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Cubic Trigonometric Rational Wat Bezier Curves

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Abstract- **A new kind of Rational cubic Bézier basis function by the blending of algebraic polynomials and trigonometric polynomials using weight method is presented, named WAT Rational Bézier basis are used to developed rational cubic Bezier curve. Here weight coefficients are also shape parameters, which are called weight parameters. The interval [0, 1] of weight parameter values can be extended to [-2,2.33]and the corresponding WAT Bézier curves and surfaces are defined by the introduced new rational basis functions. The WAT Bézier curves inherit most of properties similar to those of Cubic Bézier curves, and can be adjusted easily by using the shape parameter λ. With the shape parameter chosen properly, the defined curves can express exactly any plane curves or space curves defined by parametric equation based on{1, sint, cost, sint2t, cos2t} and circular helix with high degree of accuracy without using rational form. Examples are given to illustrate that the curves and surfaces can be used as an efficient new model for geometric design in the fields of CAGD. This WAT Bézier curves much closer to the control point compare to the other curves. The geometric effect of the of this weight parameter is discussed.**

Index Terms- **Bézier curves and surfaces, trigonometric polynomial, shape parameter, G2 and C2 continuity.**

1. INTRODUCTION

Computer aided geometric design (CAGD) studies the construction and manipulation of curves and surfaces using polynomial, rational, piecewise polynomial or piecewise rational methods. Among many generalizations of polynomial splines, the trigonometric splines are of particular theoretical interest and practical importance. In recent years, trigonometric splines with shape parameters have gained wide spread application in particular in curve design. Bézier form of parametric curve is frequently used in CAD and CAGD applications like data fitting and font designing, because it has a concise and geometrically significant presentation. Smooth curve representation of scientific data is also of great interest in the field of data visualization. Key idea of data visualization is the graphical representation of information in a clear and effective manner. In the recent past, a number of authors and references have contributed to the shape-preserving interpolation. In [1-6], different polynomial methods, which are used to generate the shape-preserving interpolant, have been considered. In this paper, we present a class of new different trigonometric polynomial rational basis functions with a parameter based on the space Ω=span {1, sin$t$, cos$t$, sint2t, cos2t}, and the corresponding curves and tensor product surfaces named WAT Rational trigonometric Bézier curves and surfaces are constructed based on the introduced new idea of rational basis functions. The WAT Rational Cubic trigonometric Bézier curves not only inherit most of the similar properties to Cubicc Bézier curves, but also can express any plane curves or space curves defined by parametric equation based on {1, sin$t$, cos$t$, sint2t, cos2t} including some quadratic curves such as the circular arcs, parabolas, cardioid exactly and circular helix with high degree of accuracy under the appropriate conditions..

In this paper, we present a WAT Rational Cubic Trigonometric Bézier curves with weight parameter based on the blending space span. Also the change range of the curves is wider than that of C-Bézier curves. The paths of the given curves are line segments. Some transcendental curves can be represented by the WAT with the shape parameters and control points chosen properly. The rest of this paper is organized as follows. Section 2, defines the WAT-Bezier rational basis Functions and the corresponding curves and surfaces, theirs properties are discussed. In section 3, we discussed the shape control of the WAT Rational Cubic Trigonometric Bézier curves. In section 4 Approximation of WAT Rational Cubic Trigonometric Bézier curves to the ordinary WAT Cubic Trigonometric Bézier curve and ordinary Cubic Bézier curves are presented. In section 5 we discussed continuity conditions of

WAT-Rational Bezier curves. In section 6, we show the representations of some curves. Besides, some examples of shape modeling by using the WAT-Bezier surfaces are presented also. The conclusions are given in section 7.

