Role of Dynamic Programming for Data Analysis & Mathematical Application

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Abstract- **Dynamic programming is a valuable mathematical method used to render a series of interrelated decisions. It calls for a standardized method to evaluate the best mix of decisions. Proper investment decision-making is vital to the investor's progress in their attempts to keep up with the dynamic market climate. Mitigation of risk management plays a critical function, because customers are already actively subjected to the volatile decision-making climate. The volatility (and risk) of an investment is growing with a growth in the amount of competitive investors joining the sector. As a consequence, the estimated return on investment (ROI) of a decision also contributes to a high degree of ambiguity. The goal is to devise a dynamic mathematical programming model for an investment decision, incorporating this complexity in a probabilistic manner. The Dynamic Programming Strategy Iteration algorithm is used to solve the problem. Our simulation test demonstrates that the algorithm is capable of enabling us to make an effective investment decision**

Index terms- **Investment decision, dynamic programming, uncertainty, Data Analysis, Mathematical Application**

I.INTRODUCTION

Dynamic Programming is a mathematical technique that is applicable to many problems. It has been used to solve problems in areas such as allocation, cargo loading, replacement, sequencing, scheduling and inventory. However, dynamic programming is an "approach" to the solution of problems rather than a single algorithm that can be used to solve all of these types of problems. Thus, a separate algorithm is needed for each type of problem. Many problems fall into the general category of allocation problems, so a single dynamic programming algorithm could be used to solve problems such as investing in securities, allocating money for advertising, or assigning men to jobs. On the other hand, a different algorithm or formulation of equations would be needed to an equipment replacement problem.

II. CASE STUDY

The dynamic programming approach involves the optimization of multistage decision processes. That is, it basically divides a given problem into stages or sub problems and then solves the sub problems sequentially until the initial problem is optimality set forth by Bellman.

It states that An optimal policy has the properly that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.

This principle of optimality is an extremely powerful concept and should become more meaningful to the reader as the dynamic programming algorithms are developed for the examples.

Although dynamic programming can be used to determine the optimal solution for a large variety of problems, it is by no means the most efficient method to be use in all cases. Experience and ingenuity are the guiding factors in determining when to use dynamic programming. In addition, there are a number of problems for which dynamic programming algorithms have been developed, but because of time and storage limitations of present-day computers, only the very small problems can be solved. One example is the traveling salesman problem formulated. Dynamic programming can be used to solve the 10-15, and 25-city problems but is out of the question for large problems involving 50 or more cities.

Several examples are presented in Investment problem to illustrate the underlying concepts of dynamic programming for a particular type of problem. In each example a detailed step-by-step development of the solution using the concept of dynamic programming is followed by a concise dynamic programming algorithm and the corresponding computer program to solve the problem. Our first problem deals with the allocation or money to invest programs.

The P.P.C. is concerned with planning and controlling all aspects of production.

III. INVESTMENT PROBLEM

Consider the general problem of allocating a fixed amount of resource (money) to a number of activities (investment programs) in such a way that total is maximized. More specifically, for simplicity, suppose only 8 units of money available for allocation in unit amounts to three investment programs. The return function for each program is tabulated in Table 1. The function g_i (x) represents the return from investing x units of money in the investment program (i=1, 2 and 3).

Table 1. Return function, g_i (x)

x	0	1	2	3	4	5	6	7	8
$g_1(x)$	0	5	15	40	80	90	95	98	100
$g_2(x)$	0	5	15	40	60	70	73	74	75
$g_3(x)$	0	4	26	40	45	50	51	52	53

The return from each program is independent of the allocation to the oilier programs, For example, $g_2(5) = 70$ is the return tiom investing 5 unils in program 2 regardless of how remaining units are allocated to the other two programs.

The other assumptions needed to apply dynamic programming are:

1. The returns from all programs be measured in a common unit. Note that the unit of measure for x need not be the same as the unit of measure for the returns. It may be that each unit of x represents $5, and the returns may be in actual dollar amounts.
2. The total return is the sum of the individual returns.

With these assumptions, the problem can be embedded in a more general problem that can be solved sequentially in stage. Each stage is a sub problem which, when solved, provides information for the next stage. The solution to the sub problem at the final Stage is also the solution to the original problem. This is basically the dynamic programming approach to the solution of the problem. We will used this principle to determine how many units of money should be allocated to each investment program to maximize the total return.

