Evaluation of Ultimate Capacity of a Rectangular Solid Slab using Yield Line Analysis

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***Abstract*- In this paper, yield line theory is used to analyze regular slabs for different nine edge conditions. Two approaches are used for the computation of ultimate load-carrying capacity of a reinforced concrete slab concerning yield line theory. One is an energy method which uses the principle of virtual work and the other, an equilibrium method, which study the equilibrium of diverse part of the slab formed by the yield lines. The yield line theory is an upper bound theory and hence which pattern gives lowest load carrying capacity is the governing pattern. Finally equations are formulated by using yield line theory which are simple to apply.**

***Index terms*- Edge conditions, Equilibrium work method, Virtual work method, Failure pattern, Yield analysis**

I.INTRODUCTION

Yield line theory was first came into reality by Ingerslev (1923) where for the first time he performed this analysis on an rectangular slab simply supported by only assuming that bending moments act alone at yield lines. This theory was further developed and substantially forged by K W Johansen (1931).

In 1931, K W Johansen gave the conception an implication as lines of relative rotation of rigid slab parts, and in 1943 published this eponymous theory. Yield line analysis was adopted by Danish concrete code and set up into the curriculum at Technical University of Denmark. Wide-ranging research continued following the publications, from 1961-1962, by Wood, Jones and English translation of Johansen's thesis. Yield line theory has also been used quite extensively in the design of concrete plate elements that are required to withstand the forces generated by explosions in both domestic and military applications.

In rectangular slab based on the amount of reinforcement provided in the span directions there are 2 types of patterns available one is regular and the other is reversal pattern.

Fig.1 Rectangular Slab Yield Patterns

II. METHODS OF ANALYSIS

A. Virtual Work Method

In this method, the work done by external force during small moments of collapse mechanism is equal to the work absorbed by the plastic hinges.

Work done = energy absorbed by the yield lines.

$\sum (w \times \delta) = \sum (m \times l \times \theta)$

w is the load acting, δ is the vertical displacement of load

θ is rotation of region about axis of rotation

m is the moment of resistance of the slab per unit run

l is the length of yield line or the projected length on to the axis of rotation.

B. Equilibrium/Segmental Method

In this method external work done and internal work done of each individual segments of the slab is determined then the external and internal work done of each segment is equated and thereafter the load carrying capacity of each individual segment of slab is determined.

III. EQUATIONS FORMULATION

Equation calculation by virtual work method:

A. *For solid slab regular pattern*

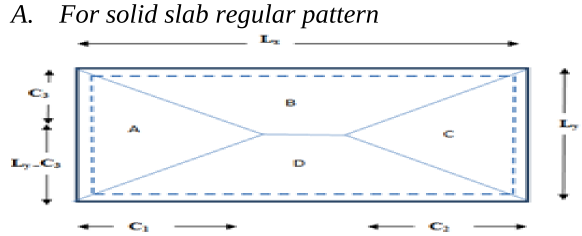

Fig 2: Solid slab regular pattern supported on 4 sides

The figure 2 is a rectangular slab with regular pattern occurs if slab is reinforced sufficiently strong in shorter span direction compared to longer span direction, the yield line pattern divides the slab into 4 segments A, B, C and D with L_x as long span and short span as L_y whereas C_1, C_2, C_3 and C_4 (L_y- C_3). By using Virtual work method the total external work in undergoing small virtual displacement is equal to the internal work done in rotation along yield lines. Let $r=L_x/L_Y$, $r_1=L_x/C_1$, $r_2=L_x/C_2$, $r_3=L_Y/C_3$, $r_4=L_Y/C_4$

Total External Work Done

For Segment 'A': $W_{e1} = \left(\frac{1}{2}\right) * (L_Y{*}C_1){*}(\frac{C_1}{3}){*}(\frac{1}{C_1}){*}W = (1/6)WL_YC_1$

For Segment 'B':

$W_{e2} = [\{(1/6)C_1C_3\} + \{(L_X - C_1 - C_2){*}(\frac{C_3}{2})\} + \{(1/6)C_2C_3\}]\ W$

