# Magneto Convection in Temperature-Sensitive Newtonian Liquids with Heat Source

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***Abstract-*** **The present paper deals with a linear stability problem of Rayleigh-Bénard convection in a finitely conducting Newtonian fluid with heat source confined between two horizontal surfaces. Heat source parameter, Thermo rheological parameter and the Chandrasekhar number influence the onset of convection. It is found that the effect of applied magnetic field is to stabilize the system whereas the effect of internal heat source and variable viscosity is to destabilize the system.**

**Index Terms- Rayleigh-Bénard Convection, Heat Source, Variable Viscosity, Linear, Magnetic Field.**

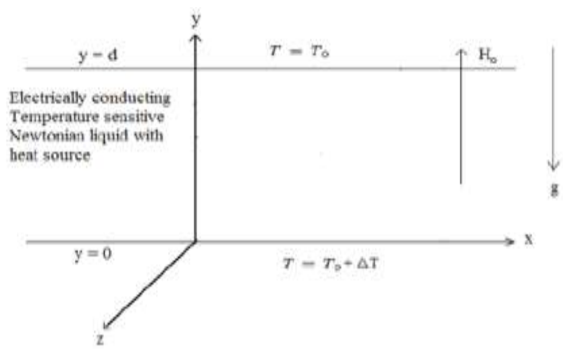
## I. INTRODUCTION

The Rayleigh-Bénard instability problem with internal heat generation, thermo rheological effect and magnetic field has received widespread attention due to their implications in heat transfer and in many engineering applications. The effect of magnetic field in detail on the onset of convection is discussed in Chandrasekhar [1], Platten and Legros [7] and Drazin and Reid [14]. The thermo rheological effect in an exponential form or in a polynomial form through truncated Taylor series expansion has been examined by Torrance and Turcotte [3]. McKenzie et al. [4] studied the convection in the earth mantle considering infinite Prandtl number. This paper reviews both the geophysical information and also the convection in the Boussinesq fluid of the infinite Prandtl number. The effect of internal heat generation on heat transport by the Rayleigh-Bénard convection was investigated by Tveitereid and Palm [5] and Clever [6]. They analyzed the two dimensional finite amplitude convective solutions and heat transfer, obtained as a function of the two Rayleigh number parameters and the wave numbers. Investigation of the stability of convective motion revealsa region in the dual Rayleigh number domain wherein stationary; two dimensional convection represents a stable solution of the equations of the motion. Riahi [8],[9] made a non-linear analysis of a fluid heated below in the presence of internal heat source, they proved that effect of increasing internal heat source parameter. They proved that the critical wave number is found to be insensitive to the changes in the micro polar fluid parameters, but sensitive to the Chandrasekhar number. Further they [13] showed that electrically conducting fluid layer with suspended particles heated from below is more stable compared to the classical electrically conducting fluid layer without suspended particles and the critical wave number is insensitive to the changes in the suspension parameters but sensitive to the changes in the Chandrasekhar number. Siddheshwar et al. [15], [16], [18], [19] studied Rayleigh-Benard and Marangoni-Bénard instability problems in the presence of thermo rheological effect by considering different constraints, and they found that effect of increasing the value of thermo rheological parameter is to destabilize the system. Recently the use of minimal representation of Fourier series expansion gained its attention in finite amplitude analysis of Rayleigh-Bénard convection (see Siddheshwar and Titus [20]) over stress free and isothermal surfaces. They found that truncated Fourier series is good enough to estimate the Eigen value in the problem and for finite amplitude analysis. Enhances the convection. Siddheshwar and Pranesh [11] studied the effect of magneto convection in micropolarfluid with free–free, isothermal, spin-vanishing boundaries.

The present paper aims at studying the convective instability in an electrically conducting temperature sensitive Newtonian liquid with heat source; we use the minimal representation half range Fourier cosine series expansion to study the onset of instability in electrically conducting Newtonian fluid with heat source/sink. The Galerkin technique is used to find out the analytical expression for the thermal Rayleigh number R_E as a function of Internal heat source parameterR_1, Thermo rheological parameter V, Chandrasekhar number Q and the wave number $\pi\alpha$.

