# PID Back stepping Controller Design for Motion Control of Segway Vehicle

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*Abstract*- **This paper proposes a PID Backstepping controller design method for motion control of Segway vehicle. The Segway vehicle consists of two wheels and chassis, etc. Based on Newton's 2nd law, modeling of wheels and chassis are proposed. A control system is designed based on the modeling of wheels and chassis. The control system consists of three loops composed of balance control loop, rotation control loop for chassis and position control loop for Segway vehicle. The balance control loop and the rotation control loop are designed using Lyapunove functions and using Backstepping method. The position control loop is designed using PD controller. The effectiveness of the proposed control system is shown by simulation results.**

*Index Terms*- **Inverted pendulum, Segway vehicle, Backstepping, PID controller.**

## 1. INTRODUCTION

The stabilization of an unstable nonlinear inverted pendulum system in the upright position is obtained using a PID Backstepping control strategy design technique. Studying and designing controllers for an inverted pendulum system is a suitable way to prove the controller's performance. The inverted pendulums are also used in the study of wheeled motion and balancing mechanisms. The inverted pendulum system can use in medical field as a wheel chair. Inverted pendulum systems can also be used for industrial purpose like a crane.

To test several challenging control algorithms from classical control theories to intelligent control, the inverted pendulum system is used. For example, M. Sasaki et al. [1] proposed the human assisting robots that were simple and lightweight vehicles used as personal ground transportation devices. X. Ruan et al. [2] proposed a two loop cascade adaptive controller using Backstepping and fuzzy neural network for two wheeled self-balancing robot. A. Benaskeur et al. [3] proposed nonlinearLyapunove-based controller to stabilize the inverted pendulum. J. Li et al. [4] presented the method of path planning for two wheeled inverted pendulum mobile robot in the known environment. The path planner based on optimization is proposed.

Because fast response and strong robustness are the main advantages of Backstepping control technique, several researches using a Backstepping controller have been studied in [5~7]. So it is feasible to use Backstepping approach to design a compatible controller for two-wheeled self-balancing robot when the mathematical model of robot is identified.

This paper proposes a control system by using PID Backstepping controller design method for motion control of Segway vehicle. The Segway vehicle consists of two wheels and chassis, etc. Modeling of wheels and chassis based on Newton's 2$^{nd}$ law are proposed. Based on the modeling of wheels and chassis, a control system is designed. The control system consists of three loops composed of balance control loop, rotation control loop for chassis and position control loop for Segway vehicle. The balance control loop and the rotation control loop are designed using Lyapunove functions and using Backstepping method. The position control loop is designed using PD controller. The simulation results show the effectiveness of the proposed control system.

## 2. DESCRIPTION AND MODELING OF SYSTEM

The Segway vehicle is composed of inverted pendulum and chassis with two coaxial wheels. Two encoders are used to measure the motor speed. A clinometer sensor is used to detect the angle deviation of the Segway in vertical axis. Fig. 1 shows the picture of the Segway.

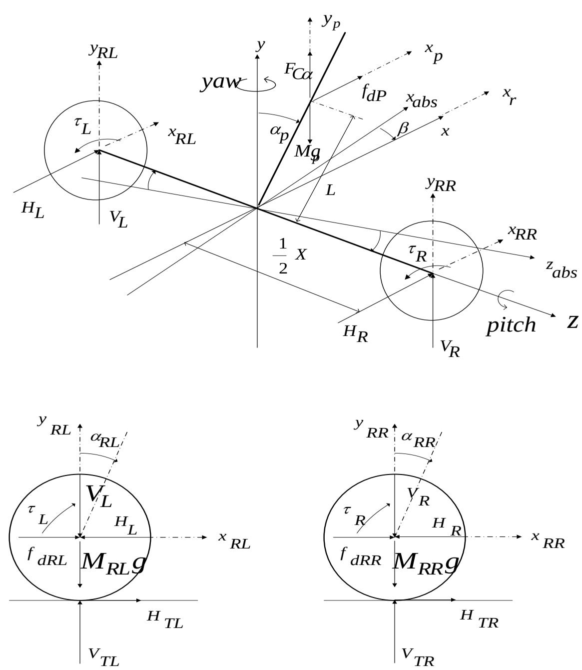
Fig.1 Picture of the Segway

This parameters for the mobile inverted pendulum used in this paper are given in Table 1.

