Optimal solution of Fuzzy Game Problem by using Hexadecagonal Fuzzy Numbers

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Abstract- **The fuzzy set theory has been applied in almost every business enterprise as well as day to day activity. Ranking of fuzzy numbers plays an important role in decision making process. In this paper, we introduced a fuzzy game problem (FGP) in which the values of payoff matrix are represented by hexadecagonal fuzzy numbers. By using ranking to payoffs we convert the fuzzy game problem into crisp valued game problem, which can be solved by traditional method.**

Index Terms- fuzzy numbers, hexadecagonal fuzzy Numbers, fuzzy Game Problem, fuzzy ranking.

1. INTRODUCTION

Ranking fuzzy numbers plays an important role in decision making process. It was firstly proposed by Zadeh [11]. Bellman and Zadeh [1] elaborated on the concept of decision making in the fuzzy environment. Later on, fuzzy methodologies have been successfully applied in a wide range of real world situations. Jain [2] was the first to propose method of ranking fuzzy numbers for decision making in fuzzy situations. Yager [10] used the concept of centroids in the ranking of fuzzy numbers.

In a game problem each players attempts to take best decision by selection various strategies from the set of available strategies. The traditional game theory assumes the existence of exact payoffs to solve competitive situations. Game theory is applicable to situations such as two players struggling to win at chess, candidates fighting an election, firms struggling to maintain their market shares etc,.. Studied in[4] Narmata, Dr Umesh Chandra Gupta and Dr Neha Ishesh Thakur on "solution of fuzzy game problem using Dodecagonal fuzzy number ",[3]R .Jahir Hussain and A. Priya," Solving fuzzy game problem using hexagonal fuzzy numbers" and [7]R. senthil kumar and k.Gnanaprakash on "solving fuzzy game of order 3x3 using octagonal fuzzy numbers " and et al. In this paper, we discussed fuzzy game problem in which the values of payoff matrix are represented by hexadecagonal fuzzy numbers. After applying ranking, there fuzzy numbers are converted into crisp problem.

2. PRELIMINARIES

2.1 Fuzzy set:[4]

Let X = {x} denote a collection of objects denoted generically by x. Then a fuzzy set $\tilde{A}$ in X is a set of ordered pairs $\tilde{A} = \{(x,\mu_{\tilde{A}}(x);\ x \in X\}$ where $\mu_{\tilde{A}}(x)$ is termed as the grade of membership of x in A and $\mu_{\tilde{A}} : X \rightarrow M$ is a function from X to a space M which is called membership space. When M contains only two points, 0 and 1, A is non fuzzy and its membership function becomes identical with the characteristic function of a non fuzzy set.

2.2 Normal set:[4]

A Fuzzy set $\tilde{A}$ of universe set X is normal if and only if $\sup_{x\in X} \mu_{\tilde{A}}(x) = 1$.

2.3 Convex set[3]

A fuzzy set $\tilde{A}$ in universal set X is called convex iff

$$\mu_{\tilde{A}}(x_1\lambda + (1-\lambda)x_2) \geq \min[\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)]$$

for all $x_1, x_2 \in X$ and $\lambda \in [0,1]$

2.4 Definition: [7]

A fuzzy set $\tilde{A}$ of universal set is a fuzzy number iff it is normal and convex.

2.5 Definition:[6]

A fuzzy number $\tilde{A} = (m, n, \alpha, \beta)_{LR}$ is said to be an LR flat fuzzy number if its membership function is given by:

$$\mu_{\tilde{A}}(x) = \begin{cases} L\left(\frac{m-x}{\alpha}\right), & x \leq m,\ \alpha > 0 \\ R\left(\frac{x-n}{\beta}\right), & x \geq n,\ \beta > 0 \\ 1 & , m \leq x \leq n \end{cases}$$

L and R are called reference functions, which are continuous, non-increasing functions that defining the left and right shapes of $\mu_{\tilde{A}}(x)$ respectively and L(0)=R (0) =1.