## II. MATHEMATICAL FORMULATION

Consider an electrically conducting Newtonian liquid confined between two infinite, parallel horizontal planes of depth b between x=0 and x=b that supports a temperature gradient ΔT>0in the presence of transverse magnetic field H0 (see Fig. 1). We assume the Oberbeck-Boussinesq approximation is valid and consider only small-scale convective motions (Lorenz) and the boundaries are assumed to be stress-free and isothermal. Density, dynamic viscosity and heat source are assumed to be temperature-dependent. The basic governing equations of the flow are




$\nabla.\vec{q} = 0$ (1)

$\nabla.\vec{H} = 0$ (2)

$\rho_0[\frac{\partial\vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q}] = -\nabla p + \rho(T)\vec{g}\,\nabla.[\mu_f(T)(\nabla\vec{q} + \nabla\vec{q}^{\,Tr})] + \mu_m(\vec{H}.\nabla)\vec{H}$ (3)

$\frac{\partial T}{\partial t} + (\vec{q}.\nabla)T = \chi\nabla^2 T + Q_1(T - T_0)$ (4)

$\frac{\partial\vec{H}}{\partial t} + (\vec{q}.\nabla)\vec{H} = (\vec{H}.\nabla)\vec{q} + \gamma_m\nabla^2 \mathrm{H}$ (5)

$\rho(T) = \rho_0[1 - \beta(T - T_0)]$ (6)

$\mu_f(T) = \mu_0 e^{-\delta(T-T_0)}$ (7)

To make finite amplitude analysis, we consider the following perturbations

$\vec{q} = q_b + \vec{q}\,'T = T_b + T'\vec{H} = H_b + H'()$

$\rho = \rho_b + \rho', \rho = p_b + p', \mu_f = \mu_{fb} + \mu'_f$ (8)

where the primes indicates perturbed quantities. The basic state quantities $T_b(y)$,$\mu_{fb}(y)$, $\rho_b(y)$and$H_b(y)$ have the forms

$T_b = T_0 + \Delta T f(\frac{y}{d})$ , $\rho_b = \rho_0(1 - \beta\nabla T f\left(\frac{y}{a}\right))$,

$H_b = H_b(y)$, $p_b = -\int \rho_b\left(\frac{y}{d}\right) gd\left(\frac{y}{d}\right) + C_0$,

$\mu_{fb} = \mu_0 e^{-\beta f(\frac{y}{d})}$ (9)

Where$\left(\frac{y}{a}\right) = \frac{\sin[\sqrt{R}(1-\frac{y}{d})}{\sin\sqrt{R}}, C_0$ , is the constant of integration and$R_1 = \frac{Q_1 d^2}{\chi}$is the internal Rayleigh number. Since the flow considered is two dimensional, we introduce the stream function $\psi$' and the magnetic potential function$\phi'$ as follows:

$u' = \frac{\partial\psi'}{\partial y'}$, v'=$-\frac{\partial\psi'}{\partial x}$ (10)

$H'_x = \frac{\partial\phi'}{\partial y}$, $H'_y = -\frac{\partial\phi'}{\partial x}$ (11)

Eliminating the pressure in the equation (3) and using the equations (10) and (11) in the resulting equation, we get the following equations

$\rho_0\frac{\partial}{\partial t}(\nabla^2\psi') = \frac{\partial\mu f_b}{\partial y}\frac{\partial}{\partial y}(\nabla^2\psi') + \mu_{fb}\nabla^4\psi' + \frac{\partial' u_f}{\partial x}\frac{\partial}{\partial y}(\nabla^2\psi') + \mu'_f\nabla^4\psi' + \frac{\partial u'_f}{\partial x}\frac{\partial}{\partial x}(\nabla^2\psi') - \rho_0\alpha g\frac{\partial T'}{\partial x} + \frac{\partial(\psi',\nabla^2\psi')}{\partial(x,y)} - \frac{\partial(\phi',\nabla^2\phi')}{\partial(x,y)} + \mu_m H_b\frac{\partial}{\partial y}(\nabla^2\phi')$, (12)

$\frac{\partial T'}{\partial t} = \frac{\partial\psi'}{\partial x}\frac{dT_b}{dy} + \chi\nabla^{2}T' + Q_1 T' + \frac{\partial(\phi'.T')}{\partial(x,y)}$ (13)

$\frac{\partial\phi'}{\partial t} = H_b\frac{\partial\psi'}{\partial y} + v_m\nabla^2\phi' - \frac{\partial(\phi',\psi')}{\partial(x,y)}$(14)

we non-dimentionalize the equation (12),(13)and (14) using the following definitions