Table 1 Nomenclature of mobile inverted pendulum

| Parameters | Description | Units |
|---|---|---|
| $\alpha_p$ | Pitch angle | [ $rad$ ] |
| $\beta$ | Yaw angle | [ $rad$ ] |
| $I_{RL}$ | Moment of inertia of the wheel | [ $kgm^2$ ] |
| $M_r$ | Mass of the wheel | [ $kg$ ] |
| $I_p$ | Moment of inertia of the inverted pendulum with respect to the z axis | [ $kgm^2$ ] |
| $I_\beta$ | Moment of inertia of the inverted pendulum with respect to the y axis | [ $kgm^2$ ] |
| $M_p$ | Mass of the inverted pendulum | [ $kg$ ] |
| $R$ | Radius of wheel | [ $m$ ] |
| $L$ | Distance between the wheel's center and the pendulum's center of gravity | [ $m$ ] |
| $X$ | Distance between the wheels | [ $m$ ] |
| $x_p$ | Position of the pendulum with respect to the x axis | [ $m$ ] |
| $g$ | Gravitational acceleration | [ $m/s^2$ ] |
| $\tau_L,\tau_R$ | Input torque for left and right wheels | [ $Nm$ ] |
| $x_{RL0},x_{RR0}$ | Position of the left and right wheels | [ $m$ ] |
| $H_L,H_R$ $V_L,V_R$ | Reaction forces between left/right wheels and pendulum | [ $N$ ] |
| $\alpha_{RL0},\alpha_{RR0}$ | Rotational angle of left and right wheels | [ $rad$ ] |
| $f_{dRL},f_{dRR}$ | Disturbance forces to the center of the left/right wheels | [ $N$ ] |
| $f_{dP},F_{C\alpha}$ | Horizontal and vertical disturbance forces to the center gravity of the pendulum | [ $N$ ] |

Fig. 2 shows the free body diagram about motion of the Segway vehicle.




(a) Left wheel (b) Right wheel

Fig.2 Coordinate system of the Segway vehicle

2.1Wheel dynamics

This section describes for wheel's dynamics. The equations for the left wheel only are presented since those for the right wheel are completely similar.

$$M_r\ddot{x}_{RL} = H_{TL} - H_L \quad (1)$$

$$M_r\ddot{y}_{RL} = V_{TL} - V_L - M_r g \quad (2)$$

$$I_r\ddot{\alpha}_{RL} = \tau_L - H_{TL}R \quad (3)$$

Where $M_{RL} = M_{RR} = M_r$; $I_{RL} = I_{RR} = I_r$

Assume the wheels rotate without slipping, the following is obtained.

$$\dot{x}_{RL} = R\dot{\alpha}_{RL} \rightarrow \ddot{x}_{RL} = R\ddot{\alpha}_{RL} \rightarrow \ddot{\alpha}_{RL} = \frac{\ddot{x}_{RL}}{R} \quad (1)$$

From Eqs. (2) ~ (3), horizontal reaction force is expressed into:

$$H_{TL} = \frac{\tau_L}{R} - \frac{I_r}{R^2}\ddot{x}_{RL} \quad (2)$$

By substituting Eq. (4) into Eq. (1), the following is obtained.

$$\left(M_r R + \frac{I_r}{R}\right)\ddot{x}_{RL} = \tau_L - H_L R \quad (3)$$

Similarly, on the right wheel, the following equation is obtained.

$$\left(M_r R + \frac{I_r}{R}\right)\ddot{x}_{RR} = \tau_R - H_R R \quad (1)$$

2.2 Body dynamics

The coordinates of the Segway vehicle's body is given as follows:

$$x_P = L\sin\alpha_P + x_r \quad (1)$$

$$y_P = L\cos\alpha_P \quad (2)$$

where $x_r$ is moving displacement of Segway vehicle.

$$x_r = \frac{x_{RL} + x_{RR}}{2} \quad (3)$$

Applying the 2nd Newton's law to the chassis, the following equation is obtained.

