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Numerical Analysis of Heat and Mass Transfer on Fluid past a Sheet

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***Abstract*- In this work, a variational technique is applied to fluid flow over a sheet. The heat and mass transfer effects have been investigated and analyzed by this technique. The flow fields inside the boundary layer are approximated as polynomial functions. Euler-Lagrange equations for the functional of variational principle are constructed. The non-linear boundary layer equations are simplified as simple polynomial equations in terms of momentum, thermal and concentration boundary layer thicknesses. The temperature, concentration profiles, local heat and mass transfer rates are analyzed and are compared with existing numerical results. The comparison shows remarkable accuracy.**

Index term- Gyarmati's variational principle; boundary layer; heat and mass transfer; thermophoresis; thermal radiation.

I. INTRODUCTION

The prime objective of this work is to analyse the heat transfer enhancement of fluid flow over a sheet with radiation effect by using the field of thermodynamics of irreversible processes and to obtain numerical solution to heat and mass transfer with the help of a variation technique based on the Governing Principle of Dissipative Processes (GPDP).In many industrials, extrusion is an important process in manufacturing of products. The quality of these products solely depends on the heat transfer rate at the stretching sheets. Sakiad is analyzed the boundary layer. Flow over a moving continuous solid surfaces. Crane [2] found a closed form exact solution for Sakiadis problem. The effects of magneto hydrodynamics and thermal radiation on convective heat transfer play vital role in the phenomena of electrically conducting fluid past a heated surface and thermal processes involving high temperatures such as power generators, nuclear power plants etc. Swati analyzed these effects on boundary layer flow over an exponentially stretching sheet. In recent years, nano fluid which is a mixture of nano-sized particles suspended in a conventional fluid is used to enhance the heat transfer rate. The benefits of nano fluids are theoretically investigated by Choi. The xplanation for abnormal convective heat transfer enhancement in nanofluids. an analysis for laminar nanofluid flow over a stretching sheet. the boundary layer flow of a nanofluid over a non-isothermal stretching sheet with magnetic and radiation effects. As suggested in Buongiorno model the two important slip mechanisms Brownian motion and thermophoresis effects are considered in this boundary layer flow over a non-isothermal stretching sheet through quiescent nanofluidin the presence of radiation and constant magnetic flux density. Gyarmati's variation technique has been employed and the results are given for the temperature profile, concentration profile, the local Nusselt number (heat transfer)and the Sherwood number (mass transfer) for various values of Prandtl number Pr, magnetic parameter ξ, wall temperature parameter n, radiation parameter Nr, the slip parameters Nb (Brownian effect), Nt (thermophoresis effect) and Lewis number Le. The present results are compared with known numerical results and are found to be quite in agreement. The intention of this research work is to establish the fact that Gyarmati's principle is one of the exact and most general variational techniques in solving heat and mass transfer problems. Chandrasekar [8], Chandrasekar and Kasiviswanathan [9] have already applied Gyarmati'svariational principle for steady and unsteady heat transfer and boundary layer flow problems.

II. THE GOVERNING BOUNDARY LAYER EQUATIONS

The system of steady, two dimensional and laminar boundary layer flow of a fluid over a sheet with velocity U0 in x-direction is considered. The leading edge of the sheet is at x = 0 and the sheet is parallel to the x-axis. It is assumed that as y →, the quiescent fluid is with ambient temperature T and concentration C. By Boundary layer-Boussinesq approximations and with the assumption that all fluid properties are constants, the boundary layer equations in the presence of thermal radiation are considered as follows,

$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$ (1)

$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=v\frac{\partial^2 u}{\partial y^2}-\frac{kb_0^2}{b_f}u$ (2)

$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial y^2}\right)-\frac{1}{(pc)_f}\left(\frac{\partial q_r}{\partial y}\right)+\tau\left[D_b\left(\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}\right)+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right]$ (3)

$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\left(\frac{\partial^2 C}{\partial y^2}\right)+\frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial y^2}\right)$ (4)

subject to the boundary conditions

$y\to 0 \Rightarrow u=U_0=ax, v=0, T=T_0=T_\infty+Ax^n, C=C_0$

$y\to\infty \Rightarrow u=0, v=0, T=T_\infty, C=C_\infty$ (5)

where u, v, T, C are the velocity of the fluid in the longitudinal direction, transverse direction, temperature and concentration of the fluid respectively. The symbols υ, κ, B0, f ρ , f c , τ, DB, DT, T0 are respectively the kinematic viscosity, electrical conductivity, externally imposed magnetic field in the y-direction, density and specific heat of the fluid, c is the ratio of nanoparticle heat capacity and base fluid heat capacity, It is assumed that the temperature of the sheet T0 is greater than the ambient temperature Tλ. Using Rosseland approximation, the radiative heat flux is described by $q_r=\frac{-4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}$, where σ*, k*are the Stefan-Boltzmann constant and mean absorption coefficient respectively. Linearization of the radiation (4 T) in terms of temperature difference between the atmospheric level and the main flow as follows,

