IFSGb-continuous mappings in intuitionistic fuzzy topological spaces

Angelin Tidy.G[1], Francina Shalini.A[2]
[1]*Research Scholar, Department of Mathematics, Nirmala College for Women*
[2]*Assistant Professor, Department of Mathematics, Nirmala College for Women*

Abstract- **In this paper is to define and study the concepts of intuitionistic fuzzy sgb-continuous mappings and intuitionistic fuzzy sgb-irresolute mappings on intuitionistic fuzzy topological spaces. Further relationship between intuitionistic fuzzy sgb-continuous mapping with other intuitionistic fuzzy continuous mappings a established. And intuitionistic fuzzy slightly sgb-continuous functions, we investigate some of their properties.**

Index Terms- Intuitionistic fuzzy topology, Intuitionistic fuzzy sgb-continuous mappings, Intuitionistic fuzzy sgb-irresolute mappings and Intuitionistic fuzzy slightly sgb-continuous functions.

1. INTRODUCTION

As a generalization of fuzzy sets, the concepts of intuitionistic fuzzy sets were introduced by Atanassov [4]. Recently, Coker [5] introduced the basic definitions and properties of intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. In 2017, Angelin Tidy and Francina Shalini [2] introduced sgb-continuous and sgb-irresolute in topological spaces. In this paper we introduce intuitionistic fuzzy sgb-continuous mappings and intuitionistic fuzzy sgb-irresolute mappings. And we introduce and study the concepts of intuitionistic fuzzy slightly sgb-continuous in intuitionistic fuzzy topological space.

2. PRELIMINARIES

Definition 2.1: [4] Let X be a nonempty fixed set. An intuitionistic fuzzy set (briefly IFS) A is an object of the form $A = \{\langle x, \mu(x), \nu(x)\rangle : x \in X\}$, where μ and ν are degrees of membership and non-membership of each $x \in X$, respectively, and $0 \leq \mu(x) + \nu(x) \leq 1$ for each $x \in X$. A class of all the IFS's in X is denoted as IFS(X). When there is no danger of confusion, an IFS $A = \{\langle x, \mu(x), \nu(x)\rangle : x \in X\}$ may be written as $A = \langle \mu_A, \nu_A\rangle$.

Definition 2.2: [4] Let X be a nonempty set and $A = \langle \mu_A, \nu_A\rangle$, $B = \langle \mu_B, \nu_B\rangle$ IFSs in X. Then
(1) $A \subseteq B$ if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$, for all $x \in X$,
(2) $A = B$ if $A \subseteq B$ and $B \subseteq A$,
(3) $\bar{A} = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}$,
(4) $A \cap B = \{\langle x,_A \wedge \mu_B, \nu_A \wedge \nu_B\rangle : x \in X\}$ [15],
(5) $A \cup B = \{\langle x,_A(x) \vee \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle : x \in X\}$[15].

Definition 2.3: [4] IFS's $\tilde{0}$ and $\tilde{1}$ are defined as $\tilde{0} = \{\langle x, 0, 1\rangle : x \in X\}$ and $\tilde{1} = \{\langle x, 1, 0\rangle : x \in X\}$, respectively.

Definition 2.4: [10] Let $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$. An intuitionistic fuzzy point (IFP for short) $p_{(\alpha,\beta)}$ of X is an IFS of X defined by

$$p_{(\alpha,\beta)}(x) = \begin{cases} (\alpha, \beta) & \text{if } x = p, \\ (0, 1) & \text{otherwise.} \end{cases}$$

Definition 2.5: [5] An intuitionistic fuzzy topology (IFT for short) on a nonempty set X is a family of IFSs in X satisfying the following axioms:
(1) $\tilde{0}, \tilde{1} \in \tau$,
(2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,
(3) $\cup G_i \in \tau$ for any arbitrary family $\{G_i : i \in J\} \subseteq \tau$.
In this case, the pair (X, τ) is called an intuitionistic fuzzy topological space (briefly, IFTS) and members of τ are called intuitionistic fuzzy open (briefly, IFO) sets. The complement $\bar{A}$ of an IFO set A is called an intuitionistic fuzzy closed (IFC) set in X. Collection of all IFO (resp., IFC) sets in IFTS X is denoted as IFO(X) (resp., IFC(X)).

Definition 2.6: [5] Let (X, τ) be an IFTS and A = ⟨μ_A, ν_A⟩ an IFS in X. Then the fuzzy interior and fuzzy closure of A are denoted and defined as
Cl A = ⋂ {K : K is an IFC set in X and A ⊆ K} ,
Int A = ⋃ {G : G is an IFO set in X and G ⊆ A} .

Definition 2.7: [10] Let $p_{(\alpha,\ \beta)}$ be an IFP in IFTS X. An IFS A in X is called an intuitionistic fuzzy neighborhood (IFN) of $p_{(\alpha,\ \beta)}$ if there exists an IFOS B in X such that $p_{(\alpha,\beta)}$ ∈ B ⊆ A.