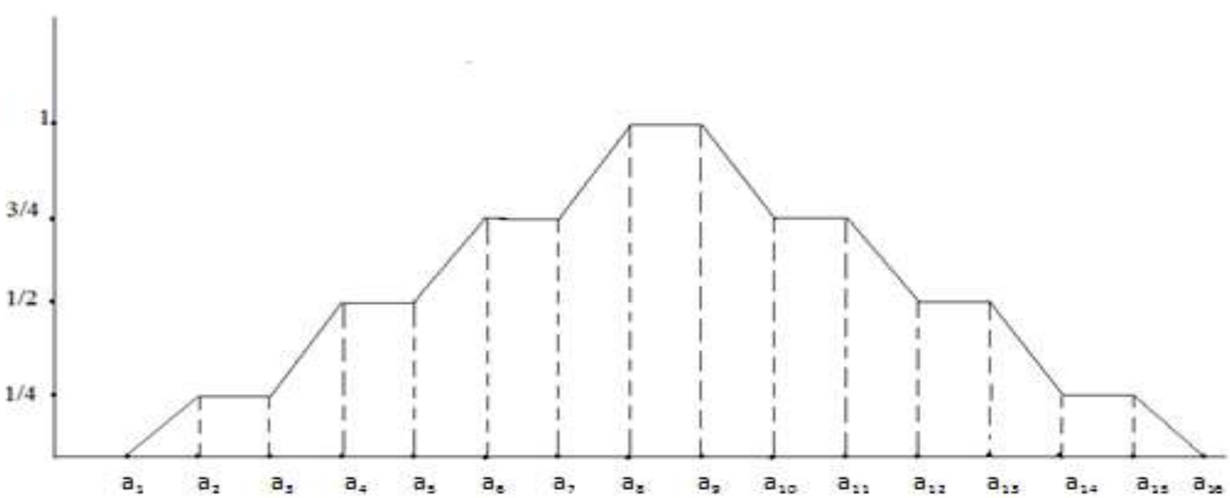
3. HEXADECAGONAL FUZZY NUMBERS

A generalized fuzzy numbers $\tilde{A}_{HD} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16})$ is said to be hexadecagonal fuzzy number if its membership function $\mu_{\tilde{A}_{HD}}(x)$ is given below:

$$\mu_{X_{HD}}(x) = \begin{cases} 0 & x \leq a_1 \\ \frac{1}{4}\left(\frac{x - a_1}{a_2 - a_1}\right) & a_2 \leq x \leq a_3 \\ \frac{1}{2}\left(\frac{x - a_3}{a_4 - a_3}\right) & a_3 \leq x \leq a_4 \\ \frac{1}{2} & a_4 \leq x \leq a_5 \\ \frac{3}{4}\left(\frac{x - a_5}{a_6 - a_5}\right) & a_5 \leq x \leq a_6 \\ \frac{3}{4} & a_6 \leq x \leq a_7 \\ \left(\frac{x - a_7}{a_8 - a_7}\right) & a_7 \leq x \leq a_8 \\ 1 & a_8 \leq x \leq a_9 \\ \left(\frac{a_{10} - x}{a_{10} - a_3}\right) & a_9 \leq x \leq a_{10} \\ \frac{3}{4} & a_{10} \leq x \leq a_{11} \\ \frac{3}{4}\left(\frac{a_{12} - x}{a_{12} - a_{11}}\right) & a_{11} \leq x \leq a_{12} \\ \frac{1}{2} & a_{12} \leq x \leq a_{13} \\ \frac{1}{2}\left(\frac{a_{14} - x}{a_{14} - a_{13}}\right) & a_{13} \leq x \leq a_{14} \\ \frac{1}{4} & a_{14} \leq x \leq a_{15} \\ \frac{1}{4}\left(\frac{a_{16} - x}{a_{16} - a_{15}}\right) & a_{15} \leq x \leq a_{16} \\ 0 & a_{16} \leq x \end{cases}$$

where $0 < k_1 < k_2 < k_3 < 1$

3.2 Graphical representation of hexadecagonal fuzzy number:



3.3 Ranking of hexadecagonal fuzzy number:

Let $\widetilde{A}$ be a normal hexadecagonal fuzzy number. The value

$$M^{HD}{}_0(\widetilde{A}) = \frac{1}{2}\int_0^{k_1}(l_1(r)+l_2(r))dr + \frac{1}{2}\int_{k_1}^{k_2}(g_1(s)+g_2(s))dr + \frac{1}{2}\int_{k_2}^{k_3}(h_1(t)+h_2(t))dr + \frac{1}{2}\int_{k_3}^{1}(f_1(h)+f_2(h))dr$$