2. WAT- RATIONAL BASIS FUNCTIONS, WAT-BEZIER CURVES AND SURFACES

2.1 The Construction of the WAT-Bézier Rational Basis Functions

The WAT cubic Bezier basis function defined by Xie et. al. [ ] are as follows:

Definition 2.1.1: For $0 \le \lambda \le 1$, the following four functions of $t \in [0, 1]$, are defined as WAT cubic Bezier basis functions

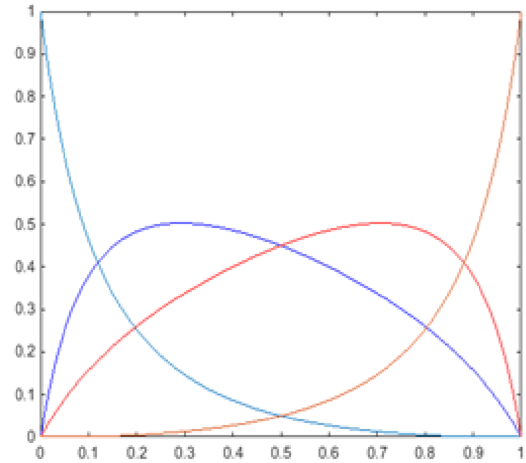
$$
\begin{aligned}
b_0(t,\lambda) &= \lambda(1-t)^3 + (1-\lambda)\frac{\pi(1-t)-\sin\pi t}{\pi},\\
b_1(t,\lambda) &= 3\lambda(1-t)^2 t + (1-\lambda)\left(\frac{1}{2}+t+\frac{1}{2}\cos\pi t+\frac{\sin\pi t}{\pi}\right),\\
b_2(t,\lambda) &= 3\lambda(1-t)t^2 + (1-\lambda)\left(\frac{1}{2}-t-\frac{1}{2}\cos\pi t+\frac{\sin\pi t}{\pi}\right),\\
b_4(t,\lambda) &= \lambda t^3 + (1-\lambda)\frac{\pi t-\sin\pi t}{\pi},
\end{aligned}
\tag{1}
$$

Definition 2.1.2: For $t \in [0, 1]$ and $0 \le \lambda \le 1$, we define a WAT rational cubic Bezier basis functions with a shape parameter λ:

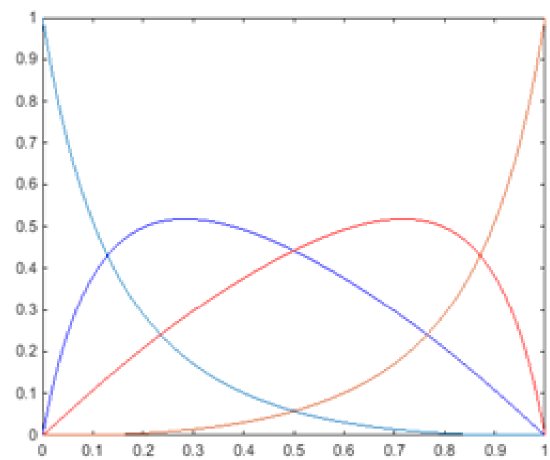
$$
\begin{aligned}
b_0(t,\lambda) &= \frac{w_0\left[\lambda(1-t)^3 + (1-\lambda)\frac{\pi(1-t)-\sin\pi t}{\pi}\right]}{\sum_{i=0}^{3} w_i c_i(t)},\\
b_1(t,\lambda) &= \frac{w_1\left[3\lambda(1-t)^2 t + (1-\lambda)\left(-\frac{1}{2}+t+\frac{1}{2}\cos\pi t+\frac{\sin\pi t}{\pi}\right)\right]}{\sum_{i=0}^{3} w_i c_i(t)},\\
b_2(t,\lambda) &= \frac{w_2\left[3\lambda(1-t)t^2 + (1-\lambda)\left(\frac{1}{2}-t-\frac{1}{2}\cos\pi t+\frac{\sin\pi t}{\pi}\right)\right]}{\sum_{i=0}^{3} w_i c_i(t)},\\
b_3(t,\lambda) &= \frac{w_3\left[\lambda t^3 + (1-\lambda)\frac{\pi t-\sin\pi t}{\pi}\right]}{\sum_{i=0}^{3} w_i c_i(t)},
\end{aligned}
$$

(2)

Where $b_i(t,\lambda)$ (i=0, 1, 2, 3) are the WAT cubic Bezier basis functions defined in eq. (1).