Definition 2.8: [3] An IFS A = {⟨x, μ_A(x), ν_A(x) ⟩ / x ∈ X} in an IFTS (X, τ) is said to be
1) intuitionistic fuzzy b- closed set[2] (IFbCS) if cl(int(A)) ⋂int(cl(A)) ⊆ A,
2) intuitionistic fuzzy α-closed set[7] (IFαCS) if cl(int(cl(A))) ⊆ A.

Definition 2.9: [3] An IFS A = {⟨x, μ_A(x), ν_A(x) ⟩ / x ∈ X} in an IFTS (X, τ) is said to be
1) intuitionistic fuzzy b open set[2](IFbOS) if A ⊆int(cl(A)) ⋃ cl(int(A)),
2) intuitionistic fuzzy α-open set[7] (IFαOS) if A ⊆int(cl(int(A))).

Definition 2.10: [3] An IFS A = {⟨x, μ_A(x), ν_A(x) ⟩ / x ∈ X} in an IFTS (X, τ) is said to be
1) intuitionistic fuzzy generalized αclosed set[10] (IFGαCS) if αcl(A) ⊆ U whenever A ⊆ U and U is an IFαOS in (X,τ),
2) intuitionistic fuzzy α generalized semi closed set[8] (IFαGSCS) if αcl(A) ⊆ U whenever A ⊆ U and U is an IFSOS in (X,τ),

Definition 2.11: [3] Let (X, τ) be an IFTS and A = ⟨ x, μ_A, ν_A⟩ be an IFS in (X, τ). Then the intuitionistic fuzzy b closure of A (bcl(A)) and intuitionistic fuzzy b interior of A (bint(A)) are defined as
1) bint(A) = ⋃ { G / G is an IFbOS in X and G ⊆ A},
2) bcl (A) = ⋂ { K / K is an IFbCS in X and A ⊆ K }.

Definition 2.12: [3] An IFS A is said to be an intuitionistic fuzzy semi generalized b-closed set (IFSGbCS) if bcl(A) ⊆ U whenever A ⊆ U and U is an IFSOS in (X,τ).

An IFS A is said to be an intuitionistic fuzzy semi generalized b-open set (IFSGbOS) in (X,τ) if the complement A^c is an IFSGbCS in (X,τ).

Definition 2.13:[1] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an intuitionistic fuzzy totally continuous if in verse image of every intuitionistic fuzzy open set in Y is an intuitionistic fuzzy clopen set in X.

Defnition 2.14: Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an
(1) intuitionistic fuzzy continuous (IF continuous in short) if f^{-1}(B) ∈ IFOS(X)for every B ∈ σ [6],
(2) intuitionistic fuzzy α-continuous (IF α-continuous in short) if f^{-1}(B) ∈ IFαOS(X) for every B ∈ σ [8],

Defnition 2.15: Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an
(1) intuitionistic fuzzy generalized α-continuous(IFGα-continuous in short) if f^{-1}(B) is an IFGαCS for every IFCS B of (Y, σ)[9].
(2) intuitionistic fuzzy α-generalized semi continuous(IFαGS-continuous in short) if f^{-1}(B) is an IFαGSCS for every IFCS B of (Y, σ)[7]

Defnition 2.16: Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an
(1) intuitionistic fuzzy irresolute (IF irresolute in short) if f^{-1}(B) ∈ IFCS(X) for every IFCS B in Y[11],
(2) intuitionistic fuzzy generalized irresolute (IFG irresolute in short) if f^{-1}(B) is IFGCS in X for every IFGCS B in Y[11].

3. INTUTIONISTIC FUZZY SEMI GENERALIZED b-CONTINUOUS MAPPINGS

In this section we introduce intuitionistic fizzy semi generalized b-continuous mapping and study some of its properties.
Definition 3.1: A mapping f: (X,τ) → (Y,σ) is called an intuitionistic fuzzy semi generalized b-continuous (IFSGb continuous) if f^{-1}(B) is an IFSGbCS in (X,τ) for every IFCS in (Y,σ).
Example 3.2: Let X={a,b}, Y={u,v}, G_1= ⟨x, (0.7, 0.8), (0.3, 0.2) ⟩ and G_2= ⟨x, (0.5, 0.6), (0.4, 0.3) ⟩. Then τ = {0~, G_1, 1~} and σ = {0~, G_2, 1~} are IFTs on X and Y respectively. Define a mapping f: (X,τ)

$\rightarrow (Y,\sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IFSGb continuous mapping.

Theorem 3.3: Every IF continuous mapping is an IFSGb continuous mapping.

Proof: Let f: $(X,\tau) \rightarrow (Y,\sigma)$ be an IF continuous mapping. Let A be an IFCS in Y. Since f is an IF continuous mapping, $f^{-1}(A)$ is an IFCS in X. Since every IFCS is an IFSGbCS, $f^{-1}(A)$ is an IFSGbCS in X. Hence f is an IFSGb continuous mapping.

Example 3.4: IFSGb continuous mapping $\nrightarrow$ IF continuous mapping.

