Cordial Labeling of Graphs

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Abstract- **Let G={V,E} be a graph. A mapping f : V(G)→{0,1} is called Binary Vertex Labeling. A Binary Vertex Labeling of a graph G is called a Cordial Labeling if |v_f (0)-v_f (1)|≤1 and |e_f (0)-e_f (1)|≤1. A graph G is Cordial if it admits Cordial Labeling. Here, we prove that Sunlet graph (S_n) and Shell graph C_((n,n-3)) are Cordial and the Splitting graphs of them are also Cordial.**

Index Terms- Cordial labeling, Splitting graph.

INTRODUCTION

All graphs considered here are finite, simple and undirected. The origin of graph labelings can be attributed to Rosa [8]. For all terminologies and notations we follow Harary [4]. Following definitions are useful for the present study.

Definition 1.1: For each vertex v of a graph G, take a new vertex v'. Join v' to all the vertices of G adjacent to v. The graph S(G) thus obtained is called splitting graph of G [5].

Definition 1.2: The assignment of values subject to certain conditions to the vertices of a graph is known as graph labeling [5].

Definition 1.3: Let G = {V, E} be a graph. A mapping f: V(G)→{0,1} is called Binary Vertex Labeling and f(v) is called the label of the vertex v of G under f.

For an edge e = uv, the induced edge labeling f^*: E(G) → {0,1} is given by f^*(e) = $|f(u) - f(v)|$. Let $v_f(0), v_f(1)$ be the number of vertices of G having labels 0 and 1 respectively under f and let $e_f(0), e_f(1)$ be the number of edges having labels 0 and 1respectively under f^* [5].

Definition 1.4: A Binary Vertex Labeling of a graph G is called a Cordial Labeling if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is Cordial if it admits Cordial Labeling [5].

Definition 1.5: The n-Sunlet graph is the graph on 2n vertices is obtained by attaching n-pendant edges to the cycle C_n and it is denoted by S_n.

Definition 1.6: A shell graph is defined as a cycle C_n with (n-3) chords sharing a common end point called the apex. Shell graph are denoted as $C_{(n,n-3)}$ [7].

MAIN RESULTS

Theorem 2.1: The graph S_n is cordial.

Proof: Let G be S_n. The vertices of S_n are $v_1, v_2, \dots, v_{2n}$.The edges are $e_1, e_2, \dots, e_{2n}$.

The vertex labeling f : V (G) →{0,1} is given below:

$$f(v_i) = \begin{cases} 1, if \;\; d(v_i) = 3 \\ 0, if \;\; d(v_i) = 1 \end{cases}$$

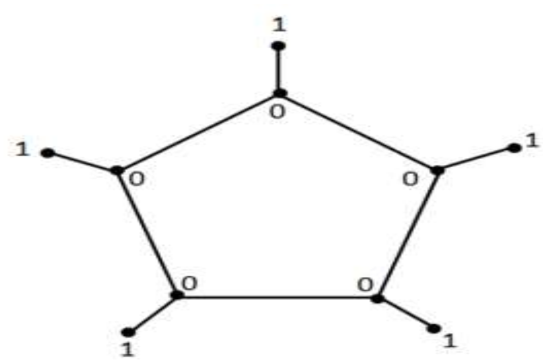

Fig 1: cordial labeling of Sunlet graph S_n

Here, $v_f(0) = v_f(1)$ and $e_f(0) = e_f(1)$ for all n.

Therefore the graph S_n satisfies the conditions $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Hence S_n is cordial.

Theorem 2.2: The graph S(S_n) is cordial.

Proof: Let G be S_n. The vertices of S_n are $v_1, v_2, \dots, v_{2n}$.Then *S(G)* has the vertices $v_1, v_2, \dots, v_{2n}, v'_1, v'_2, \dots, v'_{2n}$.

The vertex labeling f : V (G) →{0,1} is given below:

$$f(v_i) = \begin{cases} 1, if \;\; d(v_i) = 3 \\ 0, if \;\; d(v_i) = 1 \end{cases}$$

and

$$f(v_i') = \begin{cases} 1, if \ \ d(v_i) = 1 \\ 0, if \ \ d(v_i) = 3 \end{cases}$$

$v_f(0) = v_f(1)$ and $e_f(0) = e_f(1)$ for all n.

Therefore the graph S_n satisfies the conditions $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Hence $S(S_n)$ is cordial.