(a)



(b)

Figure 1. (a) For $\lambda = 0.5$, (b) For $\lambda = 0.75$

2.1.3The Properties of the WAT Rational Cubic Basis Functions

Theorem 1: The basis functions (2.1) have the following properties:

Straight calculation testifies that these WAT-Bézier bases have the properties similar to the cubic Bernstein basis as follows.

1) Properties at the endpoints:

$$b_0(0,\lambda)=1, b_i^{(j)}(0,\lambda)=0,$$

$$b_3(1,\lambda)=1, b_{i-3}^{(j)}(1,\lambda)=0,$$

Where j = 0, 1, 2…i-1, i = 1, 2, 3 and

$$b_i^{(0)}(t,\lambda)=b_i(t,\lambda)$$

2) Symmetry:

$$b_1(t,\lambda)=b_2(1-t,\lambda)$$
$$b_0(t,\lambda)=b_3(1-t,\lambda)$$

3) Partition of unity:

$$\sum_{i=0}^{3} b_i(t,\lambda) = 1$$

4) Nonnegativity:

$$b_i(t,\lambda) \geq 0; i = 0,1,2,3.$$

According to the method of extending definition interval of C-curves in Ref., the interval [0, 1] of weight parameter values can be extended to $\left[-2, \frac{14\pi^2 + 8\pi - 96}{8\pi - 54}\right]$ where $\frac{14\pi^2 + 8\pi - 96}{8\pi - 54}$ = 2.33.

2.2 WAT- RATIONAL BÉZIER CURVES

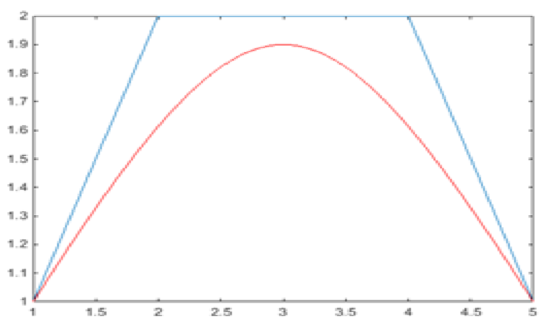
2.2.1 The Construction of the WAT- Rational Bézier Curves

Definition 2.2.1 Given points P_k(k = 0, 1, 2, 3) in R^2 or R^3 , then

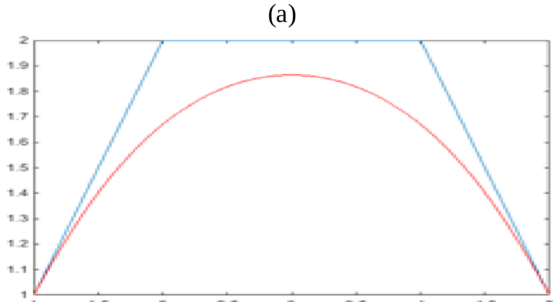
$$R(t,\lambda) = \frac{\sum_{i=0}^{3} P_i w_i b_i(t,\lambda)}{\sum_{i=0}^{3} w_i b_i(t,\lambda)}$$

;t $\in$[0,1] for i = 0, 1, 2,3. $\lambda \in$ [-2, 2.33], (2)

This $R(\lambda, t)$ is called WAT- Rational Bezier curve and $b_i(t,\lambda) \geq 0; i = 0,1,2,3.$ are the WAT-Bezier basis.



