# Optimal Solution of Fuzzy Game Problem Using Heptagonal Fuzzy Numbers

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***Abstract*- In this paper, we introduce a new concept of fuzzy game problem using heptagonal fuzzy numbers. Also, we convert the fuzzy valued game problem to crisp valued game problem using ranking method, which can be solved using row-minima and column maximum. Further, we discuss the solution of such fuzzy games with saddle point by Minimax - Maximin principle.**

**Index Terms- Fuzzy number, Membership function, Heptagonal fuzzy number, ranking of fuzzy number , saddle point.**

## I. INTRODUCTION

The mathematical treatement of the game theory was made available in1944. When John von newmann and Oscar Morgenstern [7] published the famous article "theory of games and economic behaviour. Game theory has played an important role in the fields of decision making theory, economics, management etc. In 2016, Anandhi, studied an optimal solution for solving fuzzy pentagonal transportation problem. In 2017 Monisha and Sangeetha have discussed the solution of fuzzy game problem using pentagonal fuzzy number. The purpose of this paper is to introduced a new concept of fuzzy game problem using heptagonal fuzzy numbers. Also, we convert the fuzzy valued game problem to crisp valued game problem using ranking method, which can be solved using row-minima and column maximum. Further, we discuss the solution of fuzzy game with saddle point by minimax-maximax principle.

## II. PRELIMINARIES

### 2.1. FUZZY SETS:[6]

A fuzzy set is characterized by a membership function mapping elements of a domain, space, or universe of discourse X to the unit interval [0, 1]. (i.e.) $A=\{x, \mu A(x); x \in X\}$ here $\mu A: X \rightarrow [0,1]$ is a mapping called the degree of membership function of the fuzzy set A and $\mu A(x)$ is called the membership value of x in the fuzzy set A. These membership grades are often represented by real numbers ranging from [0, 1].

### 2.2. FUZZY NUMBERS: [6]

A fuzzy set A defined on the set of real numbers R is said to be a fuzzy number if its Membership function $\mu A: R \rightarrow [0, 1]$ has the following characteristics.

(i)A is normal. It means that there exists an $x \in R$ such that $\mu A(x) = 1$.

(ii)A is convex. It means that for every $x1, x2 \in R$, $\mu A(\lambda x1 + (1 - \lambda) x2) \geq \text{Min} \{\mu A(x1), \mu A(x2)\}$, $\lambda \in [0,1]$

(iii)$\mu$ A is upper semi-continuous. (iv) Supp (A) is bounded in R.

### 2.3. SADDLE POINT: [6]

A saddle point of a payoff matrix is that position in the payoff matrix. Where maximum of row minima, co-inside with the minimum of the column maxima. The pay off at the saddle point is called the value of the game denoted by gamma. The saddle point need not be unique. We denote the maximin value of the game by gamma ($\gamma$) and the minimax value of the game by Ȳ. The game is said to be fair. If γ = 0 = Ȳ. The game is said to be strictly determinable. If γ = γ= Ȳ

## III. HEPTAGONAL FUZZY NUMBER

### 3.1. DEFINITION:

A heptagonal fuzzy number of a fuzzy set A is defined as ={a1,a2,a3,a4,a5,a6,a7}, where a1,a2,a3,a4,a5,a6,a7 are real numbers and its membership function is given by

$$\begin{cases} \frac{1}{4}\left(\frac{x-a_1}{a_2-a_1}\right), & \text{for } a_1 \le x \le a_2; \\ \frac{1}{4}, & \text{for } a_2 \le x \le a_3; \\ \frac{1}{4}+\frac{3}{4}\left(\frac{x-a_2}{a_3-a_2}\right), & \text{for } a_3 \le x \le a_4; \\ \frac{1}{4}+\frac{3}{4}\left(\frac{a_4-x}{a_5-a_4}\right), & \text{for } a_4 \le x \le a_5; \\ \frac{1}{4}, & \text{for } a_5 \le x \le a_6; \\ \frac{1}{4}\left(\frac{a_7-x}{a_7-a_6}\right), & \text{for } a_6 \le x \le a_7; \\ 0, & \text{otherwise}; \end{cases}$$