$$M^{HD}{}_0(\widetilde{A}) = \frac{1}{4}\{(a_1+a_2+a_{15}+a_{16})k_1 + (a_3+a_4+a_{13}+a_{14})(k_2-k_1) + (a_5+a_6+a_{11}+a_{12})(k_3-k_2) + (a_3+a_4+a_{13}+a_{14})(1-k_3)$$

where $0 < k_1 < k_2 < k_3 < 1$

4. MATHEMATICAL FORMULATION OF FUZZY GAME PROBLEM

Consider two person zero sum fuzzy game in which all the entries in the payoffs matrix are hexadecagonal fuzzy numbers. Player A is always assumed to be gainer and player B is always looser.The payoff matrix m×n is

$$A = \begin{bmatrix} \widetilde{a}_{11}\ \widetilde{a}_{12}\ldots\ldots\widetilde{a}_{1n} \\ \widetilde{a}_{21}\ \widetilde{a}_{22}\ldots\ldots\widetilde{a}_{2n} \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ \widetilde{a}_{m1}\ \widetilde{a}_{m2}\ldots\ldots\widetilde{a}_{mn} \end{bmatrix}$$

4.2 NUMERICAL EXAMPLES:

4.2.1 Consider the following fuzzy game problem with payoff as hexadecagonal fuzzy numbers

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} (-3,-2,-1,0,1,2,3,4,5,6,7,8,9,10,11,12) & (2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17) & (-8,-6,-4,-2,0,2,4,6,8,10,12,14,16,18,20,22) \\ (-5,-4,-3,-2,-1,0,1,2,3,4,5,6,7,8,9,10) & (0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15) & (-3,-1,1,3,5,7,9,11,13,15,17,19,21,23,25,27) \\ (5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20) & (2,3,5,6,8,9,11,12,14,15,17,18,20,23,24) & (0,2,4,6,8,10,12,14,16,18,20,22,24,26,28,30) \end{bmatrix} \end{array}$$

SOLUTION:

Hexadecagonal fuzzy number $\widetilde{A}$ is calculated as

$$M^{HD}{}_0(\widetilde{A}) = \frac{1}{2}\int_0^{k_1}(l_1(r)+l_2(r))dr + \frac{1}{2}\int_{k_1}^{k_2}(g_1(s)+g_2(s))dr + \frac{1}{2}\int_{k_2}^{k_3}(h_1(t)+h_2(t))dr + \frac{1}{2}\int_{k_3}^{1}(f_1(h)+f_2(h))dr$$

$$M^{HD}{}_0(\widetilde{A}) = \frac{1}{4}\{(a_1+a_2+a_{15}+a_{16})k_1 + (a_3+a_4+a_{13}+a_{14})(k_2-k_1) + (a_5+a_6+a_{11}+a_{12})(k_3-k_2) + (a_3+a_4+a_{13}+a_{14})(1-k_3)$$

where $0 < k_1 < k_2 < k_3 < 1$

step 1:

Convert the given fuzzy problem into a crisp value problem. This problem is done by taking the value of k as 0.4. We obtain the value of $M^{HD}{}_0(a_{ij})$

$a_{11}= (-3,-2,-1,0,1,2,3,4,5,6,7,8,9,10,11,12)$	$M^{HD}{}_0(a_{11}) = \frac{1}{4}\begin{bmatrix}(-3-2+11+12)(0.25)+(-1+0+9+10)(0.5-0.25)+ \\ (1+2+7+8)(0.75-0.5)+(3+4+5+6)(0.75)\end{bmatrix}$ $=\frac{1}{4}[4.5+4.5+4.5+4.5]$ $=4.5$

