

Analysis of Reversible Logic Gates

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Abstract — In the past few years reversible logic functions has emerged as an important research area. Other fields such as low power design, optical computing and quantum computing benefit directly from achieved improvements. Reversible logic has become one of the most promising research areas in the past few decades and has found its applications in several technologies; such as low power CMOS, nanotechnology, cryptography, digital signal processing and optical computing. Implementing the reversible logic has the advantages of reducing quantum cost and the number of garbage outputs. The purpose of this paper is to give a frame of reference, understanding and overview of reversible gates. In this paper various logic gates and its applicability on logic design have been discussed. A reversible circuit connects reversible gates without fan-outs and loops. Therefore, such circuits contain equal numbers of input and output wires, each going through an entire circuit. Reversible logic is gaining interest of many researchers due to its low power dissipating characteristic.

Index Terms- Reversible; Feynman; Toffoli ; Fredkin; Peres.

I. INTRODUCTION

Energy dissipation is one of the major issues in present day technology. Energy dissipation due to information loss in high technology circuits and systems constructed using irreversible hardware was demonstrated by R. Landauer in the year 1960. In 1973, Bennett, showed that in order to avoid $kT \ln 2$ joules of energy dissipation in a circuit it must be built from reversible circuits. According to Moore's law the numbers of transistors will double every 18 months. Thus energy conservative devices are the need of the day. The amount of energy dissipated in a system bears a direct relationship to the number of bits erased during computation. Reversible circuits are those circuits that do not lose information. The current irreversible technologies will dissipate a lot of

heat and can reduce the life of the circuit. The reversible logic operations do not lose information and dissipate very less heat. Synthesis of reversible logic circuit differs from the combinational one in many ways. Firstly, in reversible circuit there should be no fan-out, that is, each output will be used only once. Secondly for each input pattern there should be unique output pattern. Finally, the resulting circuit must be acyclic. Any reversible circuit design includes only the gates that are the number of gates, garbage outputs and the quantum cost.

Reversible logic has received great attention in the recent years due to their ability to reduce the power dissipation which is the main requirement in low power VLSI design. It has wide applications in DNA computing, low power CMOS and Optical information processing, quantum computation and nanotechnology. Irreversible hardware computation results in energy dissipation due to information loss. Reversible logic design differs significantly from traditional combinational logic design approaches. In reversible logic circuit the number of input lines must be equal the number of output lines, each output will be used only once and the resulting circuit must be acyclic. The output lines that are not used further are termed as garbage outputs. One of the most challenging tasks is to reduce these garbage's. Any reversible logic gate realizes only the functions that are reversible. But many of the Boolean functions are not reversible. Before realizing these functions, we need to transform those irreversible functions into reversible one. Any transformation algorithm that converts an irreversible function to a reversible one introduces input lines that are set to zero in the circuit's input side. These inputs are termed as constant inputs. Therefore, any efficient reversible logic design should minimize the garbage's as well as constant inputs. Computing systems give off heat when voltage levels change from positive to negative:

bits from zero to one. Most of the energy needed to make that change is given off in the form of heat. Rather than changing voltages to new levels, reversible circuit elements will gradually move charge from one node to the next. This way, one can only expect to lose a minute amount of energy on each transition. Reversible computing strongly affects digital logic designs. Reversible logic elements are needed to recover the state of inputs from the outputs. It will impact instruction sets and high-level programming languages as well. Eventually, these will also have to be reversible to provide optimal efficiency.

II. IRREVERSIBLE & REVERSIBLE LOGIC GATES

Conventional logic dissipates a significant amount of energy because information bits are lost during logic operations. Reversible logic is being considered as an alternative of traditional logic since reversible computing does not lose any information. According to Frank, reversible logic can recover a fraction of energy that can reach up to 100%. C.H. Bennett proved that the energy loss problem can be avoided if the circuits are built using reversible gates. A reversible gate is a logical cell that has the same number of inputs and outputs, inputs and outputs have a one-to-one mapping. Several reversible gates have been designed till date. The important basic reversible logic gates are Feynman gate which is the only 2*2 reversible gates. There is also Toffoli gate, Frekdin gate, Peres gate, all of which can be used to realize important combinational functions and all are 3*3 reversible gates.