(X,Y) $= (\frac{x}{d},\frac{y}{d})$, $\tau = \frac{x}{d^2}$t, $\psi = \frac{\psi'}{x'}$

$\phi = \frac{\phi'}{dH_b}$, $\theta = \frac{T'}{\Delta T}$ (15)

and obtain

$\frac{1}{pr}\frac{\partial}{\partial\tau}(\nabla^2\psi) = \frac{\partial\mu_{fb}}{\partial Y}\frac{\partial}{\partial Y}(\nabla^2\psi) + \mu_{fb}\nabla^4\psi + \frac{\partial\mu_f}{\partial Y}\frac{\partial}{\partial Y}(\nabla^2\psi) + \mu_f{}'\nabla^4\psi + \frac{\partial\mu_f{}'}{\partial X}\frac{\partial}{\partial X}(\nabla^2\psi) + R_E\frac{\partial\Theta}{\partial X} + \frac{1}{pr}\frac{\partial(\psi,\nabla^2\psi}{\partial(X,Y)} + \mathrm{Q}\,p_m\frac{\partial}{\partial Y}(\nabla^2\phi) - \mathrm{Q}\,p_m\frac{\partial(\phi,\nabla^{2\phi}}{\partial(x,y)}$ (16)

$\frac{\partial\Theta}{\partial\tau} = -\frac{\partial\psi}{\partial x}f'(\mathrm{Y}) + \nabla^2\Theta + R_1\Theta + \frac{\partial(\Theta,\psi)}{\partial(x,y)}$ (17)

$\frac{\partial\phi}{\partial\tau} = \frac{\partial\psi}{\partial y} + p_m\nabla^2\phi + \frac{\partial(\phi,\psi)}{\partial(x,y)}$ (18)

Where,$p_m = \frac{v_m}{x}$ represents the magnetic prandtl number, pr $= \frac{v}{x}$ is prandtl number, $R_E = \frac{\alpha g\Delta T d^3}{x^v}$ is the thermal Rayleigh number,Q= $\frac{\mu_{mH_b^2 d^2}}{p_{0vv_m}}$is chantharasekhar number and

$f'(Y) = 1 + 2\sum_{n=1}^{\alpha}\frac{R_1}{R - n^2\pi^2}\cos(n\pi Y)$

The boundary conditions for solving the equations (16),(17) and (18) are

$\psi = \nabla^{2\psi} = D\phi = \Theta = 0$ at Y=0,1 (19)

In the next section, the linear stability analysis is performed using galerkin technique which is of great utility in finding the engin value of the problem

## III. LINEAR STABILITY ANALYSIS

In order to study the linear theory, the linearized version of equations (16),(17) and (18) is considered along with the boundary conditions (19). This means that the Jacobians $\frac{\partial(\phi,\psi)}{\partial(x,y)}$ and $\frac{\partial(\Theta,\psi)}{\partial(x,y)}$ in equations (16), (17) and (18) are neglected. In the view of the principle of exchange of stabilities being valid, the solution of the linearized system is assumed to be periodic waves of the form

$\psi$ (X,Y) $= \psi_o \sin(\pi\alpha X)\sin(\pi Y)$

$\Theta$ (X,Y) $= \Theta_o \cos(\pi\alpha X)\sin(\pi Y)$

$\phi$ (X,Y) $= \phi_o \cos(\pi\alpha X)\sin(\pi Y)$ (20)

Where $r_E = \frac{R_E}{R_{E_c}}$, $\eta_1^2 = \pi^2(1+\alpha^2)$

$$R_E = \frac{\eta_1^2\ (\eta_1^2 - R_1)(4\pi^2 - R_1)}{4\pi^2\alpha^2}\left(\frac{1+\alpha^2}{2}a_o + \frac{1-\alpha^2}{2}a_2 + \frac{Q}{\eta_1^2}\right) \quad (21)$$

$$a_o = 2\mu_o \int_0^1 \frac{v}{e^{\sin[\sqrt{R1}]}} \sin[\sqrt{R_1}(y-1)]\,\mathrm{dY}$$

$$a_2 = 2\mu_o \int_0^1 \frac{v}{e^{\sin[\sqrt{R1}]}} \sin[\sqrt{R_1}(y-1)]\cos(2\pi Y)\,dY$$

In equation (20), Ł is the scaled horizontal wave number. The quantities $\psi_o$,$\Theta_o$ and respectively, amplitudes of the stream function, temperature, and the magnetic stream function. Substituting Eqn.(20) into the linear version of Eqns. (16) to (18) and integrating the above equation with respect to X in $\left[0 \quad \frac{2\pi}{\pi\alpha},\right]$ and also with respect to Y in $[0, \quad 1]$ set of homogeneous equations in $\psi_o$,$\Theta_o$ and $\phi_o$is obtained. In obtaining the non-trivial solution of the linear system, the above expression of the critical Rayleigh number is obtained.