By Taylor series, we obtain $T^4=T_\infty^4+4T_\infty^3(T-T_\infty)+6T_\infty^2(T-T_\infty)^2+\cdots$We assume that the lowest temperature differences within the main flow. Therefore the higher order terms are to be neglected, hence $T^4=4T_\infty^4T-3T_\infty^3$. Thus, we have$q_r=\frac{-16T_\infty^3\sigma^*}{3k^*}\frac{\partial T}{\partial y}$.

III. GYARMATI'SVARIATIONAL PRINCIPLE

On the basis of irreversible thermodynamics, Gyarmati's "Governing Principle of Dissipative Processes" is given in its energy picture (Gyarmati [10,11]) as

$\delta\int v\,(T\sigma-T\psi-T\phi)dV=0$ (6)

Here the energy dissipation $T_\sigma$and dissipation potentials$T\psi, T\phi$ are given by $T_\sigma=p_{12}\frac{\partial u}{\partial y}-j_q\frac{\partial lnT}{\partial y}-j_c\frac{\partial C}{\partial y}$,

$T\psi=\frac{1}{2}\left[L_s\left(\frac{\partial u}{\partial y}\right)^2+L_\lambda\left(\frac{\partial lnT}{\partial y}\right)^2+L_c\left(\frac{\partial C}{\partial y}\right)^2\right]$ and $T\phi=\frac{1}{2}\left[R_sp_{12}^2+R_\lambda J_q^2+R_cJ_c^2\right]$, where $p_{12}\left(=-\frac{\partial u}{\partial y}\right)$, $J_q\left(=-L_\lambda\frac{\partial T}{\partial y}\right)$ and $J_c\left(=-\frac{\partial C}{\partial y}\right)$are heat, momentum and concentration fluxes respectively. The constants L's, R's represent conductivities and resistances. It is well known that 'lnT ' is the proper state variable instead of T when the governing principle assumes energy picture. The variation principle (6) for the present problem takes the form

$$\delta\int_0^1\int_0^\infty\left\{-J_q\frac{\partial lnT}{\partial y}-P_{12}\frac{\partial u}{\partial y}-J_c\frac{\partial C}{\partial y}-\frac{1}{2}\left[L_\lambda\left(\frac{\partial lnT}{\partial y}\right)^2+L_s\left(\frac{\partial u}{\partial y}\right)^2+L_c\left(\frac{\partial C}{\partial y}\right)^2\right]-\frac{1}{2}\left[R_\lambda J_q^2+R_sP_{12}^2+R_cJ_C^2\right]\right\}dx\,dy=0$$

in which 'l ' is the representative length of the stretching sheet.

IV. METHOD OF SOLUTION

The velocity, temperature and concentration fields inside the respective boundary layers are assumed as the following trial polynomials,

$\frac{u}{U_0}=1-\frac{2y}{d_1}+\frac{2y^3}{d_1^3}-\frac{y^4}{d_1^4}$for $y<d_1$, $u=U_\infty$ for $y\geq d_1$

$\theta=\frac{T-T_\infty}{T_0-T_\infty}=1-\frac{2y}{d_2}+\frac{2y^3}{d_2^3}-\frac{y^4}{d_2^4}$ for $y<d_2$, $T=T_\infty$ for $y\geq d_2$

$\phi=\frac{C-C_\infty}{C_0-C_\infty}=1-\frac{3y}{d_3}+\frac{3y^2}{d_3^2}-\frac{y^3}{d_3^3}$ for $y>d_3$, $C=C_0$ for $y\geq d_3$ (8)

These profiles satisfy the following conditions y→ 0 ⟹u=U_0=ax,v=0,T=T_∞+Ax^n,C=C_0,$\frac{\partial^2 u}{\partial y^2}$=$\frac{\partial^2 T}{\partial y^2}$=$\frac{\partial^2 C}{\partial y^2}$=0

y→ ∞ ⟹u=0,T=T_∞,C=C_∞,$\frac{\partial u}{\partial y}$=$\frac{\partial T}{\partial y}$=$\frac{\partial C}{\partial y}$=0,$\frac{\partial^2 u}{\partial y^2}$=$\frac{\partial^2 T}{\partial y^2}$=$\frac{\partial^2 C}{\partial y^2}$=0

(9)