Let X={a,b}, Y={u,v}, $G_1 = \langle x, (0.7, 0.8), (0.3, 0.2) \rangle$ and $G_2 = \langle x, (0.5, 0.6), (0.4, 0.3) \rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: $(X,\tau) \rightarrow (Y,\sigma)$ by $f(a) = u$ and $f(b) = v$. Since the IFS $A = \langle x, (0.4, 0.3), (0.5, 0.6) \rangle$ is IFCS in Y, $f^{-1}(A)$ is an IFSGbCS but not IFCS in X. Therefore f is an IFSGb continuous mapping but not an IF continuous mapping.

Theorem 3.5: Every IFα continuous mapping is an IFSGb continuous mapping.

Proof: Let f: $(X,\tau) \rightarrow (Y,\sigma)$ be an IFα continuous mapping. Let A be an IFCS in Y. Then by the hypothesis $f^{-1}(A)$ is an IFαCS in X. Since every IFαCS is an IFSGbCS, $f^{-1}(A)$ is an IFSGbCS in X. Hence f is an IFSGb continuous mapping.

Example 3.6: IFSGb continuous mapping $\nrightarrow$ IFα continuous mapping.

Let X={a,b}, Y={u,v}, $G_1 = \langle x, (0.2, 0.3), (0.7, 0.6) \rangle$ and $G_2 = \langle x, (0.3, 0.4), (0.6, 0.5) \rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: $(X,\tau) \rightarrow (Y,\sigma)$ by $f(a) = u$ and $f(b) = v$. Since the IFS $A = \langle x, (0.6, 0.5), (0.3, 0.4) \rangle$ is IFCS in Y, $f^{-1}(A)$ is an IFSGbCS but not IFαCS in X. Therefore f is an IFSGb continuous mapping but not an IFα continuous mapping.

Theorem 3.7: Every IFGα continuous mapping is an IFSGb continuous mapping.

Proof: Let f: $(X,\tau) \rightarrow (Y,\sigma)$ be an IFGα continuous mapping. Let A be an IFCS in Y. Then by the hypothesis $f^{-1}(A)$ is an IFGαCS in X. Since every IFGαCS is an IFSGbCS, $f^{-1}(A)$ is an IFSGbCS in X. Hence f is an IFSGb continuous mapping.

Example 3.8: IFSGb continuous mapping $\nrightarrow$ IFGα continuous mapping.

Let X={a,b}, Y={u,v}, $G_1 = \langle x, (0.3, 0.4), (0.7, 0.5) \rangle$ and $G_2 = \langle x, (0.6, 0.4), (0.3, 0.5) \rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: $(X,\tau) \rightarrow (Y,\sigma)$ by $f(a) = u$ and $f(b) = v$. Since the IFS $A = \langle x, (0.3, 0.5), (0.6, 0.4) \rangle$ is IFCS in Y, $f^{-1}(A)$ is an IFSGbCS but not IFGαCS in X. Therefore f is an IFSGb continuous mapping but not an IFGα continuous mapping.

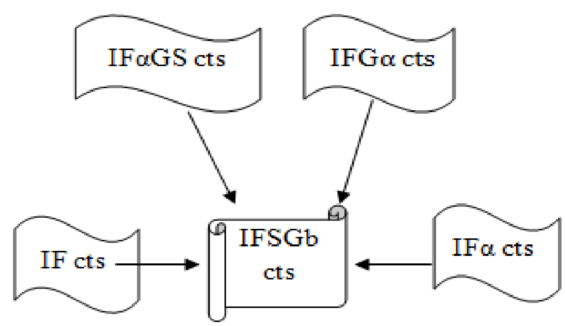
Theorem 3.9: Every IFαGS continuous mapping is an IFSGb continuous mapping.

Proof: Let f: $(X,\tau) \rightarrow (Y,\sigma)$ be an IFαGS continuous mapping. Let A be an IFCS in Y. Then by the hypothesis $f^{-1}(A)$ is an IFαGSCS in X. Since every IFαGSCS is an IFSGbCS, $f^{-1}(A)$ is an IFSGbCS in X. Hence f is an IFSGb continuous mapping.

Example 3.10: IFSGb continuous mapping $\nrightarrow$ IFαGS continuous mapping.

Let X={a,b}, Y={u,v}, $G_1 = \langle x, (0.3, 0.4), (0.7, 0.5) \rangle$ and $G_2 = \langle x, (0.5, 0.4), (0.4, 0.4) \rangle$. Then $\tau = \{0_\sim, G_1, 1_\sim\}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping f: $(X,\tau) \rightarrow (Y,\sigma)$ by $f(a) = u$ and $f(b) = v$. Since the IFS $A = \langle x, (0.4, 0.4), (0.5, 0.4) \rangle$ is IFCS in Y, $f^{-1}(A)$ is an IFSGbCS but not IFαGSCS in X. Therefore f is an IFSGb continuous mapping but not an IFαGS continuous mapping.

Remark 3.11: We obtain the following diagram from the results we discussed above.



Theorem 3.12: A mapping f: $(X,\tau) \rightarrow (Y,\sigma)$ is IFSGb continuous if and only if the inverse image of each IFOS in Y is an IFSGbOS in X.