For Segment 'C': $W_{e3} = (1/6)WL_YC_2$

For Segment 'D':

$W_{e4} = [\{(1/6)C_1\,(L_Y - C_3)\} + \{(L_X - C_1 - C_2){*}\left(\frac{L_Y - C_3}{2}\right)\} + \{(1/6)C_2(L_Y - C_3)\}]\ W$

Total External Work Done = $W_{e1}+W_{e2}+W_{e3}+W_{e4}$

T.E.W.D. $= W\,(\frac{L_Y^2}{6})[3(L_X/L_Y) - (C_2/L_Y) - (C_1/L_Y)]$

$= WL_y^2/6[3r-(r/r_1)-(r/r_2)]$

$= W(\frac{L_Y^2}{6})r\ [-(\frac{1}{r_1})-(\frac{1}{r_2}) +3]$

Total External Work Done = $W(\frac{L_Y^2}{6})r\ [-(\frac{1}{r_1})-(\frac{1}{r_2}) +3]$

(a) For Continuous Support (CS) condition:

Internal Work done or Energy Absorbed $=\sum mL\theta$

For Segment 'A': $E.A_1= (\frac{mL_y}{C_1})(K_x' + K_x)$

For Segment 'B': $E.A_2= (\frac{mL_x}{C_3})(K_y' + K_y)$

For Segment 'C': $E.A_3= (\frac{mL_y}{C_2})(K_x' + K_x)$

For Segment 'D': $E.A_4= (\frac{mL_x}{(L_y - C_3)})(K_y' + K_y)$

Total Energy Absorbed = $E.A_1+E.A_2+E.A_3+E.A_4$

$= m[(K_x' + K_x)(\frac{r_1}{r} + \frac{r_2}{r}) + (K_y' + K_y)(rr_3 + \frac{rr_3}{(r_3-1)})]$

Equating *Total External Work Done* and *Total Energy Absorbed*

$$\frac{WrL_y^2}{6}\left[3 - \frac{1}{r_1} - \frac{1}{r_2}\right] = m[(K_x' + K_x)(\frac{r_1}{r} + \frac{r_2}{r}) + (K_y' + K_y)(rr_3 + \frac{rr_3}{(r_3 - 1)})]$$

$$\frac{WrL_y^2}{6} = \frac{m[(K_x' + K_x)(\frac{r_1}{r} + \frac{r_2}{r}) + (K_y' + K_y)(rr_3 + \frac{rr_3}{(r_3-1)})]}{\left[3 - \frac{1}{r_1} - \frac{1}{r_2}\right]}$$

$$S = \frac{WL_y^2}{m} = \frac{6}{r}\left|\frac{[(K_x' + K_x)\left(\frac{r_1}{r} + \frac{r_2}{r}\right) + (K_y' + K_y)\left(rr_3 + \frac{rr_3}{(r_3-1)}\right)]}{\left[3 - \frac{1}{r_1} - \frac{1}{r_2}\right]}\right|$$

(B) For solid slab reversal pattern:

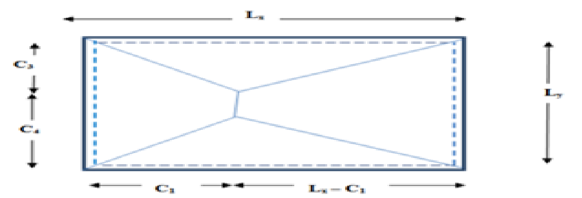

Fig 3: Solid Slab Reversal Pattern supported on 4 Sides

The figure 3.1 is a rectangular slab with reversal pattern occurs if slab is reinforced sufficiently strong in longer span direction compared to shorter span direction, the yield lines divides the slab into 4 segments A, B, C and D with L_x as long span and short span as L_y whereas C_1, C_2, C_3 and C_4 (L_y- C_3). By using Virtual work method the total external work in undergoing small virtual displacement is equal to the internal work done in rotation along yield lines.