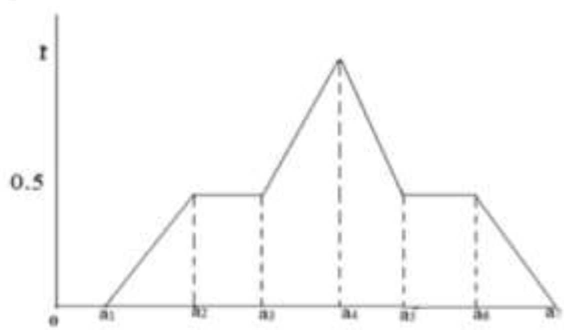


Fig 3.2 Graphical representation of heptagonal fuzzy numbers

## IV. MATHEMATICAL FORMULATION OF A FUZZY GAME PROBLEM [6]

Let player A have m strategies A1, A2...Am and player B have n strategies B1, B2,...,Bn. Here, it is assumed that each player has his choices from amongst the pure strategies. Also it is assumed that player A is always the gainer and player B is always the loser. That is, all payoff are assumed in terms of player A. Let $a$ij be the payoff which player A gains from player B if player A chooses strategy $A$i and player B chooses strategy $B$j . Then the payoff b matrix to player A.

### 4.1.PROCEDURE FOR SOLVING FUZZY GAME PROBLEM

We shall present a solution to fuzzy game problem involving strategies of the players using triangular fuzzy numbers.

Step 1: Check whether a saddle point exists in the problem. If it exists, the solution can be obtained directly. If the saddle point does not exist, go to the next step.

Step 2: Comparison of column strategies.

a) If elements of Column A ≤ elements of Column B, Column A strategy dominates over column B strategy. Hence delete column B strategy from the pay off matrix.
b) Compare each column strategy with all possible column strategies and delete inferior strategies as for as possible.

Step 3: Comparison of row strategies.

a) If elements of Row A ≥ elements of Row B, Row A strategy dominates over Row B strategy. Hence delete Row B strategy from the pay off matrix.
b) Compare each row strategy with all possible row strategies and delete inferior strategies as for as possible.
c) The Game may reduce to a single cell giving information about the value of the game and optimal strategies of players. If not go to step 4.
d) Step 4: Dominance need not to be based on the superiority of pure strategies only. A given strategy can be dominated if it is inferior to an average of two or more other pure strategies.

### 4.2.NUMERICAL EXAMPLE:

Consider the following fuzzy game problem

| | $B_1$ | $B_2$ | $B_3$ | $B_4$ | $B_5$ |
|---|---|---|---|---|---|
| $A_1$ | (5,7,9,11,13,15,17) | (1,3,5,7,9,11,13) | (3,5,7,9,11,13,15) | (8,10,12,14,16,18,20) | (1,4,7,10,13,16,19) |
| $A_2$ | (5,6,7,8,9,10,11) | (1,2,3,4,5,6,7) | (5,8,11,14,17,20,2) | (2,4,6,8,10,12,14) | (3,6,9,12,15,18,21) |
| $A_3$ | (6,8,10,12,14,16,18) | (2,3,4,5,6,7,8) | (4,6,8,10,12,14,16,) | (4,5,6,7,8,9,10) | (0,1,2,3,4,5,6) |
| $A_4$ | (5,6,7,8,9,10,11) | (0,2,4,6,8,10,12) | (1,3,5,7,9,11,13) | (6,7,8,9,10,11,12) | (3,4,5,6,7,8,9) |

SOLUTION:

Using ranking function

R($\bar{A}$) $= \dfrac{P_{1+}P_2+P_3+P_4+P_5+P_6+P_7}{7}$

R(5,7,9,11,13,15,17) $= \dfrac{5+7+9+11+13+15+17}{7} = \dfrac{77}{7} = 11$

$R(1,3,5,7,9,11,13) = \dfrac{1+3+5+7+9+11+13}{7} = \dfrac{49}{7} = 3$

R(3,5,7,9,11,13,15) $= \dfrac{3+5+7+9+11+13+15}{7} = \dfrac{63}{7} = 9$

R(8,10,12,14,16,18,20) $= \dfrac{8+10+12+14+16+18+20}{7} = \dfrac{98}{7} = 14$

R(1,4,7,10,13,16,19) $= \dfrac{1+4+7+10+13+16+19}{7} = \dfrac{70}{7} = 10$

R(5,6,7,8,9,10,11) $= \dfrac{5+6+7+8+9+10+11}{7} = \dfrac{56}{7} = 8$

R(1,2,3,4,5,6,7) $= \dfrac{1+2+3+4+5+6+7}{7} = \dfrac{28}{7} = 4$

R(5,8,11,14,17,20,23) $= \dfrac{5+8+11+14+17+20+23}{7} = \dfrac{98}{7} = 14$

R(2,4,6,8,10,12,14) $= \dfrac{2+4+6+8+10+12+14}{7} = \dfrac{56}{7} = 8$

R(3,6,9,12,15,18,21) $= \dfrac{3+6+9+12+15+18+21}{7} = \dfrac{84}{7} = 12$

R(6,8,10,12,14,16,18) $= \dfrac{6+8+10+12+14+16+18}{7} = \dfrac{84}{7} = 12$

R(2,3,4,5,6,7,8) $= \dfrac{2+3+4+5+6+7+8}{7} = \dfrac{35}{7} = 5$

R(4,6,8,10,12,14,16) $= \dfrac{4+6+8+10+12+14+16}{7} = \dfrac{70}{7} = 10$

R(4,5,6,7,8,9,10) $= \dfrac{4+5+6+7+8+9+10}{7} = \dfrac{49}{7} = 7$

R(0,1,2,3,4,5,6) $= \dfrac{0+1+2+3+4+5+6}{7} = \dfrac{21}{7} = 3$

R(5,6,7,8,9,10,11) $= \dfrac{5+6+7+8+9+10+11}{7} = \dfrac{56}{7} = 8$

$$R(0,2,4,6,8,10,12) = \frac{0+2+4+6+8+10+12}{7} = \frac{42}{7} = 6$$

$$R(1,3,5,7,9,11,13) = \frac{1+3+5+7+9+11+13}{7} = \frac{49}{7} = 7$$

$$R(6,7,8,9,10,11,12) = \frac{6+7+8+9+10+11+12}{7} = \frac{63}{7} = 9$$

$$R(3,4,5,6,7,8,9) = \frac{3+4+5+6+7+8+9}{7} = \frac{42}{7} = 6$$

| | | | | | | Row minimax |
|---|---|---|---|---|---|---|
| $A_1$ | 11 | 7 | 9 | 14 | 10 | 7 |
| $A_2$ | 8 | 4 | 14 | 8 | 12 | 4 |
| $A_3$ | 12 | 5 | 10 | 7 | 3 | 3 |
| $A_4$ | 8 | 6 | 7 | 9 | 6 | 6 |
| Column Maximum | 12 | 7 | 14 | 14 | 12 | |

Minimum=maximum=7

Saddle point is (7,7)

Saddle point ath the position

Value of the game

## V. CONCLUSION

In this paper, we consider some operations of hepatagonal fuzzy number. The solution of such fuzzy game with saddle point by minimax-maximin principle is discussed.

## REFERENCE


[1] R. Anandhi, "An optimum solution for solving fuzzy pentagonal transportation problem," International Journal: Scientific Research and Development, vol. 11, pp. 2321-0613, 2016.

[2] S. Chandrasekaran, "Ranking of heptagon number using zero suffix method," International Journal: Science and Research, vol. 4, pp. 2319-7064, 2013.

[3] S. Chandrasekaran, "Fuzzy transportation problem of hexagon number with alpha cut and ranking technique," International Journal: Scientific Engineering and Applied Science, vol. 5, pp. 2395-3470, 2015.

[4] R . Jansi and K. Mohana, "To solve fuzzy sequencing problem using heptagonal fuzzy number, Pascal's triangular graded mean technique", international journal of innovative research in technology, vol.5, issue 6, pp.240-243, 2017.

[5] K. Khadhirvel and K. Balamurugan, "Method for solving the transportation problem using trapezoidal fuzzy numbers," International Journal: Engineering and Applications, vol. 5, pp. 2154-2158, 2012.

[6] P.Monisha and K.Sangeetha, "to solve fuzzy game problem using pentagonal fuzzy numbers", international journal for modern trends in science and technology, vol.03, issue 09,September 2017,pp.-152-154

[7] J.V.Newmann and O.Morgenstern, theory of games and economic behaviour, Princeton university press, Princeton, new jersey, 1947

[8] A. Thamaraiselvi and R. Santhi, "Optimal solution of fuzzy transportation problem using hexagonal fuzzy numbers," International Journal: Scientific and Engineering Research, vol. 6, pp. 2229-5518, 2015.

[9] L. A. Zadeh, "Fuzzy sets," Information and Control, vol. 8, pp. 338-353, 1965.