(a)



(b)

Figure 2.(a) For λ = 0.5, (b) For λ = 0.75

2.2.2 The Properties of the WAT- Rational Bezier Curve

From the definition of the basis function some properties of the WAT- Rational Bezier curve can be obtained as follows:

Theorem 2 : The WAT- Rational Bezier curve curves (2.2.1) have the following properties:

- Terminal Properties

$$R(0,\lambda) = P_{0,} \quad R(1,\lambda) = P_{4,}$$

$$R'(0,\lambda) = \frac{(\lambda+2)w_1}{w_0}(P_1 - P_0)$$

$$R'(1,\lambda) = \frac{(\lambda+2)w_2}{w_3}(P_3 - P_2) \quad (3)$$

- Symmetry: Assume we keep the location of control points P_i (i = 0,1,2,3,4) fixed, invert their orders, and then the obtained curve coincides with the former one with opposite directions. In fact, from the symmetry of WAT-Bezier base functions, we have
- R(1- t , λ , P_3, P_2, P_1, P_0) = R(t , λ , P_0, P_1, P_2, P_3) ; t ∈ [0,1] , λ∈ [-2, 2.33],
- Geometric Invariance: The shape of a WAT- Rational Bezier curve is independent of the choice of coordinates, i.e. (2.2.1) satisfies the following two equations:

R(t; λ; P_0 + q , P_1 +q, P_2 +q, ,P_3 +q) = R(1- t; λ; P_3, P_2, P_1, P_0) + q ;

R(t; λ; P_0 *T , P_1 *T , P_2*T , ,P_3 *T) = R(1- t; λ; P_3, P_2, P_1, P_0) * T ;

Where q is arbitrary vector in R^2 or R^3 and T is an arbitrary d * d matrix, d = 2 or 3.

- Convex Hull Property: The entire WAT-Rational Bezier curve segment lies inside its control polygon spanned by P_0, P_1, P_2, P_3.

2.3 WAT Rational Bézier Surfaces

Definition 2.3 Given the control mesh [P_{rs}] (r = i … ,i + 2; s = j…, j + 2),(i = 0, 1 ,… , n – 1; j = 0,1,…, m −1) , Tensor product WAT- Rational Bézier surfaces can be defined as

$$R_{i,j}(u,v) = \frac{\sum_{r=i}^{3}\sum_{s=j}^{3} w_i b_i(u,\lambda_1) b_i(v,\lambda_2) P_{rs}(u,v)}{\sum_{r=i}^{3}\sum_{s=j}^{3} w_i b_i(u,\lambda_1) b_i(v,\lambda_2)}; (u,v) \in [0,1]\times[0,1] \quad (4)$$

Where $b_i(u,\lambda_1)$ and a $b_i(v,\lambda_2)$ are WAT-Bezier base function.

3.SHAPE CONTROL OF WAT- RATIONAL BEZIER CURVE

Due to the interval [0, 1] of weight parameter values can be extended to[−2, 2.33], the change range of the WAT- Rational Bézier curve is wider than that of C-Bézier. From the Figure 3, it can be seen that when the control polygon is fixed, by adjusting the weight parameter from -2 to 2.33, the WAT- Rational Bézier curves can cross the cubic Bézier curves and reach the both sides of cubic Bézier curves, in other words, the WAT- Rational Bézier curves can range from below the C-Bézier curve to above the cubic Bézier curve. The weight parameters have the property of geometry. The larger the shape parameter is, and the more approach the curves to the control polygon is. Also, these WAT- Rational Bézier curves we defined include C-Bézier curve ($\alpha = \pi$) as special cases. So that WAT- Rational Bezier Curve have more advantages in shape adjusting than that C-Bezier curves do. Figure 3. Shows the effect on the shape of WAT-Rational Bezier Curve with altering the values of λ = -2 in green, λ = 0 in red, λ = 1 in blue and λ = 2.33 in yellow.

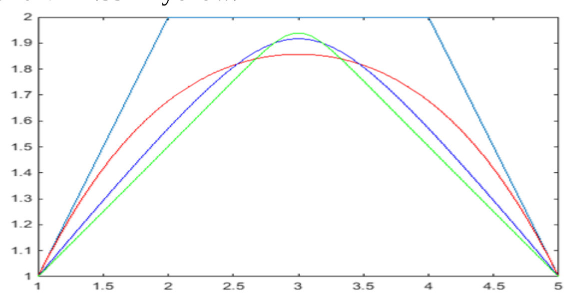


Figure 3. The effect on the shape of WAT-Rational Bezier Curve with altering the values of λ = -2 in green, 0 in blue and 1 in red.