Proof: ⇒ part

Let A be an IFOS in Y. This implies A^c is IFCS in Y. Since f is IFSGb continuous, $f^{-1}(A^c)$ is IFSGbCS in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IFSGbOS in X.

⇐ part

Let A be an IFCS in Y. Then A^c is an IFOS in Y. By hypothesis $f^{-1}(A^c)$ is IFSGbOS in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $(f^{-1}(A))^c$ is an IFSGbOS in X. Therefore

$f^{-1}(A)$ is an IFSGbCS in X. Hence f is IFSGb continuous.

Theorem 3.13: Let f: $(X, \tau) \to (Y, \sigma)$ be an IFSGb continuous mapping and g: $(Y, \sigma) \to (Z, \delta)$ be an IF continuous, then gof : $(X, \tau) \to (Z, \delta)$ is an IFSGb continuous.

Proof: Let A be an IFCS in Z. Then $g^{-1}(A)$ is an IFCS in Y, by hypothesis. Since f is an IFSGb continuous mapping, $f^{-1}(g^{-1}(A))$ is an IFSGbCS in X. Hence gof is an IFSGb continuous mapping.

Definition 3.14: Let (X, α) be an IFTS. The semi generalized b-closure (sgbcl(A) in short) for any IFS A is defined as follows. $sgbcl(A) = \cap \{K \mid K$ is an IFSGbCS in X and $A \subseteq K\}$. If A is IFSGbCS, then sgbcl(A) = A.

Theorem 3.15: Let f: $(X, \tau) \to (Y, \sigma)$ be an IFSGb continuous mapping. Then the following conditions are hold.

(1) $f(sgbcl(A)) \subseteq cl(f(A))$, for every IFS A in X.

(2) $sgbcl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$, for every IFS B in Y.

Proof: (1) Since cl(f(A)) is an IFCS in Y and f is an IFSGb continuous mapping, $f^{-1}(cl(f(A)))$ is IFSGbCS in X. That is $sgbcl(A) \subseteq f^{-1}(cl(f(A)))$. Therefore $f(sgbcl(A)) \subseteq cl(f(A))$, for every IFS A in X.

(2) Replacing A by $f^{-1}(B)$ in (1) we get $f(sgbcl(f^{-1}(B))) \subseteq cl(f(f^{-1}(B))) \subseteq cl(B)$. Hence $sgbcl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$, for every IFS B in Y.

Remark 3.16: The composition of two IFSGb continuous mappings need not be IFSGb continuous as can be seen from the following example.

Example 3.17: Let X = {a, b} , Y = {u, v} and Z = {s, t}. Let $\tau = \{0_\sim, G_1, 1_\sim\}$, $\sigma = \{0_\sim, G_2, 1_\sim\}$ and $\delta = \{0_\sim, G_3, 1_\sim\}$ be IFTs on X, Y and Z respectively where $G_1 = \langle x, (0.2, 0.4), (0.7, 0.5) \rangle$, $G_2 = \langle x, (0.3, 0.5), (0.6, 0.5) \rangle$ and $G_3 = \langle x, (0.3, 0.2), (0.6, 0.7) \rangle$. Define f: $(X, \tau) \to (Y, \sigma)$ by f(a) = u and f(b) = v and g: $(Y, \sigma) \to (Z, \delta)$ by g(u) = s and g(v) = t. Then f and g are IFSGb continuous mappings. Since A = is an IFCS in Z, $f^{-1}(A)$ is not an IFSGbCS in X. Therefore the composition map g ∘ f: $(X, \tau) \to (Z, \delta)$ is not an IFSGb continuous.

Definition 3.18: An IFTS(X, τ) is said to be an intuitionistic fuzzy $T^*_{\frac{1}{2}}$ space (in short$IFT^*_{\frac{1}{2}}$) if every IFSGbCS of (X, τ) is an IFCS of (X, τ).

Theorem 3.19: Let f: (X, τ) → (Y, σ) be an IFSGb continuous mapping. Then f is an IF continuous mapping if X is an $IFT^*_{\frac{1}{2}}$ space.

Proof: Let V be an IFCS in Y. Then $f^{-1}(V)$ is an IFSGbCS in X, by hypothesis. Since X is an $IFT^*_{\frac{1}{2}}$ space, $f^{-1}(V)$ is an IFCS in X. Hence f is an IF continuous mapping.

Theorem 3.20: Let f : (X, τ) → (Y, σ) be an IFGSGb continuous mapping and g: (Y, σ) → (Z, δ) be an IFSGb continuous mapping and Y is an $IFT^*_{\frac{1}{2}}$ space. Then gof : (X, τ) → (Z, δ) is an IFGSGb continuous mapping.

Proof: Let V be an IFCS in Z. Then $g^{-1}(V)$ is an IFSGbCS in Y, by hypothesis. Since Y is an $IFT^*_{\frac{1}{2}}$ space, $g^{-1}(V)$ is an IFCS in Y. Therefore $f^{-1}(g^{-1}(V))$ is an IFSGbCS in X, by hypothesis. Hence gof is an IFSGb continuous mapping.