a_{12}= (2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17)	$M^{HD}{}_0(a_{12})=\frac{1}{4}\begin{bmatrix}(2+3+16+17)(0.25)+(4+5+14+15)(0.5-0.25)+\\(6+7+12+13)(0.75-0.5)+(8+9+10+11)(0.75)\end{bmatrix}$ $=\frac{1}{4}[9.5+9.5+9.5+9.5]$ $=9.5$
a_{13}= (-8,-6,-4,-2,0,2,4,6,8,10,12,14,16,18,20,22)	$M^{HD}{}_0(a_{13})=\frac{1}{4}\begin{bmatrix}(-3-1+25+27)(0.25)+(1+3+21+23)(0.5-0.25)+\\(5+7+17+19)(0.75-0.5)+(9+11+13+15)(0.75)\end{bmatrix}$ $=\frac{1}{4}[12+12+12+12]$ $=12$
a_{21}= (-5,-4,-3,-2,-1,0,1,2,3,4,5,6,7,8,9,10)	$M^{HD}{}_0(a_{21})=\frac{1}{4}\begin{bmatrix}(-5-4+9+10)(0.25)+(-3-2+7+8)(0.5-0.25)+\\(-1+0+5+6)(0.75-0.5)+(1+2+3+4)(1-0.75)\end{bmatrix}$ $=\frac{1}{4}[2.5+2.5+2.5+2.5]$ $=2.5$
a_{22}= (0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15)	$M^{HD}{}_0(a_{22})=\frac{1}{4}\begin{bmatrix}(0+1+14+15)(0.25)+(2+3+12+13)(0.5-0.25)+\\(4+5+10+11)(0.75-0.5)+(6+7+8+9)(1-0.75)\end{bmatrix}$ $=\frac{1}{4}[7.5+7.5+7.5+7.5]$ $=7.5$
a_{23}= (-3,-1,1,3,5,7,9,11,13,15,17,19,21,23,25,27)	$M^{HD}{}_0(a_{23})=\frac{1}{4}\begin{bmatrix}(-8-6+20+22)(0.25)+(-4-2+18+18)(0.5-0.25)+\\(0+2+12+14)(0.75-0.5)+(4+6+8+10)(1-0.75)\end{bmatrix}$ $=\frac{1}{4}[7+7+7+7]$ $=7$
a_{31}= (5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20)	$M^{HD}{}_0(a_{31})=\frac{1}{4}\begin{bmatrix}(5+6+19+20)(0.25)+(7+8+17+18)(0.5-0.25)+\\(9+10+15+16)(0.75-0.5)+(11+12+13+14)(1-0.75)\end{bmatrix}$ $=\frac{1}{4}[12.5+12.5+12.5+12.5]$ $=12.5$
a_{32}= (2,3,5,6,8,9,11,12,14,15,17,18,20,23,24)	$M^{HD}{}_0(a_{32})=\frac{1}{4}\begin{bmatrix}(2+3+23+24)(0.25)+(5+6+20+21)(0.5-0.25)+\\(8+9+17+18)(0.75-0.5)+(11+12+14+15)(1-0.75)\end{bmatrix}$ $=\frac{1}{4}[13+13+13+13]$ $=13$
a_{33}= (0,2,4,6,8,10,12,14,16,18,20,22,24,26,28,30)	$M^{HD}{}_0(a_{33})=\frac{1}{4}\begin{bmatrix}(0+2+28+30)(0.25)+(4+6+24+26)(0.5-0.25)+\\(8+10+20+22)(0.75-0.5)+(12+14+16+18)(1-0.75)\end{bmatrix}$ $=\frac{1}{4}[15+15+15+15]$ $=15$

Since the condition

$a_1+a_2+a_{15}+a_{16}=a_3+a_4+a_{13}+a_{14}=a_5+a_6+a_{11}+a_{12}=a_7+a_8+a_9+a_{10}$

is satisfied by all the hexadecagonal numbers for any value of k.

Step 2:

The payoff matrix is

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} 4.5 & 9.5 & 12 \\ 2.5 & 7.5 & 7 \\ 12.5 & 13 & 15 \end{bmatrix} \end{array}$$

Minimum of 1st row = 4.5

Minimum of 2nd row = 2.5

Minimum of 3rd row =12.5

Maximum of 1st column=12.5

Maximum of 2nd column=13

Maximum of 3rd column=15

Max min=12.5 Min max=12.5

It has a saddle point values of game=12.5

4.2.2 consider the fuzzy game problem

$$\begin{array}{cc} & B \\ A & \begin{bmatrix} (-3,-1,1,3,5,7,9,11,13,15,17,19,21,23,25,27) & (1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16) & (2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17) \\ (-3,-2,-1,0,1,2,3,4,5,6,7,8,9,10,11,12) & (-8,-6,-4,-2,0,2,4,6,8,10,12,14,16,18,20,22) & (-3,-2,-1,0,1,2,3,4,5,6,7,8,9,10,11,12) \\ (0,2,4,6,8,10,12,14,16,18,20,22,24,26,28,30) & (-5,-4,-3,-2,-1,0,1,2,3,4,5,6,7,8,9,10) & (-10,-8,-6,-4,-2,0,2,4,6,8,10,12,14,16,18,20) \end{bmatrix} \end{array}$$

