial T}{\partial Y} \right)_{Y=0}}{1 - \varepsilon_1 X} dX \quad \text{and}$$

$$\overline{Sh} = -Gr^{1/4} \int_0^1 \frac{\left(\frac{\partial C}{\partial Y} \right)_{Y=0}}{1 - \varepsilon_2 X} dX \tag{12}$$

III. RESULTS AND DISCUSSION

The reduced unsteady, nonlinear and coupled Eqs. (6)–(9) with conditions Eq.(10) are solved numerically by using the implicit finite difference method known as Crank-Nicolson type scheme (for more details about this method, one can refer to work of Loganathan et al. [13] and citations therein). In order to verify the accuracy of the present results, the velocity profiles of the present problem are compared with the existing solution of Gebhart and Pera [14] in the absence of doubly stratified porous medium with *Pr=0.71 Sc=0.94, N=1.0, Ec = 0.0, Sr = 0.0, Df = 0.0* and *M=0.0*. The effects of various parameters on the physical quantities are analyzed by taking fixed values of *Da=0.1, Pr=0.71, Sc=0.22, Ec=0.1,* and *N=0.5*. These computations are carried out at *t=1.5*.

Figure 3.1 represent the variations of non-dimensional velocity, temperature and concentration.

Figures from 3.1 (a) to 3.1 (c) show the velocity, temperature and concentration profiles with an increase in Dufour number and simultaneous decrease in Soret number. The present analysis has also shown that the flow field is appreciably influenced by the Dufour and Soret effects in the case of unsteady convection flow under consideration.

The variation of non-dimensional velocity, temperature and concentration for different values of thermal and solutal stratification parameter are plotted in Fig. 3.2. Figures from 3.2 (a) to 3.2 (c) show the velocity, temperature and concentration profiles.

Figure 3.3 exhibit the variation of non-dimensional velocity, temperature and concentration for different values of Darcy and magnetic parameters. Lorentz force is a resistive type of force which is generated due to transverse magnetic field and it has a tendency to slow down the motion of the fluid. The velocity, temperature and concentration profiles obtained in Figs. 3.3 (a) to 3.3 (c) show a qualitatively agreement with the expected ones.

IV. CONCLUSION

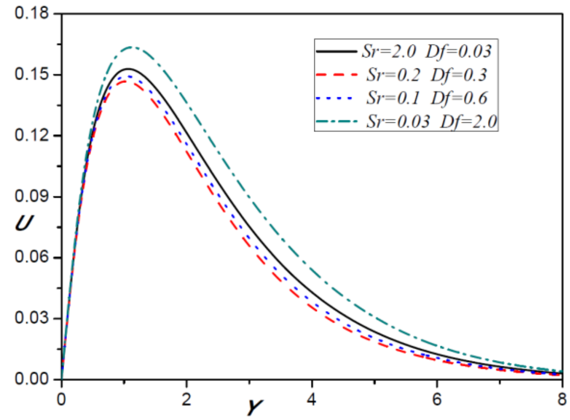
This paper analyses the unsteady MHD free convective flow in a doubly stratified porous medium with Soret and Dufour effects. The resulting equations are solved numerically by implicit finite difference method known as Crank-Nicolson type. The main findings are summarized as follows:

(a) With the influence of Soret and Dufour parameters, the velocity, temperaturer increase, but concentration decrease.

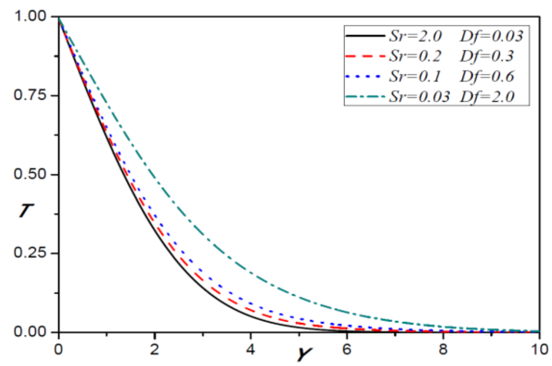
(b) As the thermal stratification parameter increases, the concentration ncrease, while the velocity, temperature display the reverse trend.

(c) For increasing of the solutal stratification parameter, the display the same increasing trend, but the velocity, concentration show a decreasing trend.

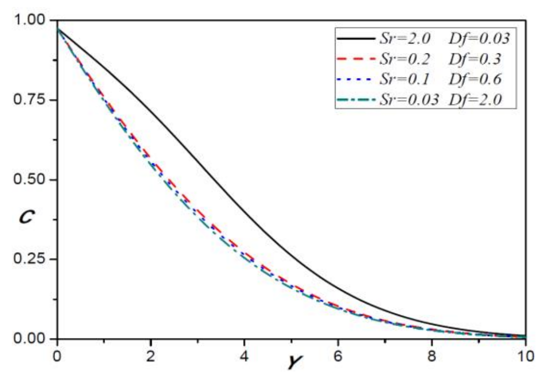
(d) The velocity increase, whereas the temperature and concentration decrease with an increase in the value of Darcy parameter. But initially there is no effect; later the velocity decrease whereas both the temperature and concentration increase with the rise in magnetic parameter.