Theorem 3.21: Let f : (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y.Then the following conditions are equivalent if X is an$IFT^*_{\frac{1}{2}}$ space.

i. f is an IFSGb continuous mapping

ii. $f^{-1}(B)$ is an IFSGbCS in X for every IFCS B in Y

iii.$cl(int(f^{-1}(A))) \cap int(cl(f^{-1}(A))) \subseteq f^{-1}(cl(A))$ for every IFS A in Y.

Proof : (i) ⇒ (ii) is obvious from the Definition 3.1.

(ii) ⇒ (iii) Let A be a IFS in Y. Then cl(A) is an IFCS in Y. By hypothesis, $f^{-1}(cl(A))$ is an IFSGbCS in X. Since X is an $IFT^*_{\frac{1}{2}}$ space, $f^{-1}(cl(A))$ is an IFCS in X. Therefore $cl(f^{-1}(cl(A))) = f^{-1}(cl(A))$. Now $cl(int(f^{-1}(A))) \cap int(cl(f^{-1}(A))) \subseteq (cl(f^{-1}(cl(A))) = f^{-1}(cl(A))$.

(iii) ⇒ (i) Let A be an IFCS in Y. By hypothesis $cl(int(f^{-1}(A))) \cap int(cl(f^{-1}(A))) \subseteq f^{-1}(cl(A)) = f^{-1}(A)$. But $f^{-1}(A) \subseteq cl(f^{-1}(A))$ always. This implies $f^{-1}(A)$ is an IFCS in X and hence it is an IFSGbCS. Thus f is an IFSGb continuous mapping.

Theorem 3.22: Let f: (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y.Then the following conditions are equivalent if X is an IFT$_{\frac{1}{2}}^{*}$ space.

i. f is an IFSGb continuous mapping
ii. If B is an IFOS in Y then $f^{-1}(B)$ is an IFSGbOS in X
iii.$f^{-1}(int(B)) \subseteq int(cl(f^{-1}(A))) \cup cl(int(f^{-1}(A)))$ for every IFS B in Y.
Proof: (i) ⇒ (ii): is obviously true.
(ii) ⇒ (iii): Let B be any IFS in Y. Then int(B) is an IFOS in Y. Then $f^{-1}(int(B))$ is an IFSGbOS in X. Since X is an IFT$_{\frac{1}{2}}^{*}$ space, $f^{-1}(int(B))$ is an IFOS in X. Therefore $f^{-1}(int(B)) = int(f^{-1}(int(B)))$. Now $int(cl(f^{-1}(A))) \cup cl(int(f^{-1}(A))) \supseteq int(f^{-1}(int(B))) = f^{-1}(int(B))$. Hence $f^{-1}(int(B)) \subseteq int(cl(f^{-1}(A))) \cup cl(int(f^{-1}(A)))$.
(iii) ⇒ (i): Let B be an IFCS in Y. Then its complement B^c is an IFOS in Y. By hypothesis
$f^{-1}(B^c) = f^{-1}(int(B^c)) \subseteq int(cl(f^{-1}(B^c))) \cup cl(int(f^{-1}(B^c)))$. This implies $f^{-1}(B^c) \subseteq int(cl(f^{-1}(B^c))) \cup cl(int(f^{-1}(B^c)))$. But $int(cl(f^{-1}(B^c))) \cup cl(int(f^{-1}(B^c))) \subseteq f^{-1}(B^c)$ always. Hence $f^{-1}(B^c)$ is an IFOS in X. Since every IFOS is an IFSGbOS, $f^{-1}(B^c)$ is an IFSGbOS in X. Therefore
$f^{-1}(B)$ is an IFSGbCS in X. Hence f is an IFSGb continuous mapping.

4. INTUTIONISTIC FUZZY SEMI GENERALIZED b-IRRESOLUTE MAPPINGS

In this section we introduce intuitionistic fuzzy semi generalized b-irresolute mappings and study some of its characterizations.

Definition 4.1: A mapping f: (X ,τ) → (Y,σ) is called an intuitionistic fuzzy semi generalized b-irresolute(IFSGb irresolute) mapping if $f^{-1}(A)$ is an IFSGbCS in (X, τ) for every IFSGbCS A of (Y, σ).

Theorem 4.2: Let f: (X ,τ) → (Y,σ) be an IFSGb irresolute, then f is an IFSGb continuous mapping.

Proof: Let f be an IFSGb irresolute mapping. Let A be any IFCS in Y. Since every IFCS is an IFSGbCS, A is an IFSGbCS in Y. By hypothesis $f^{-1}(A)$ is an IFSGbCS in X. Hence f is an IFSGb continuous mapping.
Example 4.3: IFSGb continuous mapping ↛ IFSGb irresolute mapping.