4. APPROXIMABILITY

Control polygon provides an important tool in geometric modeling. It is an advantage if the curve being modeled tends to preserve the shape of its control polygon. Now we show the relations of the cubic trigonometric rational B-spline curve and cubic Bézier curves corresponding to their control polygon.

Theorem 3: Suppose P_0,P_1,P_2 and P_3 are not collinear; the relationship between cubic trigonometric rational B-spline curve $R(t,\lambda)$ (2.2.1) and the Rational cubic Bézier curve , t ∈ [0, 1] with the same control points P_i (i=0,1,2,3) are given by

$$B(t) = \frac{\sum_{i=0}^{3} P_i w_i B_i(t)}{\sum_{i=0}^{3} w_i B_i(t)} \tag{5}$$

Where $B_i(t) = \sum_{j=0}^{3} P_i(3, j)(1-t)^{3-j} t^j$ are Bernstein polynomial and the scalar w_i are the weight function.

$$R(\frac{1}{2},\lambda) - P^* = a\left[B(\frac{1}{2},\lambda)\right] - P^*; \tag{6}$$

Where $a = \dfrac{112-70\lambda}{14\pi^2+8(1-\lambda)(\pi+2)}$,

$$b = \frac{80-80\lambda+16\lambda\pi-16\pi+14\pi^2}{14\pi^2+8(1-\lambda)(\pi+2)}$$

,and $P^* = \dfrac{P_0+P_3}{2}$

Proof: we assume that $w_0 = w_3 = 1$ and $w_1 = w_2 = 2$ then the ordinary Rational cubic Bezier curve (5) takes the form

$$B(t) = \frac{(1-t)^3P_0 + 6(1-t)^2tP_1 + 6(1-t)t^2P_2 + t^3P_3}{(1-t)^3 + 6(1-t)^2t + 6(1-t)t^2 + t^3}$$

By simple computation, we have

$$B(0) = R\ (0,\lambda) = P_0;$$

$$B(1) = R\ (1,\lambda) = P_3\ ;$$

$$B(\frac{1}{2}) = \frac{1}{14}(P_0 + 6P_1 + 6P_2 + P_3)$$

$$B(\frac{1}{2}) - P^* = \frac{1}{14}(P_0 - P_1 - P_2 + P_3)$$

Where $P^* = \dfrac{P_0+P_3}{2}$

And for $w_0 = w_3 = 1$ and $w_1 = w_2 = 2$, we have

$$R(\frac{1}{2},\lambda) - P^* = \left[\frac{112-70\lambda}{14\pi^2+8(1-\lambda)(\pi+2)}\right]B(\frac{1}{2}) - \left[\frac{80-80\lambda+16\lambda\pi-16\pi+14\pi^2}{14\pi^2+8(1-\lambda)(\pi+2)}\right]P^*$$

$$R(\frac{1}{2},\lambda) - P^* = a\left[B(\frac{1}{2},\lambda)\right] - P^*;$$

Where $a = \dfrac{112-70\lambda}{14\pi^2+8(1-\lambda)(\pi+2)}$,

$$b = \frac{80-80\lambda+16\lambda\pi-16\pi+14\pi^2}{14\pi^2+8(1-\lambda)(\pi+2)}$$

and

$$P^* = \frac{P_0+P_3}{2}$$

Equation (6) holds. These equations show that Cubic WAT- Rational Bezier Curve can be made closer to the control polygon by altering the values of shape parameters.

Corollary 3.1: The quadratic trigonometric Bézier Curves with tension parameter is closer to the control polygon

that the cubic Bezier curve if and only if

$$\lambda \in \left[-2, \frac{14\pi^2 + 8\pi - 96}{8\pi - 54}\right]$$

Corollary 3.2: when a = b = 1 the Cubic WAT-Rational Bezier Curve can be closer to the cubic Rational Bézier Curve, i.e. $B(\frac{1}{2}) = R\ (\frac{1}{2}, \lambda);$

Figure 4. show the relationship among the Cubic WAT- Rational Bezier Curve (yellow line), the cubic Rational Bézier Curve with shape parameter (blue line) and the cubic Bézier Curve (green line).