SOLUTION:

Hexadecagonal fuzzy number $\tilde{A}$ is calculated as

$$M^{HD}{}_0(\tilde{A}) = \frac{1}{2}\int_0^{k_1}(l_1(r)+l_2(r))dr + \frac{1}{2}\int_{k_1}^{k_2}(g_1(s)+g_2(s))dr + \frac{1}{2}\int_{k_2}^{k_3}(h_1(t)+h_2(t))dr + \frac{1}{2}\int_{k_3}^{1}(f_1(h)+f_2(h))dr$$

$$M^{HD}{}_0(\tilde{A}) = \frac{1}{4}\{(a_1+a_2+a_{15}+a_{16})k_1 + (a_3+a_4+a_{13}+a_{14})(k_2-k_1) + (a_5+a_6+a_{11}+a_{12})(k_3-k_2) + (a_3+a_4+a_{13}+a_{14})(1-k_3)$$

where $0 < k_1 < k_2 < k_3 < 1$

Step 1:

Convert the given fuzzy problem into a crisp value problem. This problem is done by taking the value of k as 0.4

We obtain the value of $M^{HD}{}_0(a_{ij})$

$a_{11} = (-3,-1,1,3,5,7,9,11,13,15,17,19,21,23,25,27)$	$M_0^{HD}(a_{11}) = \frac{1}{4}\begin{bmatrix}(-3-1+25+27)(0.25)+(1+3+21+23)(0.25)+ \\ (5+7+17+19)(0.25)+(9+11+13+15)(0.25)\end{bmatrix}$ $=\frac{1}{4}[12+12+12+12]$ $=12$
$a_{12} = (-8,-6,-4,-2,0,2,4,6,8,10,12,14,16,18,20,22)$	$M_0^{HD}(a_{11}) = \frac{1}{4}\begin{bmatrix}(-8-6+20+22)(0.25)+(-4-2+16+18)(0.25)+ \\ (0+2+12+14)(0.25)+(4+6+8+10)(0.25)\end{bmatrix}$ $=\frac{1}{4}[7+7+7+7]$ $=7$
$a_{13} = (2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17)$	$M_0^{HD}(a_{11}) = \frac{1}{4}\begin{bmatrix}(2+3+16+17)(0.25)+(4+5+14+15)(0.25)+ \\ (6+7+12+13)(0.25)+(8+9+10+11)(0.25)\end{bmatrix}$ $=\frac{1}{4}[9.5+9.5+9.5+9.5]$ $=9.5$