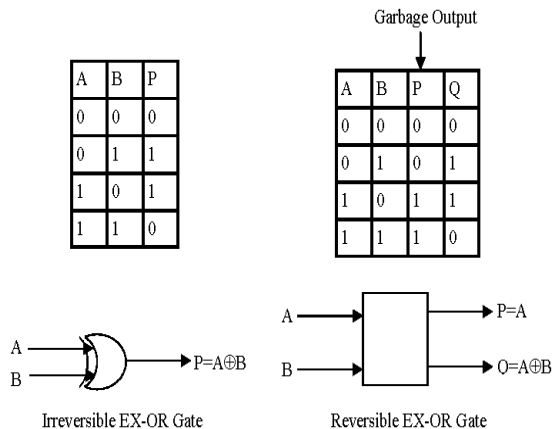


Figure 1: Irreversible & Reversible Logic Gates.

III. MOTIVATION BEHIND REVERSIBLE LOGIC

High-performance chips releasing large amounts of heat impose practical limitation on how far can we improve the performance of the system. Reversible circuits that conserve information, by non-computing bits instead of throwing them away, will soon offer the only physically possible way to keep improving performance. This computing will also lead to enhance the energy efficiency. Energy efficiency will fundamentally affect the speed of circuits such as nano circuits and hence the speed of most computing applications. Reversible computing is required to enhance the portability of devices. It will let circuit element sizes to reduce to atomic size limits and therefore devices will become more portable. Although the hardware design costs incurred in near future may be high but the power cost and performance being more dominant than logic hardware cost in today's computing era, the need of reversible computing cannot be ignored.

IV. BASIC DEFINITIONS PERTAINING REVERSIBLE LOGIC

A. The Reversible Logic Gate

The n-input and k-output Boolean function $f(x_1, x_2, x_3 \dots x_n)$ (referred to as (n, k) function) is called reversible if:

1) The number of outputs is equal to the number of inputs.

2) Each input pattern maps to unique output patterns. Reversible Gates are circuits in which number of outputs is equal to the number of inputs. And there is a one to one mapping between the vector of inputs and outputs. It helps to determine the outputs from the inputs as well as helps to uniquely recover the inputs from the outputs. This can be defined as the number of inputs that are to be maintaining constant at either 0 or 1 in order to synthesize the given logical function. Garbage Outputs, Additional inputs or outputs can be added so as to make the number of inputs and outputs equal whenever required. This also indicates the number of outputs which are not used in the synthesis of a given function. In certain cases these become mandatory to attain reversibility. Therefore garbage is the number of outputs added to make an n-input k-output function ((n; k) function) reversible. Constant inputs are used to denote the

present value inputs that are added to an (n; k) function to make it reversible. The following simple formula shows the relation between the number of garbage outputs and constant inputs. Input + constant input = output + garbage.

Quantum Cost may be defined as the cost of the circuit in terms of the cost of a primitive gate. It is calculated by the number of primitive reversible logic gates (1*1 or 2*2) required to realize the circuit. The quantum cost of a circuit is the minimum number of 2*2 unitary gates to represent the circuit keeping the output unchanged. The quantum cost of a 1*1 gate is 0 and that of any 2*2 gate is the same, which is 1 [20].

Some of the important reversible logic gates are:

i. *NOT Gate:*

The simplest Reversible gate is NOT gate and is a 1*1 gate. The Reversible 1*1 gate is NOT Gate with zero Quantum Cost is as shown in the Figure 2.



Figure 2: NOT Gate.

ii. *CNOT Gate:*

CNOT gate is also known as controlled-not gate. It is a 2*2 reversible gate. The CNOT gate can be described as: $I_v = (A, B)$; $O_v = (P=A, Q=A \oplus B)$ where I_v and O_v are input and output vectors respectively. Quantum cost of CNOT gate is 1. Figure 3 shows a 2*2 CNOT gate symbol.

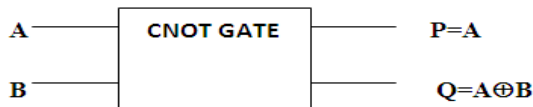


Figure 3: CNOT Gate

iii. *Feynman Gate:*

The Feynman gate which is a 2*2 gate and is also called as Controlled NOT and it is widely used for fan-out purposes. The inputs (A, B) and outputs $P=A$, $Q=A \oplus B$. It has quantum cost 1.