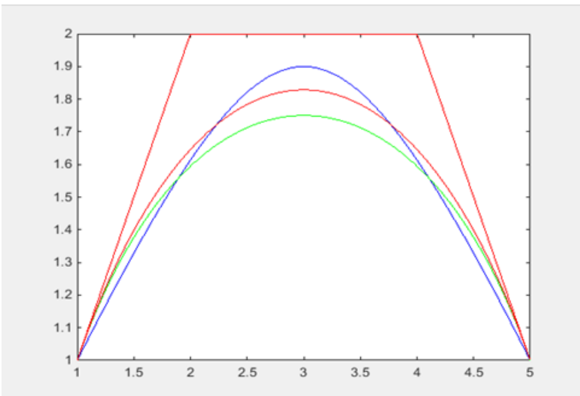


Figure 4. Relationship among the Cubic WAT-Rational Bezier Curve, the cubic WAT- Bézier Curve with shape parameter and the cubic Bézier Curve.

5. JOINTING OF WAT- RATIONAL BÉZIER CURVES

Suppose there are two segment of WAT- Rational Cubic Bezier curves

$$R(t,\lambda_1) = \frac{\sum_{i=0}^{3} P_i w_i b_i(t,\lambda_1)}{\sum_{i=0}^{3} w_i b_i(t,\lambda_1)}$$ and

$$Q(t,\lambda_2) = \frac{\sum_{i=0}^{3} Q_i w_i b_i(t,\lambda_2)}{\sum_{i=0}^{3} w_i b_i(t,\lambda_2)};$$

where $P_3 = Q_0$, parameters of $R(t,\lambda_1)$ and $Q(t,\lambda_2)$ are λ_1 and λ_2 respectively.

To achieve G^1 continuity of the two curve segments, it is required that not only the last control point of $R(t,\lambda_1)$ and the first control point of $Q(t,\lambda_2)$ must be the same, but also the direction of the first order derivative at jointing point should be the same, namely

$$R'(\lambda,1) = kQ'(\lambda,0) \quad ;(k \geq 1)$$

Substituting Eq. (3) into the above equitation, one can get

$$\frac{w_2}{w_3}(2+\lambda_1)(P_3 - P_2) = k\frac{w_1}{w_0}(2+\lambda_2)(Q_1 - Q_0)$$

Let $\delta = k\frac{w_1 w_3}{w_0 w_2}\left(\frac{2+\lambda_2}{2+\lambda_1}\right)$, substituting it into the above equitation, then

$$(P_3 - P_2) = \delta(Q_1 - Q_0)(\delta > 0)$$

Especially, for k=1, namely, $\delta = \frac{w_1 w_3}{w_0 w_2}\left(\frac{2+\lambda_2}{2+\lambda_1}\right)$

the first order derivative of two segment of curves is equal .Thus, G^1 continuity has transformed into C^1 continuity. Then we can get following theorem 4.

Theorem 4 If P_2, P_3 and Q_0, Q_1 is collinear and have the same directions, i.e.

$$(P_3 - P_2) = \delta\ (Q_1 - Q_0)\ \ (\delta > 0) \qquad (7)$$

Then curves of $P(t)$ and $Q(t)$ will reach G^1 continuity at a jointing point and when $\delta = \frac{w_1 w_3}{w_0 w_2}\left(\frac{2+\lambda_2}{2+\lambda_1}\right)$ they will get C^1 continuity.

Then we will discuss continuity conditions of G^2 when $\lambda_1 = \lambda_2 = 1$.