Let X = {a, b}, Y = {u, v}, G_1 =⟨x, (0.2, 0.4), (0.7, 0.5) ⟩ and G_2 =⟨x, (0.5, 0.3), (0.4, 0.6) ⟩ .Then τ ={0~ ,G_1 ,1~} and σ = {0~ ,G_2 ,1~} are IFTs on X and Y respectively. Define a mapping f: (X ,τ) → (Y,σ) by f(a) = u and f(b) = v. Then f is an IFSGb continuous. We have B = ⟨x, (0.6, 0.7), (0.3, 0.2) ⟩ is an IFSGbCS in Y but $f^{-1}(B)$ is not an IFSGbCS in X. Therefore f is not an IFSGb irresolute mapping.

Theorem 4.4: Let f: (X ,τ) → (Y,σ) and g: (Y,σ) → (Z, δ) be IFSGb irresolute mappings, then gof: (X ,τ) → (Z, δ) is an IFSGb irresolute mapping.

Proof: Let A be an IFSGbCS in Z. Then $g^{-1}(A)$ is an IFSGbCS in Y. Since f is an IFSGb irresolute mapping. $f^{-1}(g^{-1}(A))$ is an IFSGbCS in X. Hence gof is an IFSGb irresolute mapping.

Theorem 4.5: Let f: (X ,τ) → (Y,σ) be an IFSGb irresolute and g: (Y,σ) → (Z, δ) be IFSGb continuous mappings, then gof: (X ,τ) → (Z, δ) is an IFSGb continuous mapping.

Proof: Let A be an IFCS in Z. Then $g^{-1}(A)$ is an IFSGbCS in Y. Since f is an IFSGb irresolute,
$f^{-1}(g^{-1}(A))$ is an IFSGbCS in X. Hence gof is an IFSGb continuous mapping.

Theorem 4.6: Let f: (X, τ) → (Y, σ) be an IFSGb irresolute. Then f is an IF irresolute mapping if X is an IFT$_{\frac{1}{2}}^{*}$ space.

Proof: Let A be an IFCS in Y. Then A is an IFSGbCS in Y. Therefore $f^{-1}(A)$ is an IFSGbCS in X, by hypothesis. Since X is an IFT$_{\frac{1}{2}}^{*}$ space, $f^{-1}(A)$ is an IFCS in X. Hence f is an IF irresolute mapping.

Theorem 4.7: Let f: (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X and Y are IFT$_{\frac{1}{2}}^{*}$ spaces.

i. f is an IFSGb irresolute mapping
ii. $f^{-1}(B)$ is an IFSGbOS in X for each IFSGbOS in Y
iii. $f^{-1}(int\ B) \subseteq int\ (f^{-1}(B))$ for each IFS B of Y
iv. $cl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$ for each IFS B of Y.

Proof: (i) ⇒ (ii): is obvious from the Definition 4.1.

(ii) ⇒ (iii): Let B be any IFS in Y and int(B) ⊆ B. Also f^{-1}(int (B)) ⊆ f^{-1}(B). Since int(B) is an IFOS in Y, it is an IFSGbOS in Y. f^{-1}(int(B)) is an IFSGbOS in X, by hypothesis. Since X is an $IFT_{\frac{1}{2}}^{*}$ space, f^{-1}(int (B)) is an IFOS in X. Hence f^{-1}(int(B))
= int(f^{-1}(int (B))) ⊆ int(f^{-1}(B)).
(iii) ⇒ (iv): It is obvious by taking complement.
(iv) ⇒ (i): Let B be an IFSGbCS in Y. Since Y is $IFT_{\frac{1}{2}}^{*}$ space, B is an IFCS in Y and cl(B) = B. Hence f^{-1}(B) = f^{-1}(cl(B)) ⊇ cl(f^{-1}(B)). But clearly f^{-1}(B) ⊆ cl(f^{-1}(B)). Therefore cl(f^{-1}(B)) = f^{-1}(B). This implies f^{-1}(B) is an IFCS and hence it is an IFSGbCS in X. Thus f is an IFSGb irresolute mapping.

Theorem 4.8: Let f : (X, τ) → (Y, σ) be an IFSGb irresolute mapping from an IFTS X into an IFTS Y. Then f^{-1}(B) ⊆ cl(int(f^{-1}(B))) if X is an $IFT_{\frac{1}{2}}^{*}$ space.

Proof: Let B be an IFSGbOS in Y. Then by hypothesis f^{-1}(B) is an IFSGbOS in X.
Since X is an $IFT_{\frac{1}{2}}^{*}$ space, f^{-1}(B) is an IFOS in X. Therefore int(f^{-1}(B)) = f^{-1}(B) and f^{-1}(B) ⊆ cl(f^{-1}(B)) = cl(int(f^{-1}(B))). Hence f^{-1}(B) ⊆ cl(int(f^{-1}(B))).

Theorem 4.9: Let f: (X, τ) → (Y, σ) be an IFSGb irresolute mapping from an IFTS X into an IFTS Y. Then f^{-1}(B) ⊆ cl(int(f^{-1}(int(B)))) for every IFSGbOS B in Y, if X and Y are $IFT_{\frac{1}{2}}^{*}$ spaces.