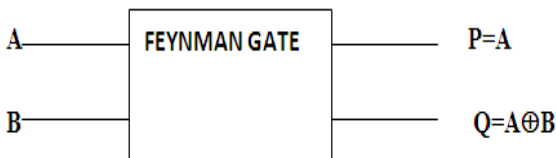


Figure 4: Feynman Gate

iv. *Toffoli Gate:*

Figure 5 shows a 3*3 Toffoli gate. The input vector is I (A, B, C) and the output vector is O(P,Q,R). The outputs are Fig 5 shows a 3*3 Toffoli gate. The input vector is I (A, B, C) and the output vector is O(P,Q,R). The outputs are: $P=A$, $Q=B$ and $R=AB \oplus C$ □C. Quantum cost of a Toffoli gate is 5[5].

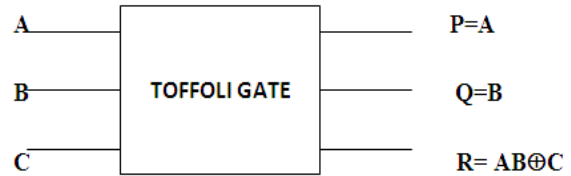


Figure 5: Toffoli Gate

v. *Fredkin Gate:*

Figure 6 shows a 3*3 Fredkin gate. The input vector is I (A, B, C) and the output vector is O (P, Q, R). The outputs are $P=A$, $Q=A'B \oplus AC$ and $R=A'C \oplus AB$. Quantum cost of a Fredkin gate is 5[6].

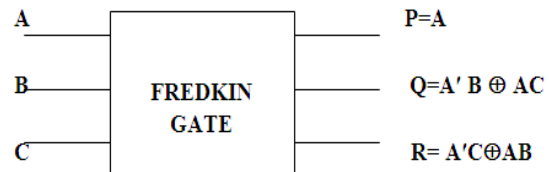


Figure 6: Fredkin Gate

vi. *Peres Gate:*

Figure 7 shows a 3*3 Peres gate. The input vector is I (A, B, C) and the output vector is O (P, Q, R). The outputs are $P = A$, $Q = A \oplus B$ and $R=AB \oplus C$. Quantum cost of a Peres gate is 4[7].

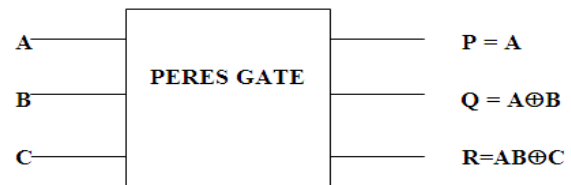


Figure 7: PERES Gate

V. APPLICATIONS OF REVERSIBLE GATES

Reversible computing may have variety of applications in computer security and transaction processing, but the main long-term benefit will be felt very well in those areas which require high energy efficiency, speed and performance. It includes the area such as

- a) Low power CMOS.
- b) Quantum computer.
- c) Nanotechnology.

- d) Optical computing.
- e) DNA computing.
- f) Computer graphics.
- g) Communication.
- h) Design of low power arithmetic and data path for digital signal processing (DSP).
- i) Field Programmable Gate Arrays (FPGAs) in CMOS technology.
- j) The potential application areas of reversible computing include the following
- k) Bio Molecular Computations
- l) Laptop/Handheld/Wearable Computers
- m) Spacecraft
- n) Implanted Medical Devices
- o) Wallet “smart cards”
- p) “Smart tags” on inventory
- q) Prominent application of reversible logic lies in quantum computers.
- r) Quantum gates perform an elementary unitary operation on one, two or more two–state quantum systems called qubits.
- s) Any unitary operation is reversible and hence quantum networks also.
- t) Quantum networks effecting elementary arithmetic operations cannot be directly deduced from their classical Boolean counterparts (classical logic gates such as AND or OR are clearly irreversible).
- u) Thus, Quantum computers must be built from reversible logical components.

VI. CONCLUSION

The reversible circuits form the basic building block of quantum computers. This paper presents the primitive reversible gates which are gathered from literature and this paper helps researchers/designers in designing higher complex computing circuits using reversible gates. The paper can further be extended towards the digital design development using reversible logic circuits which are helpful in quantum computing, low power CMOS, nanotechnology, cryptography, optical computing, DNA computing, digital signal processing (DSP), quantum dot cellular automata, communication, and computer graphics. The reversible logic circuits play a very important role in design of low power digital circuits of a future computer.

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