$$M_P\ddot{x}_P = H_L + H_R \quad (4)$$

$$M_P\ddot{y}_P = V_L + V_R - M_P g \quad (5)$$

Taking moments about the center of gravity yields the torque equation as follows:

$$I_p\ddot{\alpha}_P = (V_L + V_R)L\sin\alpha_P - (H_L + H_R)L\cos\alpha_P \quad (6)$$

From Eqs. (6)~ (13),the dynamic equation of Segway vehicle is given as follows:

$$\begin{aligned}&\left(2M_r R + \frac{2I_r}{R} + M_P R\right)\ddot{x}_r + \\ &\qquad M_P RL\left(\ddot{\alpha}_P\cos\alpha_P - \dot{\alpha}_P^2\sin\alpha_P\right) = \tau_L + \tau_R\end{aligned} \quad (7)$$

$$\left(I_p + M_P L^2\right)\ddot{\alpha}_P - M_P gL\sin\alpha_P + M_P L\cos\alpha_P\ddot{x}_r = 0 \quad (8)$$

The moment of the segway with respect to $y$ axis is given by:

$$I_\beta\ddot{\beta} = (H_L - H_R)\frac{X}{2} \quad (9)$$

From Eqs. (6)~ (7) and Eq. (16) the following is obtained. (10)

$$\left[\frac{2I_\beta R}{X} + \left(M_r R^2 + I_r\right)X\right]\ddot{\beta} = \tau_L - \tau_R$$

From Eqs. (14) ~ (15) and (17), the dynamic equations of the Segway vehicle are described as follows:

$$\begin{aligned}&\left(M_P RL\cos\alpha_P - \phi_1\phi_2\sec\alpha_P\right)\ddot{\alpha}_P + \phi_1 g\tan\alpha_P \\ &\qquad - M_P RL\dot{\alpha}_P^2\sin\alpha_P = \tau_L + \tau_R\end{aligned} \quad (11)$$

$$\cos\alpha_P\ddot{x}_r = g\sin\alpha_P - \phi_2\ddot{\alpha}_P \quad (12)$$

$$\phi_3\ddot{\beta} = \tau_L - \tau_R \quad (13)$$

where

$$\begin{cases}\phi_1 = 2M_r R + \dfrac{2I_r}{R} + M_P R \\ \phi_2 = \dfrac{\left(I_p + M_P L^2\right)}{M_P L} \\ \phi_3 = \dfrac{2I_\beta R}{X} + \left(M_r R^2 + I_r\right)X\end{cases} \quad (14)$$

## 3. CONTROLLER DESIGN

The structure of the control system of the Segway vehicle has three control loops composed of balance control loop, rotation control loop and position control loop as shown in Fig.3.

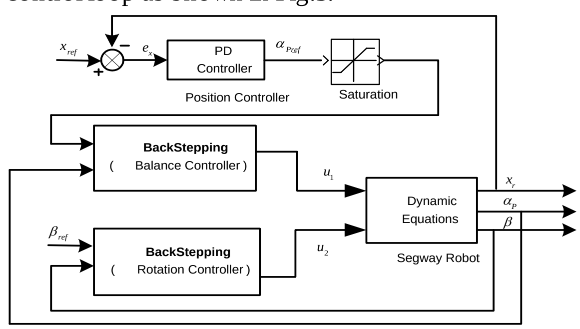



Fig. 3 Structure of the control system for Segway vehicle

Based on the dynamic equations, Eq. (18) and Eq. (20), balance controller and rotation controller are designed using Backstepping control method; the stabilization of the system is guaranteed by Lyapunove function.