It is now clear that $R_E$ defined by Eqn. (20) is the critical Rayleigh number of the marginal stationary state. The scaled critical wave number for the preferred mode satisfies the following equation:

$$(a_0 - a_2)\pi^4\alpha_c^6 + \frac{1}{2}(3a_0\pi^4 - a_2\pi^4 - R_1 a_0\pi^2 + R_1 a_2\pi^2)\alpha_c^4 - \frac{1}{2}(a_0 + a_2)\pi^4 + 2Q\pi^2 - R_1 a_0\pi^2 - R_1 a_2\pi^2 + 2QR_1 = 0. \quad (22)$$

## IV. RESULT AND DISCUSSION

The effect of a variable viscosity on Rayleigh-Bénard convection is studied in thepresence of applied magnetic field and heat source. The effect of heat source, variable viscosity and the applied magnetic field appears in the equation in the form of an internal Rayleigh number $R_E$, Thermo rheological parameter $V_{and}$ Chandrasekhar number Q. The external Rayleigh number $R_E$ is the Eigen value of the problem.

The highlight of the linear study is that the derivation of a useful analytical expression for the stationary critical Rayleigh number by using the half-range Fourier cosine series expansion for the basic non uniform temperature gradient and for the basic viscosity. The focus in the paper is on Rayleigh-Bénard convection influenced by a heat source (sink) and for this reason only those values of $R_1$ that do not allow dominance of heat source (sink) over buoyancy in effecting convection are considered. In order to understand better the results arrived at in the problem we analyze the nonlinear basic state temperature distribution, which throws light on the observed effect of heat source (sink)on the stability. A scaled, dimensionless temperature distribution is considered in the following form:

$$\theta_b\ (\mathrm{Y}) = \frac{T_b - T_0}{\Delta T} = \frac{\sin[\sqrt{R(Y-1)}}{\sin[\sqrt{R}]}$$

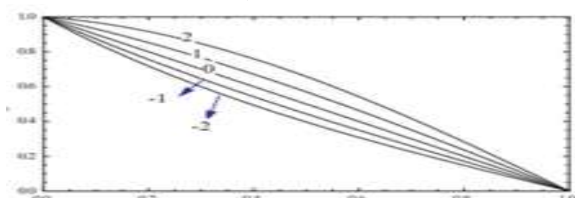



Fig.2: Temperature Profile of Quiescent Basic State for Different Values of

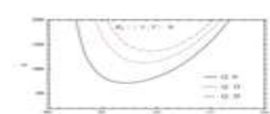


Fig.3(a)

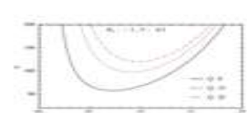


Fig.3 (b)

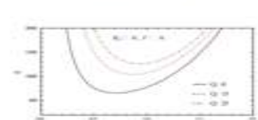


Fig.3 (c)

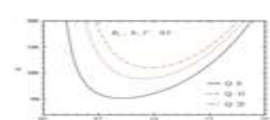


Fig.3 (d)

Fig.3 (e)

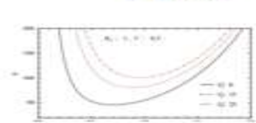


Fig.3 (f)

## V. CONCLUSION

The paper presents an analytical study of effect of temperature-sensitive viscosity, heat sourceand the magnetic field on Rayleigh-Bénard convection. Closed form expressions for α_c and R_(E_c ) are obtained asfunctions of the parameters of the problem. It is found that one can regulate the flow by suitably tuning theparameters' involved in the problem and also it is found that the effect of increasing the strength magneticfield is to stabilize the system where as the increasing the thermorheological parameter and the internalRayleigh number is to destabilize the system.

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