Step-1

Assume for a moment that program 3 is the only investment program available to invest in, since $g_3(x)$, the return function for program 3, is an increasing function of the amount invested x, we should invest the entire 8 units in program 3. The return on our investment would, of course, be:

$$f_3(8) = g_3(8) = 53$$

And the amount invested to obtain this return is:

$$d_3(8) = 53$$

Where, $f_3(8)$ is the optimal return from program 3 when 8 units are invested in it.

Step-2

Now, let

$$f_3(x) = g_3(x) \qquad x = 0,1,2, \dots \dots \dots ..7$$

Be the optimal return from program 3 when x units are invested in it, and let

$$d_3(x) \qquad x = 0,1,2, \dots \dots \dots ..7$$

Be the optimal amount to invest in program 3. This appears rather trivial, and it is; nevertheless, it is essential the optimal return from program 3 when it is the only program in order to develop the final dynamic programming solution (see Table 2).

Table 2. Results from step 1 and 2.

x	0	1	2	3	4	5	6	7	8
$f_3(x)$	0	4	26	40	45	50	51	52	53
$d_3(x)$	0	1	2	3	4	5	6	7	8

Step-3

Assume now that program 2 and 3 are the only program available, and the entire 8 units are available to invest in these two programs. Since both return functions are increasing functions of the amount invested, the entire amount should be invested. The question, then, is how many unit should be allocated to each program? We already know the optimal return from program 3 for any amount invested in it, so it is just a matter of examining each of the sums for maximum return:

$$g_2(0) + f_3(8) = 53$$
$$g_2(1) + f_3(7) = 57$$

$$g_2(2) + f_3(6) = 66$$
$$g_2(3) + f_3(5) = 90$$
$$g_2(4) + f_3(4) = 105$$
$$g_2(5) + f_3(3) = 110$$
$$g_2(6) + f_3(2) = 99$$
$$g_2(7) + f_3(1) = 78$$
$$g_2(8) + f_3(0) = 75$$

This maximum return from program 2 and 3when 8 units are available is denoted by $f_3(8)$. That is,

$$f_3(8) = \max_{z=0,1,2,\ldots\ldots8}[g_2(z) + f_3(8-z)]$$

The optimal amount 10 invest in program 2 is denoted by $d_2(8)$ and is necessarily the value of z that yield $f_2(8)$.

In this case,

$$f_2(8) = g_2(5) + f_3(3) = 110$$
$$d_2(8) = 5$$

Step-4

We retain the assumption that programs 2 and are the only programs available to invest in. but now assume that only x units are available to invest in these programs (x=0,1,2,…7) for each value of x we calculate the optimal return from these programs when x units are available; namely,

$$f_2(x) = \max_{z=0,1,2,\ldots\ldots8}[g_2(z) + f_3(x-z)]$$

The amount to invest in program 2 is:

$$d_2(x) = value\ of\ z\ that\ yieds\ f_2(x)$$

We would calculate $f_2(x)\ and\ d_2(x)$ for x=0, 1, 2, …7. These values are:

$$x = 0 \quad f_2(0) = \max_{z=0}[g_2(z) + f_3(0-z)]$$
$$= g_2(0) + f_3(0)$$
$$= 0$$
$$d_2(0) = 0$$

$$x = 1 \quad f_2(1) = \max_{z=0,1}[g_2(z) + f_3(1-z)]$$
$$= max\begin{bmatrix} g_2(0) + f_3(1) \\ g_2(1) + f_3(0) \end{bmatrix}$$
$$= max\begin{bmatrix} 0+4 \\ 5+0 \end{bmatrix}$$
$$= 5$$
$$d_2(1) = 1$$

This says that if one unit of resource is available to invest in programs 2 and 3, the optimal policy is to invest it in program 2 for a total return is:

$$x = 2 \quad f_2(2) = \max_{z=0,1,2}[g_2(z) + f_3(2-z)]$$
$$= max\begin{bmatrix} g_2(0) + f_3(2) \\ g_2(1) + f_3(1) \\ g_2(2) + f_3(0) \end{bmatrix}$$
$$= max\begin{bmatrix} 0+26 \\ 5+4 \\ 15+0 \end{bmatrix}$$
$$= 26$$
$$d_2(2) = 0$$

If 2 units are available for programs 2 and 3, the optimal policy is to invest 0 units in program 2 and 2 units in program 3 for total return of 25.