The unknown parameters d_1,d_2 and d_3 are momentum, thermal and concentration boundary layer thicknesses and are to be determined by the variational principle. The trial functions (8) are substituted in the governing boundary layer equations (1-4) and on integration with respect to y and with the help of boundary conditions (9), the fluxes P_{12}, J_q and J_c are obtained. The expressions P_{12} and J_c remains same for any Prandtl number P_r. But the energy flux Jq assumes different expressions for Pr ≤ 1 and Pr ≥ 1 respectively. When Pr ≤ 1, the expression for Jq in the range d_1≤ y ≤ d_2 is determined first and the expression for Jq in the range 0 ≤ y ≤ d_1 is obtained subsequently by matching the expression J_q of the two regions at the interface. Using the expressions P_{12}, J_q and J_c along with the trial functions (8), the variation principle (7) is formulated. On integration with respect to y, the variation principle becomes as

$\delta\int_0^1 L_1\,(d_1,d_2,d_3,d_1',d_2',d_3')dx$=0 ($P_r$ ≤ 1)

$\delta\int_0^1 L_2\,(d_1,d_2,d_3,d_1',d_2',d_3')dx$=0($P_r$ ≥ 1) (10)

where the notation (′) indicates the differentiation with respect to x. These variational principles (10) are found identical when d_1=d_2. Accordingly, the Euler-Lagrange equations are

$\frac{\partial L_{1,2}}{\partial d_1}-\frac{d}{dx}\left(\frac{\partial L_{1,2}}{\partial d_1'}\right)=0, \frac{\partial L_{1,2}}{\partial d_2}-\frac{d}{dx}\left(\frac{\partial L_{1,2}}{\partial d_2'}\right)=0(P_r \gtreqless 1)$ and $\frac{\partial L_{1,2}}{\partial d_3}-\frac{d}{dx}\left(\frac{\partial L_{1,2}}{\partial d_3}\right)=0$ (11)

where $L_{1,2}$ represents the Lagrangian densities L_1 and L_2 respectively. Equations (11) are second order ordinary differential equations in terms of d_1, d_2 and d_3. The procedure for solving (11) can be considerably simplified by introducing the non-dimensional boundary layer thicknesses d_1^*,d_2^* *and* d_3^* given by d_1= $d_1^*\sqrt{\frac{v}{a}}$, d_2=$d_2^*\sqrt{\frac{v}{a}}$ and d_3 =$d_3^*\sqrt{\frac{v}{a}}$. The Euler-Lagrange equations to the variation principles (10) subject to this transformation are obtained as simple polynomial equations

$\frac{\partial L_{1,2}}{\partial d_1^*}=0, \frac{\partial L_{1,2}}{\partial d_2^*}=0(P_r \gtreqless 1)$ and $\frac{\partial L_{1,2}}{\partial d_3^*}=0$ (12)

The coefficients of these equations (12) depend on the independent parameters Pr, ξ, n, Nr, Nb, Nt and Le, where Pr = υ/α (Prandtl number), ξ = κ$B_0^2/\rho_f\,a$ (magnetic parameter), wall temperature parameter n, Nr =16$\sigma^* T_\infty^3$∞/3k^*k (radiation parameter), Nb=$\tau D_B\,(C_{0-C_\infty})/v$(Brownian motion parameter), Nt=$\tau D_B\,(T_0$-$T_\infty)/vT_\infty$(thermophoresis parameter) and Le = υ/DB (Lewis number). Equation (12)1 is a simple polynomial equation in terms of momentum boundary layer thickness whose effectsdepend on the magnetic parameter ξ. And equations $(12)_{2,3}$are coupled equations in terms of thermal andconcentration boundary layer thicknesses whose coefficients depend on d_1^* , Pr, n, Nr, Nb, Nt and Le. After obtaining the values of d_1^*, d_2^*, and and , the quantities of physical interest skin friction (shear stress), heat transfer (Nusselt number) and mass transfer (Sherwood number) are calculated

η =y$\sqrt{\frac{a}{v}}$, τ_w=$\sqrt{\frac{vx}{U_0^3}}\left(\frac{-P_{12}}{L_s}\right)_{y=0}$

$Nu_1 = \sqrt{{vx}/{U_0(T_0-T_\infty)^2}}\left(\frac{Jq}{L_\lambda}\right)_{y=0}$ and

Sh_1=$\sqrt{{vx}/{U_0(C_0-C_\infty)^2}}\left(\frac{J_c}{L_\lambda}\right)_{y=0}$

V. CONCLUSION

By GPDP, governing partial differential equations are simplified as polynomial equations This variation technique offers a practicing engineer a rapid way of obtaining heat and mass transfer rates for any combination of these parameters. The advantage involved in this technique is that the results are obtained with the high order of accuracy and the time taken to solve the problem is certainly less when compared with more conventional methods. Hence the practicing engineers and scientists can apply this unique approximate technique as a powerful tool for solving boundary layer flow, heat and mass transfer problems.

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