Proof: Let B be an IFSGbOS in Y. Then by hypothesis f^{-1}(B) is an IFSGbOS in X. Since X is an $IFT_{\frac{1}{2}}^{*}$ space, f^{-1}(B) is an IFOS in X. Therefore int(f^{-1}(B)) = f^{-1}(B). Since Y is an $IFT_{\frac{1}{2}}^{*}$ space, B is an IFOS in Y and f^{-1}(B) ⊆ cl(f^{-1}(B)) = cl(int(f^{-1}(B))) = cl(int(f^{-1}(int(B)))). Hence f^{-1}(B) ⊆ cl(int(f^{-1}(int(B)))).

Theorem 4.10: Let f: (X, τ) → (Y, σ) be onto, an IFSGb irresolute mapping and an IFC mapping from an IFTS X into an IFTS Y. If X is an $IFT_{\frac{1}{2}}^{*}$ space, then Y is also an $IFT_{\frac{1}{2}}^{*}$ space.

Proof: Let A be an IFSGbCS in Y. Then by hypothesis f^{-1}(A) is an IFSGbCS in X. Since X is an $IFT_{\frac{1}{2}}^{*}$ space, f^{-1}(A) is an IFCS in X. Since f is an IFC mapping, A is an IFCS in Y. Therefore Y is an $IFT_{\frac{1}{2}}^{*}$ space.

5. INTUTIONISTIC FUZZY SLIGHTLY SEMI GENERALIZED b-CONTINUOUS MAPPINGS

Definition 5.1: A function f : (X, τ) → (Y, σ) is called an intuitionistic fuzzy slightly semi generalized b-continuous (IF slightly sgb-continuous) if the inverse image of every IF clopen set in Y is IF sgb-open in X.
Definition 5.2: A function f : (X, τ)→(Y, σ) from a IFTS (X, τ) to another IFTS (Y, σ) is said to be an IF slightly sgb-continuous if for each IFP p(α, β) ∈ X and each IF clopen set B in Y containing f(p(α, β)), there exists an IFsgb-open set A in X such that f(A) ⊆ B.
Theorem 5.3: Let f : (X, τ) → (Y, σ) be a function from an IFTS (X, τ) to another IFTS (Y, σ) then the following statements are equivalent
1. f is an IF slightly sgb-continuous.
2. Inverse image of every IF clopen set in Y is an IF sgb-open in X.
3. Inverse image of every IF clopen set in Y is an IF sgb-closed in X.
4. Inverse image of every IF clopen set in Y is an IF sgb-clopen in X.
Proof: (1) ⇒ (2) Let B be an IF clopen set in Y and let (p(α, β)) ∈ f^{-1}(B). Since f(p(α,β) ∈ B by (1) there exists an IF sgb-open set A in X containing p(α, β) such that $A_{p(\alpha,\beta)}$ ⊆ f^{-1}(B) we obtain that $f^{-1}(B) = \bigcup_{p(\alpha,\beta)\in f^{-1}(B)} A_{p(\alpha,\beta)}$, which is an IF sgb-open in X.
(2) ⇒ (3) Let B be an IF clopen set inY ,then B^c is IF clopen. By (2) $f^{-1}(B^c) = (f^{-1}(B))^c$ is an IF sgb-open, thus f^{-1}(B) is an IF sgb-closed set.
(3) ⇒ (4) Let B be an IF clopen set in Y. Then by (3) f^{-1}(B) is IF sgb- closed set. Also B^c is an IF clopen and (3) implies $f^{-1}(B^c) = (f^{-1}(B))^c$ is an IFsgb-closed set. Hence f^{-1}(B) is an IFsgb-clopen set.
(4) ⇒ (1) Let B be an IF clopen set in Y containing f(p(α,β)). By (4), f^{-1}(B) is an IF sgb-open. Let us take A = f^{-1}(B), then f(A) ⊆B. Hence f is an IF slightly sgb-continuous.

Definition 5.4: The intersection of all IFsgb-closed sets containing an IF set A is called an IFsgb-closure of A and denoted by sgbcl(A), and the union of all

IFsgb-open sets contained in an IF set A is called an IFsgb-interior of A and denoted by sgbint(A).

Remark 5.5: If A = sgbcl(A), then A need not be an IFsgb-closed.

Remark 5.6: The union of two IFsgb-closed sets is generally not an IFsgb- closed set and the intersection of two IFsgb-open sets is generally not an IFsgb open set.

Example 5.7: Let X={a,b,c}and let τ = {0~, 1~, A, B, C } is IFT on X, where A={<x, (0,1,0), (1,0,1)>}, B={<x, (0,0,1), (1,1,0)>} and C={<x, (0,1,1), (1,0,0)>}. Then the IFSs A^c, B^c are IFsgbOSs but $A^c \cap B^c = C^c$ is not an IFsgbOS of X, since $C^c \subseteq C^c$ and $C^c \nsubseteq$ bint(C^c) = 0, And the IFSs A, B are IFsgbCSs but A ∪ B = C is not an IFsgbCS of X, since C ⊆ C and bcl(C) = 1~⊈ C.