Remember that we are solve a whole class of sub problems that will eventually lead to the solution of the original problem. Also keep in mind that we need to know the optimal policy for programs 2 and 3 for every amount of money available up to and including 8 units, so regardless of how many units (up through 8) are invested in program l. we know immediately the corresponding optimal policy for the remaining two programs.

This will be illustrated in step 5. For,

$$x = 3 \quad f_2(2) = \max_{z=0,1,2,3}[g_2(z) + f_3(3-z)]$$
$$= max\begin{bmatrix} g_2(0) + f_3(3) \\ g_2(1) + f_3(2) \\ g_2(2) + f_3(1) \\ g_2(3) + f_3(0) \end{bmatrix}$$
$$= 40$$
$$d_2(3) = 0\ or\ 3$$

If we are interested in an optimal solution rather than all optimal solutions. We need only retain $d_2(3) = 0$ or $d_2(3) = 3$ and not both. The values $f_2(x)$ and$d_2(x)$ for x = 4. 5, 6 are found in a fashion similar to that above. The complete results from step l-4 are given in Table 3.

Step-5

The final Stage is the same as the original problem. How should 8 units of money be invested in the three investment programs? It now becomes a matter of examining the results of investing z units in program 1 and 8-z units optimally in programs 2 and 3 for z= 0.1, ...,,8. That is $f_1(8)$ is the maximum or the quantities:

$$g_1(0) + f_2(8) = 110$$
$$g_1(1) + f_2(7) = 105$$
$$g_1(2) + f_2(6) = 101$$
$$g_1(3) + f_2(5) = 110$$
$$g_1(4) + f_2(4) = 140$$
$$g_1(5) + f_2(3) = 130$$
$$g_1(6) + f_2(2) = 121$$
$$g_1(7) + f_2(1) = 103$$

$$g_1(8) + f_2(0) = 100$$
$$Namely, \quad f_1(8) = g_1(4) + f_2(4) = 140$$
$$d_1(8) = 4$$

Note that $f_1(8)$ is the optimal return when 8 units are invested optimally in the three available programs, and $d_1(8)$ is the optimal amount to invest in program 1. Since $d_1(8) = 4$, this leaves 4 units to invest optimally in programs 2 and 3. But we can get this directly from table 3. Recall that $d_2(4)$ represents the optimal amount to invest in program 2 when just programs 2 and 3 are available. This leaves 0 units to invest in program 3. Thus the optimal allocation of money to the three program is:

$$d_1(8) = 4 \; units \; in \; program \; 1$$
$$d_2(4) = 4 \; units \; in \; program \; 2$$
$$d_3(0) = 0 \; units \; in \; program \; 3$$

Table 3. Results from step 1-4

x	0	1	2	3	4	5	6	7	8
$f_3(x)$	0	4	26	40	45	50	51	52	53
$d_3(x)$	0	1	2	3	4	5	6	7	8
$f_2(x)$	0	5	26	40	60	70	86	100	110
$d_2(x)$	0	1	0	0	4	5	4	4	5

Table 4. Results from step 1-5

x	0	1	2	3	4	5	6	7	8
$f_3(x)$	0	4	26	40	45	50	51	52	53
$d_3(x)$	0	1	2	3	4	5	6	7	8
$f_2(x)$	0	5	26	40	60	70	86	100	110
$d_2(x)$	0	1	0	0	4	5	4	4	5
$f_1(x)$	0	5	26	40	80	90	106	120	140
$d_1(x)$	0	0	0	0	4	5	4	4	4

For a maximum total return of

$$f_1(8) = 140$$

Note that we can check this by examining the sum of the return functions when the corresponding optimal amounts are invested in them.

$$g_1(4) + g_2(4) + g_3(0) = 80 + 60 + 0 = 140$$

Table 4 gives the results for any initial amount of money available up through 8 units. For example, if 6 units are available for investment in the three programs, the optimal allocation of these 6 unit is:

$$d_1(6) = 4$$
$$d_1(6-4) = d_2(2) = 0$$
$$d_3(2-0) = d_3(2) = 2$$

For a total return of

$$f_3(6) = 106$$

In arriving at an optimal solution of the initial problem, we have solved a whole class of problems, Keep in mind that this was a very simplified problem to illustrate the dynamic programming technique.