Total External Work Done

For Segment 'A':

$W_{e1}= [\{(1/6)C_1C_3\} + \{(L_Y - C_3 - C_4){*}(\frac{C_1}{2})\} + \{(1/6)C_4C_1\}]\ W$

For Segment 'B': $W_{e2} = (1/6)WL_XC_3$

For Segment 'C':

$W_{e3}= [\{(1/6)C_3\,(L_X - C_1)\} + \{(L_Y - C_3 - C_4){*}\left(\frac{L_X - C_1}{2}\right)\} + \{(1/6)C_4(L_X - C_1)\}]\ W$

For Segment 'D':

$W_{e4}= 1/6\,WL_XC_4$

Total External Work Done = $W_{e1}+W_{e2}+W_{e3}+W_e$

T.E.W.D.= (W/6) $[-L_XC_3 - L_XC_4 + 3L_XL_Y]$

= (W/6) $[-L_X(\frac{L_Y}{r_3}) - L_X(\frac{L_Y}{r_4}) + 3L_XL_Y]$

= $(WL_Y^2/6)\ [-(\frac{r}{r_3}) - (r/r_4) + 3r]$

= $(WL_Y^2r/6)\ [-\left(\frac{1}{r_3}\right)- (\frac{1}{r_4})+3]$

Total External Work Done= ($WL_Y^2r/6$) [-$\left(\frac{1}{r_3}\right)$-($\frac{1}{r_4}$)+3]

For Continuous Support (CS) condition:

All the four sides of the slab is continuous so both positive and negative moment acts on all sides.

Internal Work done or Energy Absorbed =$\sum mL\theta$

For Segment 'A':

E.A$_1$= $(\frac{mL_y}{C_1})(K_x' + K_x)$

For Segment 'B'

E.A$_2$= $(\frac{mL_x}{C_3})(K_y' + K_y)$

For Segment 'C'

E.A$_3$= $(\frac{mL_y}{(L_x-C_1)})(K_x' + K_x)$

For Segment 'D'

E.A$_4$= $(\frac{mL_x}{C_4})(K_y' + K_y)$

Total Energy Absorbed = E.A$_1$+E.A$_2$ +E.A$_3$+E.A$_4$

$$= m[\left(\frac{L_y}{C_1}\right)(K_x' + K_x) + \left(\frac{L_x}{C_3}\right)\left(K_y' + K_y\right) + (\frac{L_y}{(L_x-C_1)})(K_x' + K_x) + \left(\frac{L_x}{C_4}\right)(K_y' + K_y)]$$

$$= m[(K_x' + K_x)(\frac{L_y}{C_1} + \frac{L_y}{(L_x-C_1)}) + \left(K_y' + K_y\right)(\frac{L_x}{C_3} + \frac{L_x}{C_4})]$$

$$= m[(K_x' + K_x)(\frac{r_1}{r} + \frac{r_1}{r(r_1-1)}) + \left(K_y' + K_y\right)(rr_3 + rr_4)]$$

Equating *Total External Work Done* and *Total Energy Absorbed*

Equating *Total External Work Done* and *Total Energy Absorbed*

$$\frac{WrL_y^2}{6}\left[3 - \frac{1}{r_3} - \frac{1}{r_4}\right]= m[(K_x' + K_x)(\frac{r_1}{r} + \frac{r_1}{r(r_1-1)}) + \left(K_y' + K_y\right)(rr_3 + rr_4)]$$

$$\frac{WrL_y^2}{6} = \frac{m[(K_x' + K_x)(\frac{r_1}{r} + \frac{r_1}{r(r_1-1)}) + \left(K_y' + K_y\right)(rr_3 + rr_4)]}{\left[3 - \frac{1}{r_3} - \frac{1}{r_4}\right]}$$

$$S = \frac{WL_y^2}{m} = \frac{6}{r}\left|\frac{[(K_x' + K_x)(\frac{r_1}{r} + \frac{r_1}{r(r_1-1)}) + \left(K_y' + K_y\right)(rr_3 + rr_4)]}{\left[3 - \frac{1}{r_3} - \frac{1}{r_4}\right]}\right|$$

V. CONCLUSIONS

The equations which are formulated are simple and easy to compute the ultimate load carrying capacity of slabs. Similarly the equations can also be generated for all edge conditions of slab. These equations can be applicable to all lengths of rectangular slabs.

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