a_{21}= (-3,-2,-1,0,1,2,3,4,5,6,7,8,9,10,11,12)	$M_0^{HD}(a_{11}) = \frac{1}{4}\left[\begin{matrix}(-3-2+11+12)(0.25)+(-1+0+9+10)(0.25)+ \\ (1+2+7+8)(0.25)+(3+4+5+6)(0.25)\end{matrix}\right]$ $=\frac{1}{4}[4.5+4.5+4.5+4.5]$ $=4.5$
a_{22}= (-5,-4,-3,-2,-1,0,1,2,3,4,5,6,7,8,9,10)	$M_0^{HD}(a_{11}) = \frac{1}{4}\left[\begin{matrix}(-5-4+9+10)(0.25)+(-3-2+7+8)(0.25)+ \\ (-1+0+5+6)(0.25)+(1+2+3+4)(0.25)\end{matrix}\right]$ $=\frac{1}{4}[2.5+2.5+2.5+2.5]$ $=2.5$
a_{23}= (-3,-2,-1,0,1,2,3,4,5,6,7,8,9,10,11,12)	$M_0^{HD}(a_{11}) = \frac{1}{4}\left[\begin{matrix}(-3-2+11+12)(0.25)+(-1+0+9+10)(0.25)+ \\ (1+2+7+8)(0.25)+(3+4+5+6)(0.25)\end{matrix}\right]$ $=\frac{1}{4}[4.5+4.5+4.5+4.5]$ $=4.5$
a_{31}= (0,2,4,6,8,10,12,14,16,18,20,22,24,26,28,30)	$M_0^{HD}(a_{11}) = \frac{1}{4}\left[\begin{matrix}(0+2+28+30)(0.25)+(4+6+24+26)(0.25)+ \\ (8+10+20+22)(0.25)+(12+14+16+18)(0.25)\end{matrix}\right]$ $=\frac{1}{4}[15+15+15+15]$ $=15$
a_{32}= (1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16)	$M_0^{HD}(a_{11}) = \frac{1}{4}\left[\begin{matrix}(1+2+15+16)(0.25)+(3+4+13+14)(0.25)+ \\ (5+6+11+12)(0.25)+(7+8+9+10)(0.25)\end{matrix}\right]$ $=\frac{1}{4}[8.5+8.5+8.5+8.5]$ $=8.5$
a_{33}= (-10,-8,-6,-4,-2,0,2,4,6,8,10,12,14,16,18,20)	$M_0^{HD}(a_{11}) = \frac{1}{4}\left[\begin{matrix}(-10-8+18+20)(0.25)+(-6-4+16+14)(0.25)+ \\ (-2+0+10+12)(0.25)+(2+4+6+8)(0.25)\end{matrix}\right]$ $=\frac{1}{4}[5+5+5+5]$ $=5$

Since the condition

$$a_1+a_2+a_{15}+a_{16}=a_3+a_4+a_{13}+a_{14}=a_5+a_6+a_{11}+a_{12}=a_7+a_8+a_9+a_{10}$$

is satisfied by all the hexadecagonal numbers for any value of k.

Step 2:

The payoff matrix is

$$\begin{array}{cc} & B \\ A & \left[\begin{matrix}12 & 7 & 9.5 \\ 4.5 & 2.5 & 4.5 \\ 15 & 8.5 & 5\end{matrix}\right]\end{array}$$

Minimum of 1st row = 7

Minimum of 2nd row = 2.5

Minimum of 3rd row =5

Maximum of 1st column=15

Maximum of 2nd column=8.5

Maximum of 3rd column=9.5

Max min=8.5 $\neq$ Min max=7

It has no saddle point.

Row I is dominated by row II so we omit row II

$$\begin{array}{cc} & B \\ A & \left[\begin{matrix}12 & 7 & 9.5 \\ 15 & 8.5 & 5\end{matrix}\right]\end{array}$$

Column I is dominated by Column II so we omit row I

$$\begin{array}{cc} & B \\ A & \left[\begin{matrix}7 & 9.5 \\ 8.5 & 5\end{matrix}\right]\end{array}$$

$$P_1 = \frac{a_{22} - a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$
$$= \frac{5-9.5}{(7+5)-(9.5+8.5)} = \frac{-4.5}{12-18} = \frac{-4.5}{-6} = \frac{4.5}{6}$$
$$P_2 = 1 - P_1 = 1 - \frac{4.5}{6} = \frac{1.5}{6}$$
$$q_1 = \frac{a_{22} - a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$
$$= \frac{5-8.5}{(7+5)-(9.5+8.5)} = \frac{-3.5}{12-18} = \frac{-3.5}{-6} = \frac{3.5}{6}$$
$$q_2 = 1 - q_1 = 1 - \frac{3.5}{6} = \frac{2.5}{6}$$

The optimum strategies are

$$S_A = (P_1, P_2) = \left(\frac{4.5}{6}, \frac{1.5}{6}\right)$$
$$S_B = (q_1, q_2) = \left(\frac{3.5}{6}, \frac{2.5}{6}\right)$$

The Value of the game is

$$V = \frac{a_{11}a_{22} - a_{21}a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})}$$
$$= \frac{(7)(5)-(8.5)(9.5)}{(7+5)-(9.5+8.5)} = \frac{-45.75}{-6}$$
$$= \frac{45.75}{6}$$

5. CONCLUSION

In this paper a method of solving fuzzy game problem using ranking of hexadecagonal fuzzy number has been considered. The parameter K can be modified suitably by the decision maker to get the desired result. We may get different fuzzy game value for different value of k for the same fuzzy games.

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