In summary, the dynamic programming function equations for the investment problem are:

$$f_3(x) = g_3(x)$$
$$d_3(x) = x \qquad \dot{x} = 0,1, \ldots \ldots \ldots 8$$
$$f_i(x) = \max_{z=0,1,\ldots\ldots x} [g_i(z) + f_{i+1}(x-z)] \quad x$$
$$= 0,1, \ldots \ldots \ldots 8$$
$$d_i(x) = value \; of \; z \; that \; yields \; f_i(x) \; for \; i = 2,1$$

Where $f_i(x)$ is the optimal return investing x units in programs $i, i+1, \ldots \ldots, 3$ and $d_i(x)$ is the optimal amount to invest in program i when x units are available to invest in programs $i, i+1, \ldots \ldots, 3$ for $i = 1,2,3$

IV. CHAIN OF TRENDS

Suppose we want an optimal solution of a given problem. To get at the solution, it is usually more meaningful and convenient to write out the problem in mathematical terms. This mathematical description or representation of the problem is called a mathematical model of the problem. Generally, it is easier to get a "handle" on the model and solve it rather than the problem in its original nonmathematical form. If a mathematical model of a given problem can solved either analytically or numerically, the solution can then be applied to the original problem. If the mathematical model is a good representation of the problem, the solution of the model will be a good solution of the problem. On the other hand, even an exact solution of a poor model will not be good solution of the problem.

Many problems can be represented by a number or different models, but one model is usually more appropriate than others. To this end, a number of unique models with appropriate methods of solution have become well known during the past 25 years. For example, linear programming models, dynamic programming models, inventory models, and queuing models have solutions readily available. Thus, if a given problem can be modeled as (put into the mathematical form of) a certain of linear programming model. The method of solution is immediately available. The object of any operational research OR project is to determine the most appropriate mathematical model for the problem at hand, and either use available methods to solve the mode' or develop new methods of solution.

It may be that a mathematical model of a given problem cannot he constructed on the other hand, it may be possible to construct a mathematical model, but exact methods for solving the model may not be readily available or may not be amendable for computer solution because of the large amount of computer storage or time required. Consequently, an alternative in this situation is to use an intuitive or heuristic approach to solution. This approach has been used successfully to solve problems directly without formulating a mathematical model. It has been used to provide approximate and /or exact solutions or many mathematical models of problems. Quite often heuristic methods provide exact, or at least adequate, solutions of problems much faster than numerical methods that are used to solve an appropriate model of the given problem. For example. One type of problem involving the allocation of resources to activities may be formulated as an integer linear programming model; however, a heuristic method called the Hungarian method provides solution of the problem much faster in most cases.

Finally, if problem is so complex that it cannot be modelled adequately for solution by one of the available methods or if it cannot be solved adequately with a heuristic method, then the problem solver usually resorts to simulation, Of course, simulation is not the answer to all problems; nevertheless, it does have great deal of merit in studying large, complex systems where components are highly interrelated.

It is our purpose to present computer oriented algorithms for most of' the method used to solve the well-known mathematical models, as well as algorithms important heuristic methods. In most cases, a computer program the algorithm is presented so the reader can be exposed to the solution and analysis of a large variety of problems to enhance his problem-solving ability. A number of up-to-dole methods from the literature are also presented in easy-to-grasp algorithmic form.

V. APPLICATION WITH ADVANTAGES

In this section, we have basically examined the dynamic programming approach to the solution of the problem of allocating resource to activities in order to optimize the total return from all activities. In general, the resource could be people, money machines, gasoline, grocery products, etc. and activities could be jobs, investment programs, manufacturing plants, refineries, grocery stores, etc. The basic assumptions in each case are:

1. The returns from all activities are measured in a common unit.
2. The return from a specific activity is independent of the returns from the other activities.
3. The return functions are no decreasing.
4. The total return from all activities is equal to the sum of the individual returns.

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