Proposition 5.8: Every intuitionistic fuzzy sgb-continuous is an intuitionistic fuzzy slightly sgb-continuous. But the converse need not be true.

Example 5.9: Let X = {a, b}, Y = {u, v} and A = {< x, (1,0), (0,1) >}, B={< x, (0,0.8), (1,0.2)>}, C={< x, (1,0.8), (0,0.2)>}, D={< x, (0.7,0.5), (0.3,0.5) >}. Then τ = {0~, 1~, A, B, C} and σ = {0~, 1~, D} are IFTS on X and Y respectively. Define a mapping f : (X, τ) → (Y, σ) by f(a)= u and f(b) = v. Then f is an IF slightly sgb-continuous but not an IFsgb-continuous. Since $f^{-1}(D^c)$ = {< x, (0.3,0.5), (0.7,0.5)>}⊆ C (semi open set) and bcl($f^{-1}(D^c)$) =1~⊈C.

Proposition 5.10: Every intuitionistic fuzzy sgb-irresolute function is an intuitionistic fuzzy slightly sgb-continuous. But the converse need not be true.

Theorem 5.11: If f: X → Y is an IF slightly sgb-continuous and g : Y → Z is an IF totally continuous then g o f is an intuitionistic fuzzy sgb-continuous.

Proof: Let B be an IFOS in Z, since g is an IF totally continuous, g^{-1}(B) is an IF clopen set in Y. Now $(g \circ f)^{-1}$ (B)= f^{-1} (g^{-1} (B)). Since f is an IF slightly sgb-continuous, f^{-1} (g^{-1} (B)) is an IFsgbOS in X. Hence g o f is an intuitionistic fuzzy sgb-continuous.

Theorem 5.12: A mapping f:(X, f) → (Y, σ) from an IFTS (X, τ) to another IFTS (Y, σ) is an IF slightly sgb-continuous if and only if for each IFP p(α,β) in X and IF clopen set B in Y such that f(p(α,β)) ∈B, cl(f^{-1}(B)) is an IFN of IFP p(α,β) in X.

Proof: Let f be any IF slightly sgb-continuous mapping, p(α,β) be an IFP in X and B be any IF clopen set in Y such that f(p(α,β)) ∈ B. Then p(α,β) ∈ f^{-1}(B) ⊆ bcl(f^{-1}(B)) ⊆ cl(f^{-1}(B)). Hence cl(f^{-1} (B)) is an IFN of p(α,β) in X.
Conversely, let B be any IF clopen set in Y and p(α,β) be an IFP in X such that f(p(α,β)) ∈B. Then p(α,β) ∈ f^{-1}(B). According to assumption cl(f^{-1}(B)) is an IFN of IFP p(α,β) in X. So, p(α,β) ∈ f^{-1}(B) ⊆ cl(f^{-1}(B)), and by (definition of IF slightly sgb-continuous) there exists an IFsgb-open A in X such that p(α,β)∈ A ⊆ f^{-1}(B).Therefore f is an IF slightly sgb-continuous.

REFERENCES

[1] Amal M. Al- Dowais, Abdul Gawad A. Q. Al-Qubati, "On Intuitionistic Fuzzy Slightly πgb-Continuous Functions" Vol. 4, Issue 1, 2017.

[2] Angelin Tidy G and Francina Shalini A, "On softsgb-continuous functions in soft topological spaces,"International journal of engineering and computing, vol 7, issue no.4,2017.

[3] Angelin Tidy G and Francina Shalini A, "On intuitionistic fuzzy sgb-closed sets in intuitionistic fuzzy topological spaces," International journal of research trends and innovation, vol. 3, issue 7, 2018.

[4] Atanassov. K. T., Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20(1986), 87-96.

[5] Coker. D.,An introduction to fuzzy topological spaces, Fuzzy sets and systems, 88(1997), 81-89.

[6] Gurcay. H., Coker. D.,and Es. A. Haydar., On fuzzy continuity in intuitionistic fuzzy topological spaces, Jour. Of Fuzzy Math., 5(1997), 365-378.

[7] M.Jeyaraman A.Yuvarani and O.Ravi, "IFαGS continuous and IFαGS irresolute mappings," International journal of analysis and applications, vol 3, no.2(2013), 93-103.

[8] Joung Kon Jeon, Young Bae Jun, and Jin Han Park, Intuitionistic fuzzy alpha continuity and intuitionistic fuzzy precontinuity, International

Journal of Mathematics and Mathematical Sciences, 19(2005), 3091- 3101.

[9] Kalamani. D, Sakthivel. K and Gowri. C. S.,Generalized alpha closed sets in Intuitionistic fuzzy topological spaces, Applied Mathematical Sciences, 6(2012), 4691-4700.

[10] R. Renuka, V. Seenivasan, "On Intuitionistic Fuzzy Slightly Pre-continuous Functions" Vol. 86, No. 6,2013, 993-1004

[11] Santhi. R and Sakthivel. K.,Intuitionistic fuzzy generalized semi continuous mappings, Advances in Theoretical and Applied Mathematics, 5(2009), 73-82.