First, we'll discuss conditions of G^2 continuity which is required to have common curvature, namely

$$\frac{\left|R'(1) \times R''(1)\right|}{\left|R''(1)\right|^3} = \frac{\left|Q'(0) \times Q''(0)\right|}{\left|Q''(0)\right|^3} \qquad (8)$$

Let $\lambda_1 = \lambda_2 = 1$, second derivatives of two segments of curves can be get

$$R''(1,\lambda_1) = \frac{1}{w_3^2}\left\{\left[6\lambda_1 + \frac{\pi^2}{2}(1-\lambda_1)\right]w_1 w_3(P_1 - P_3) + \left[\left(-12\lambda_1 - \frac{\pi^2}{2}(1-\lambda_1) - 2(2+\lambda_1)^2\right)w_2 w_3 + 2(2+\lambda_1)^2 w_2^2\right](P_2 - P_3)\right\}$$

(9)

$$Q''(0,\lambda_2) = \frac{1}{w_0^2}\left\{\left[6\lambda_2 + \frac{\pi^2}{2}(1-\lambda_2)\right]w_0 w_2(P_2 - P_0) + \left[\left(-12\lambda_2 - \frac{\pi^2}{2}(1-\lambda_2) - 2(2+\lambda_2)^2\right)w_0 w_1 + 2(2+\lambda_1)^2 w_1^2\right](P_1 - P_0)\right\}$$

Substituting Eq. (3) and (9) into Eq. (8), simplifying it, then

$$\frac{|(P_3-P_2)\times(P_2-P_1)|}{|P_3-P_2|^3}=\frac{|(Q_1-Q_0)\times(Q_2-Q_1)|}{|Q_1-Q_0|^3} \quad (10)$$

Substituting Eq. (7) into the above equitation, one can get

$h_1 = \delta^2 h_2$

where h_1 is the distance from P_1 to P_2 P_3 and h_2 is the distance from Q_2 to Q_0 Q_1 . Hence we can get theorem 5.

Theorem 5 Let parameters λ_1 , λ_2 are all equal one, if they satisfy Eq. (7) and (10), five points P_1 , P_2 , P_3 ,Q_1 ,Q_2 are coplanar and P_1 , Q_2 are in the same side of the common tangent, then jointing of curves $R(t,\lambda_1)$ and $Q(t,\lambda_2)$ reach G^2 continuity.

6. APPLICATIONS OF WAT- RATIONAL BÉZIER CURVES AND SURFACES

Proposition 4.1 Let P_0, P_1, P_2 and P_3 be four control points. By proper selection of coordinates, their coordinates can be written in the form

$$P_0 = (0,0), P_1 = \left(\frac{1-\pi}{2}a, o\right), P_2 = \left(\frac{1-\pi}{2}a, 2a\right), P_3 = (a, 2a)\,(a \neq 0)$$

Then the corresponding WAT- Rational Bézier curve with the weight parameters $\lambda = 0$ and $t \in [0,1]$ represents an arc of cycloid.

Proof: If we take P_0, P_1, P_2 and P_3 into (2), then the coordinates of the WAT- Rational Bézier curve are

$$x(t) = a(t - \sin \pi t),$$
$$y(t) = a(1 - \cos \pi t)$$

It is a cycloid in parametric form, see Figure 6.

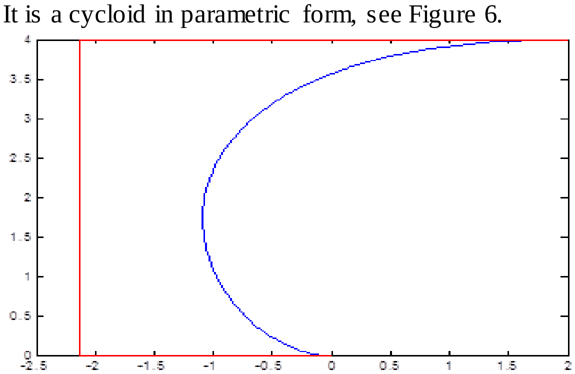


Fig. 6 The Representation of Cycloid With WAT-RATIONAL Bézier Curve

Proposition 4.2 Let P_0, P_1, P_2 and P_3 be four properly chosen control points such that

$$P_0 = (a,0,0), P_1 = \left(0,a,\frac{\pi}{2}b\right), P_2 = \left(-a,a,\frac{\pi}{2}b\right), P_3 = (-a,0,b)\,(a \neq 0, b \neq 0)$$

Then the corresponding WAT-Rational Bézier curve with the weight parameters λ=0 and $t \in [0,1]$ represents an arc of a helix.