3.1 Backstepping controller design

Backstepping controller design for Segway vehicle's balance is as follows:

Step 1a: Defining $x_1 = \alpha_P$ and $x_2 = \dot{\alpha}_P$, Eq. (18) is rewritten as follows:

$$g(x_1)\dot{x}_2 + h(x_1) = u_1 \quad (1)$$

$$h(x_1) = \phi_1 g \tan x_1 - M_P RL x_2^2 \sin x_1 \tag{2}$$

$$g(x_1) = M_P RL \cos x_1 - \phi_1 \phi_2 \sec x_1 \tag{3}$$

where $u_1 = \tau_1 + \tau_2$ is the first control input;
Using Eqs.(21) and (24), the following is obtained;

$$0 < M_p RL \cos x_1 < M_p RL < \phi_1 \phi_2 \tag{4}$$

Because $g(x_1) \neq 0$ from Eqs.(24) and (25), Eq. (22) is,

$$\dot{x}_2 = \frac{u_1 - h(x_1)}{g(x_1)} \tag{5}$$

The tracking pitch angle error of the chassis is defined as follows:

$$e_1 = x_1 - x_{1ref} \tag{6}$$

where $x_{1ref} = \alpha_{pref}$ is the reference value for $\alpha_p$.

The virtual control $x_{2*}$ is defined as:

$$x_{2*} = -k_1 e_1 + \dot{x}_{1\mathrm{ref}} \tag{7}$$

The angular velocity error between the virtual control $x_{2*}$ and $x_2$ as follows:

$$e_2 = x_2 - x_{2*} \tag{8}$$

The derivation of $e_1$ and $e_2$ are as follows:

$$\dot{e}_1 = \dot{x}_1 - \dot{x}_{1ref} = x_2 - \dot{x}_{1ref} = e_2 - k_1 e_1 \tag{9}$$

$$\dot{e}_2 = \dot{x}_2 - \dot{x}_{2*} = \dot{x}_2 + k_1 \dot{e}_1 - \ddot{x}_{1\mathrm{ref}} \tag{10}$$

Substitute Eq. (30) to Eq. (31), the following obtained.

$$\dot{e}_2 = \dot{x}_2 + k_1 e_2 - k_1^2 e_1 - \ddot{x}_{1\mathrm{ref}} \tag{11}$$

Step 2a: the Lyapunove function is defined as follows:

$$V_1 = \frac{1}{2} e_1^2 + \frac{1}{2} e_2^2 \tag{12}$$

The derivative of $V_1$ is as follows:

$$\begin{aligned}
\dot{V}_1 &= e_1 \dot{e}_1 + e_2 \dot{e}_2 \\
&= e_1 \left(e_2 - k_1 e_1\right) + e_2 \left(\dot{x}_2 + k_1 e_2 - k_1^2 e_1 - \ddot{x}_{1\mathrm{ref}}\right) \\
&= -k_1 e_1^2 - k_2 e_2^2 \\
&\quad + e_2 \left[ \frac{u_1 - h(x_1)}{g(x_1)} + \left(k_1 + k_2\right) e_2 + \left(1 - k_1^2\right) e_1 - \ddot{x}_{1\mathrm{ref}} \right]
\end{aligned} \tag{13}$$

To make $\dot{V}_1$ negative, the first control input is chosen as follows:

$$u_1 = h(x) + g(x) \left[ \ddot{x}_{1\mathrm{ref}} - \left(k_1 + k_2\right) e_2 - \left(1 - k_1^2\right) e_1 \right] \tag{14}$$

Substituting Eq. (35) to Eq. (34), the derivative of the Lyapunove function is given by:

$$\dot{V}_1 = -k_1 e_1^2 - k_2 e_2^2 \leq 0 \tag{15}$$

Similarly, the procedure of Backstepping controller design for Segway vehicle's rotation is as follows:

Step 1b: Defining $x_3 = \beta$ and $x_4 = \dot{\beta}$, the Eq. (20) is rewritten as follows:

$$\phi_3 \dot{x}_4 = u_2 \tag{16}$$

where $u_2 = \tau_L - \tau_R$.

The tracking yaw angle errors and the tracking yaw angular velocity error are defined respectively as follows:

$$e_3 = x_3 - x_{3ref} \tag{17}$$

$$e_4 = x_4 - x_{4*} \tag{18}$$

where $x_{3ref} = \beta_{ref}$ is the reference value of yaw angle $x_3$ and the virtual control $x_{4*}$ is chosen as follows:

$$x_{4*} = -k_3 e_3 + \dot{x}_{3\mathrm{ref}} \tag{19}$$

Step 2a: the Lyapunove function is defined as follows:

$$V_2 = \frac{1}{2} e_3^2 + \frac{1}{2} e_4^2 \tag{20}$$

The derivative of $V_1$ is as follows:

$$\begin{aligned}
\dot{V}_1 &= e_3 \dot{e}_3 + e_4 \dot{e}_4 \\
&= -k_3 e_1^2 - k_4 e_2^2 \\
&\quad + e_2 \left[ \frac{u_1}{\phi_3} + \left(k_3 + k_4\right) e_4 + \left(1 - k_3^2\right) e_3 - \ddot{x}_{3\mathrm{ref}} \right]
\end{aligned} \tag{21}$$

