# Contraction Mapping in Fixed Point Theorem

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## I. INTRODUCTION

Many problems in pure and applied mathematics have as their solutions the fixed point of some mapping. Therefore a number of procedures in numerical analysis and approximations theory amount to obtaining successive approximations to the fixed point of an approximate mapping. Our object in this paper to discuss about fixed point theory in metric spaces, also we established some fixed point theorems in complete metric spaces.

## II. BANACH FIXED POINT THEOREM

The Banach fixed point theorem (also known as the contraction mapping theorem or contraction mapping Principle.) is an important tool in the theory of metric spaces; it guarantees the existence and uniqueness of fixed points of certain self maps of Metric space, and provides a constructive method to find those fixed points.

2.1. Contraction mapping

Let (X, d) be a Metric space. A mapping $f : X \to X$ is called a contraction mapping if there is a real number k, $0 < k < 1$, such that

$d(fx, fy) \le k d(x, y)$ for all $x, y \in X$

The well-known Banach [1] contraction principle states that a contraction mapping of a Complete Metric space Xinto itself has a unique fixed points. This theorem has been extensively used in proving existences and uniqueness of solutions to various functional equations, particularly differential and integral equations. Because of its wide spread applicability there has been a search for generalization of the Banach contraction Principle. This celebrated principle has been generalized by many author, viz. chu. and Diaz [2], Sehgal [3]. Holmes [4]. Reich [5], Hardy and Rogers [6], Wong [7], Edelstein [11] and Kannan [10] others in various ways. In this paper we obtain yet an other generalization of this principle.

## III. MAIN RESULTS

*Theorem:* Let f be a continuous self-map defined on a complete metric space (X, d). Further, let f satisfy the following condition:

$$d(f(x), f(y)) \le \frac{\alpha d(x, f(x)).d(y, f(y))}{\eta + d(x, y)} + \beta d(x, y) \qquad (A)$$

for all $x, y \in X$, and for some $\alpha, \beta \in [0,1)$ with $a + \beta < 1$ and $\eta$ is finite. Then f has a unique fixed point in X.

Proof: Let $x_o$ be an arbitrary point of X and let $\{x_n\}$ where $x_n = f^n(x_o)$ and n is a positive integer, be the sequence of iterates of f at $x_o$. If $x_n = x_{n+1}$ for some n then the result is immediate. So let $x_n \ne x_{n+1}$ for all n. Now

$$d(x_{n+1}, x_n) = d(f(x_n), f(x_{n-1}))$$

$$\le \frac{\alpha d(x_n, f(x)).d(x_{n-1}, f(x_{n-1}))}{\eta + d(x_n, x_{n-1})} + \beta d(x_n, x_{n-1})$$

$$\le \frac{\alpha d(x_n, f(x_{n+1})).d(x_{n+1}, x_n)}{d(x_n, x_{n-1})} + \beta d(x_n, x_{n-1})$$

$= \alpha d(x_n, x_{n+1}) + \beta d(x_n, x_{n-1})$

Which implies that

$$d(x_{n+1}, x_n) < \left(\frac{\beta}{1-\alpha}\right) d(x_n, x_{n-1})$$

:
:

$$< \left(\frac{\beta}{1-\alpha}\right)^n d(x_1, x_0)$$

By the triangle inequality we have for $m \geq n$,

$d(x_n, x_m) \leq d(x_n, x_{n+l}) + d(x_{n+1}, x_{n+2}) + ... + d(x_{m-1}, x_m)$

$< (k^n + k^{n+1} + .... + k^{m-1})\, d(x_o\ f\ (x_o))$, where $k = \dfrac{\beta}{1-\alpha}$

$$< \frac{k^n}{1-k} d(x_0, f(x_0))$$

$\to 0$ if $m, n \to \infty$

Since X is complete, therefore there exists a $u \in X$ such that $x_n \to u$. Further, the continuity of f in X implies

$f(u) = f(\lim_{n\to\infty} x_n)$

$= (\lim_{n\to\infty} f(x_n)$

$= u.$

Therefore u is a fixed point of f in X. Now, if there exists another point $v \neq u$ in X such that $f(v) = v$, then

$d(v, u) = d(f(v),(u))$

$$\leq \frac{\alpha d(v, f(v)).d(u, f(u))}{\eta + d(v,u)} + \beta d(v,u)$$

$$\leq \frac{\alpha d(v.u).d(u,u)}{d(v,u)} + \beta d(v.u)$$

$= \beta d(v, u)$

$< d(v, u)$,

$(\therefore \beta < 1)$

a contradiction. Hence u is a unique fixed point of f inX.

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