Proof: Substituting P_0, P_1, P_2 and P_3 into (2) yields the coordinates of the WAT- Rational Bézier curve

$x(t) = a\cos \pi t$,

$y(t) = a\sin \pi t$,

$z(t) = bt$,

which is parameter equation of a helix, see Figure 7.

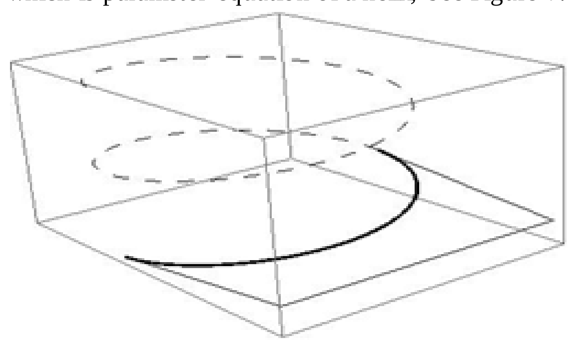


Fig.7 The Representation of Helix With WAT-Rational Bézier Curve

Proposition 4.3 Given the following four control points, P0 = (0,0),P1 = P2 = (a, π2b),P3 = (2a ,0) (ab ≠ 0).Then the corresponding WAT- Rational Bézier curve with the weight parameters $\lambda = 0$ and $t \in [0,1]$ represents a segment of sine curve.

Proof: Substituting P0, P1, P2 and P3 into (2), we get the coordinates of the WAT- Rational Bézier curve, $x(t) = at$ $y(t) = b\sin \pi t$, which implies that the corresponding WAT- Rational Bézier curve represents a segment of sine curve, see Figure5.

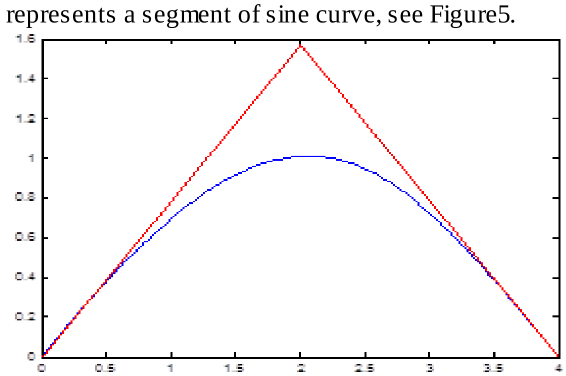


FIG.8 The Representation of Sine Curve with WAT-Rational Bézier Curves

7. CONCLUSIONS

In this paper, the WAT-Rational Bézier curves have the similar properties that cubic Bézier curves have. The jointing of two pieces of curves can reach G^2 and C^2 continuity under the appropriate conditions. The given curves can represent some special transcendental curves. What is more, the paths of the curves are linear, the WAT- Rational Bézier curves have more advantages in shape adjusting than that C-Bézier curves. Both rational methods (NURBS or Rational Bézier curves) and WAT- Rational Bézier curves can deal with both free form curves and the most important analytical shapes for the engineering. However, WAT- Rational Bézier curves are simpler in structure and more stable in calculation. The weight parameters of WAT- Rational Bézier curves have geometric meaning and are easier to determine than the rational weights in rational methods. Furthermore, some complex surfaces can be constructed by these basic surfaces exactly. While the method of traditional Cubic Bézier curves needs joining with many patches of surface in order to satisfy the precision of users for designing. Therefore the method presented by this paper can raise the efficient of constituting surfaces and precision of representation in a large extent Meanwhile, WAT-Rational Bézier curves can represent the helix and the cycloid precisely, but NURBS cannot. Therefore, WAT- Rational Bézier curves would be useful for engineering.

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