To make $\dot{V}_2$ negative, the second control input is chosen as follows:

$$u_1 = \phi_3 \left[ \ddot{x}_{3\mathrm{ref}} - \left(k_3 + k_4\right) e_4 - \left(1 - k_3^2\right) e \right] \tag{22}$$

3.2 Position controller design

When the Segway vehicle is in upright position and doesn't move, the reference angle is set to zero, $(\alpha_{Pref} = 0)$. If the Segway vehicle moves forward, the reference angle is positive, $(\alpha_{Pref} > 0)$. If the Segway vehicle moves backward, the reference angle is negative, $(\alpha_{Pref} < 0)$. A PD controller is designed to control the position of the Segway vehicle. Control law for the PD controller is as follows:

$$\begin{cases} e_x = x_{ref} - x_r \\ \alpha_{pref} = K_{px} e_x + K_{dx} \dot{e}_x \end{cases} \tag{1}$$

where $e_x$ is the displacement error of Segway vehicle and $x_{ref}$ is the reference value for $x_r$

## 4. SIMULATION RESULTS

To examine the effectiveness of the proposed controllers, simulations have been done for the motion control of Segway vehicle. In the simulation, the numerical parameter values, and the initial values are given in Table 1 and Table 2.

Table 1 Numerical parameter values an and initial values

| Parameters | Values | Units |
|---|---|---|
| $M_p$ | 26 | [kg] |
| $M_r$ | 10.7 | [kg] |
| $X$ | 0.566 | [m] |
| $L$ | 0.14 | m |
| $g$ | 9.81 | [m/s$^2$] |
| $I_r$ | 0.8 | [$kgm^2$] |
| $I_\beta$ | 3.2 | [$kgm^2$] |
| $I_p$ | 2.4 | [$kgm^2$] |
| $\alpha_0$ | 0.2 | [rad] |
| $x_0$ | 0 | [m] |
| $\beta_0$ | 0 | [rad] |
| $\alpha_{pref}$ | 0 | [rad] |
| $x_{ref}$ | 10 | [m] |
| $\beta_{ref}$ | 1 | [rad] |
| $R$ | 0.22 | m |

The gains of Backstepping controllers are determined with $k_1 = k_2 = 5$; $k_3 = k_4 = 5$ and the gains of PID controller is determined with $k_p = 0.05$ and $k_d = 0.1$. Fig. 4 shows the yaw angle. The setting time is about 1.8 second. Fig. 5 shows the pitch angle. The dot line indicates reference pitch angle, $\alpha_{ref}$ obtained from the PD controller. The continuous line is the pitch angle, $\alpha_p$. In Fig. 5 the pitch angle tracks the reference pitch angle very well and it goes to zero after 12 second. It is the same time when the Segway vehicle gets to desired position in Fig. 6. Fig. 7 shows the control inputs $u_1$ and $u_2$. The first control input, $u_1$, converges to zero after 2 second while the second control input, $u_2$, converges to zero after 6 second and $u_2$ is changed quickly and largely because of variation of the reference pitch angle.

Fig. 8 shows the Segway vehicle used in experiment and Fig. 9 shows program in Simulink for controlling the Segway vehicle using the proposed controller.