(a)

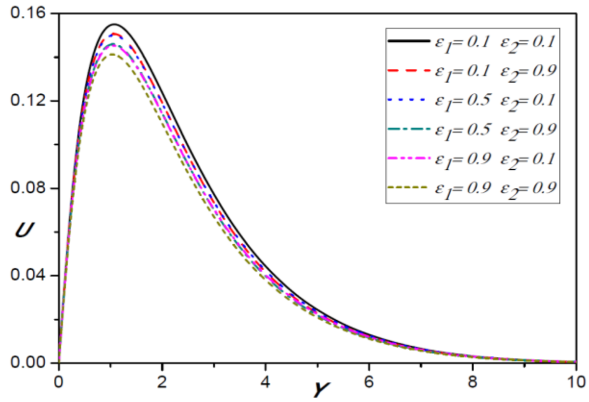


(b)

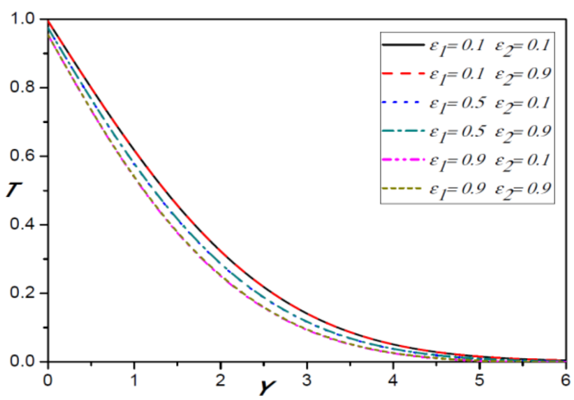


(c)

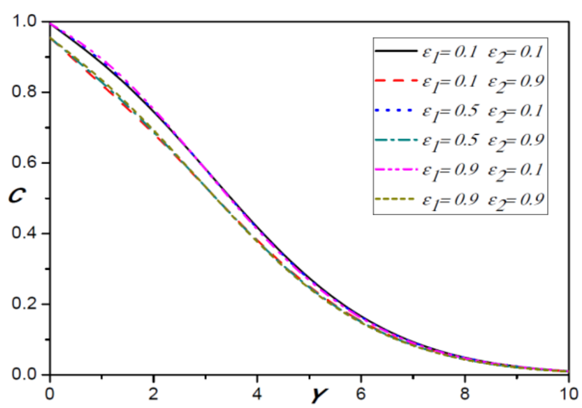
Fig-3.1: Effect of Soret and Dufour parameters on a) Velocity, b) Temperature and c) Concentration profiles at X=1.



(a)

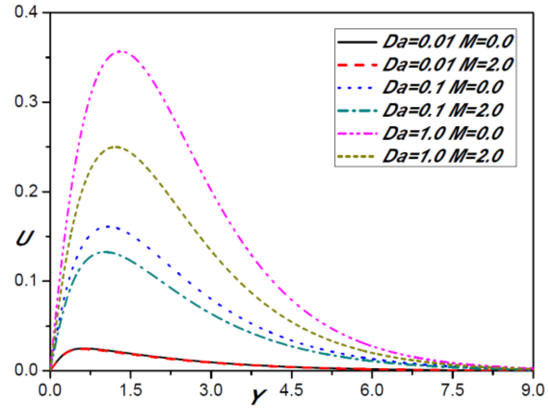


(b)

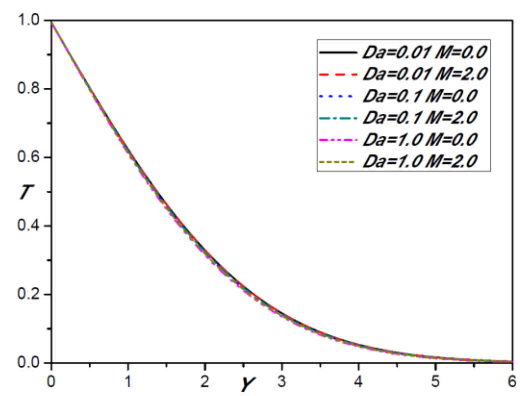


(c)

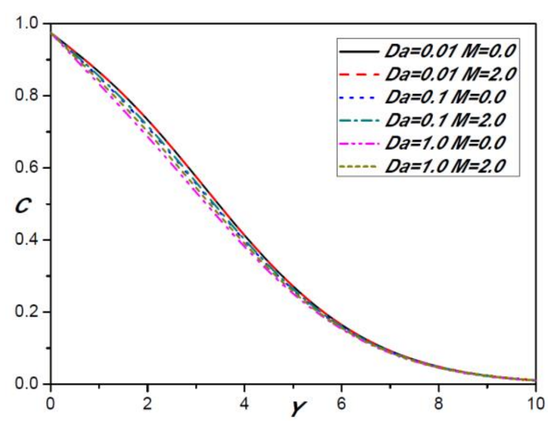
Fig-3.2: Effect of thermal and solutal stratification parameters on a) Velocity, b) Temperature and c) Concentration profiles at X=1.



(a)



(b)



(c)

Fig-3.3: Effect of Darcy and magnetic parameters on a) Velocity, b) Temperature and c) Concentration profiles at X=1.

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