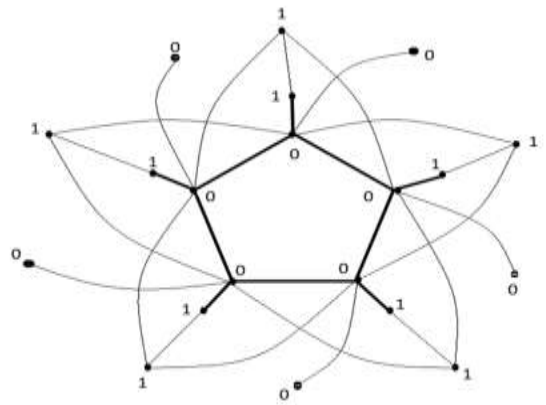

Fig 2: cordial labelling of splitting graph of Sunlet graph $S(S_n)$

Theorem 2.3: The graph $C_{(n,n-3)}$ is cordial.

Proof: Let G be $C_{(n,n-3)}$.. The vertices of $C_{(n,n-3)}$ are $v_1, v_2, \dots, v_n$. The edges are $e_1, e_2, \dots, e_n$.

The vertex labeling f : V (G) →{0,1} is given below:

Case i: *n is even*

$$f(v_i) = \begin{cases} 0, if \ 1 \leq i \leq n/2 \\ 1, otherwise \end{cases}$$

$v_f(0) = v_f(1)$ and $e_f(1) = e_f(0) + 1$.

Case ii: *n is odd*

$$f(v_i) = \begin{cases} 0, if \ 1 \leq i \leq (n-1)/2 \\ 1, otherwise \end{cases}$$

$v_f(0) = v_f(1)$ and $e_f(1) = e_f(0) + 1$.

Therefore the graph $C_{(n,n-3)}$ satisfies the conditions $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Hence $C_{(n,n-3)}$ is cordial.

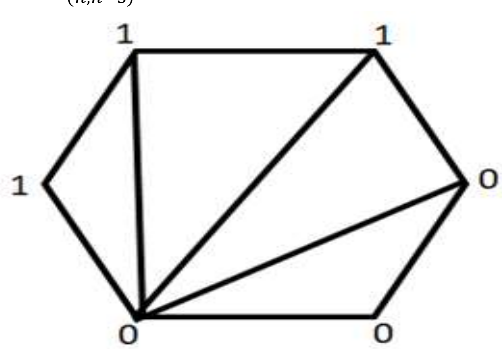

Fig 3. Cordial labeling of shell graph $C_{(n,n-3)}$

Theorem 2.4: The graph $S(C_{(n,n-3)})$ is cordial.

Proof: Let G be $C_{(n,n-3)}$.. The vertices of $C_{(n,n-3)}$ are $v_1, v_2, \dots, v_n$.Then *S(G)* has the vertices $v_1, v_2, \dots, v_n, v_1', v_2', \dots, v_n'$.

The vertex labeling f : V (G) →{0,1} is given below:

Case i: *n is even*

$$f(v_i) = \begin{cases} 0, if \ 1 \leq i \leq n/2 \\ 1, otherwise \end{cases}$$

and

$$f(v_i') = \begin{cases} 1, if \ 1 \leq i \leq n/2 \\ 0, otherwise \end{cases}$$

$v_f(0) = v_f(1)$ and $e_f(0) = e_f(1) + 1$.

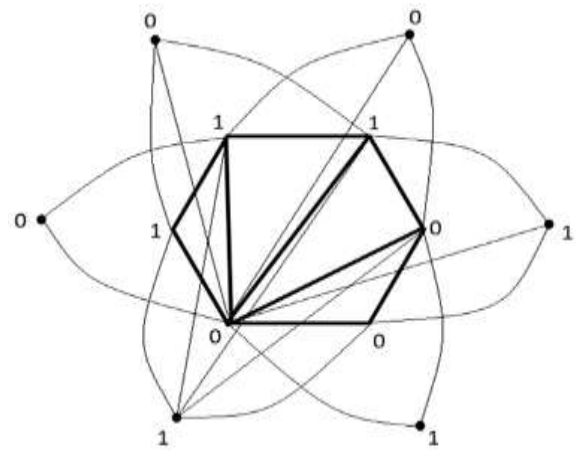

Fig 4. Cordial labeling of splitting graph of shell graph $S(C_{(n,n-3)})$

Case ii: *n is odd*

$$f(v_i) = \begin{cases} 0, if \ 1 \leq i \leq (n-1)/2 \\ 1, otherwise \end{cases}$$

and

$$f(v_i') = \begin{cases} 1, if \ 1 \leq i \leq (n+1)/2 \\ 0, otherwise \end{cases}$$

$v_f(0) = v_f(1)$ and $e_f(0) = e_f(1)$.

Therefore the graph $S(C_{(n,n-3)})$ satisfies the conditions $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Hence $S(C_{(n,n-3)})$ is cordial.

CONCLUSION

In this paper, we have tried to obtain the splitting graph of Sunlet graph and Shell graph and also we have labeled them and their base graphs using cordial labeling.

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