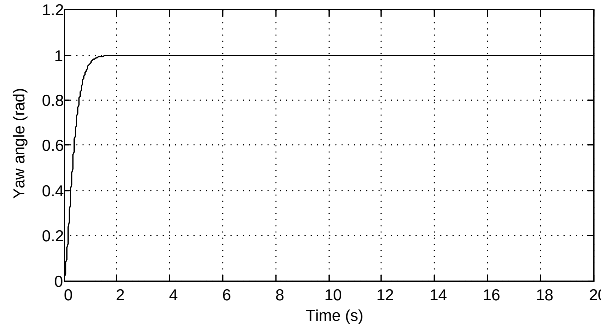



Fig. 4 Yaw angle, $\beta$

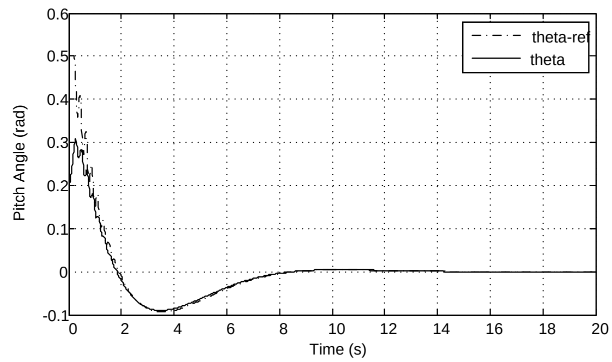



Fig. 5 Pitch angle, $\alpha_P$

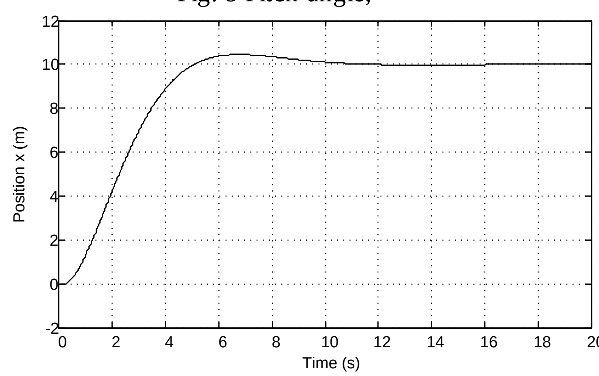



Fig. 6 Segway position, $x_r$

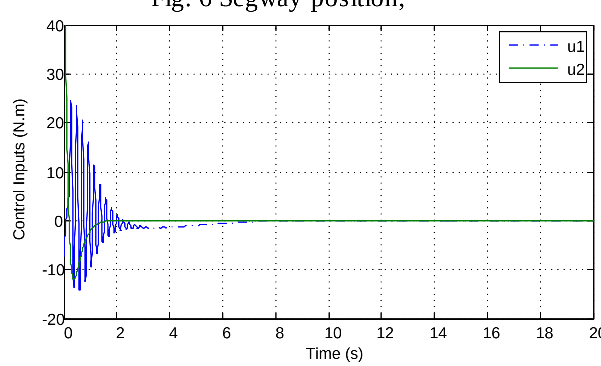



Fig. 7 Control inputs, $u_1, u_2$

Fig. 8 Segway vehicle used in Experiment

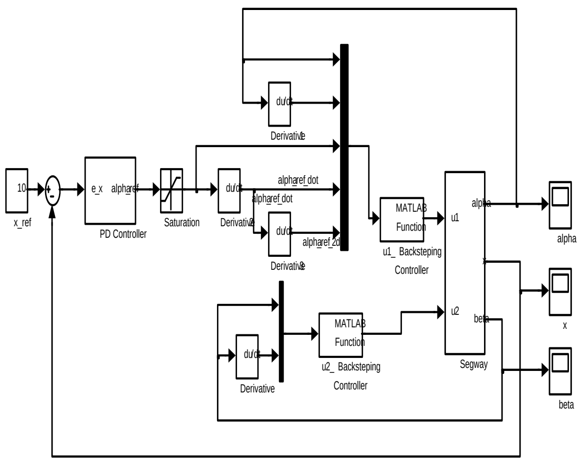



Fig. 9 Simulink program for controlling the Segway vehicle using the proposed controller.

## 5. CONCLUSION

This paper presented the balance control and motion control of Segway vehicle. We used PID Backstepping controller. The control system consists of three loop, balance control loop, rotation control loop and position control loop. Based on the dynamic equations, the first two control loops is designed based on stabilization of Lyapunove function. The position control loop is achieved by using PD controller. The simulation results have shown that the proposed control system has good performance in terms of good balance and stability of motion.

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