Theoretical Model for a Special Type of Distance Based Clustering of Feature Points Based on Distance to Complement Feature Point or Orthogonal Feature Point of Each Feature Point {Version 3}

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Abstract- **In this research investigation, the author has detailed a scheme for a special type of distance based clustering of feature points of concern based on distance to complement feature point or orthogonal feature point of each feature point**

Index terms- **Clustering, Distance Based Clustering**

I.INTRODUCTION

There have been many propositions regarding Clustering Models, the major among them being [1], [2], [3], [4], [5]. Also, there have been a few propositions on Overlapping Clustering Models [6], [7].

II.PROPOSED THEORETICAL MODEL

Notion of the Complement of A Given Vector [8]

For any given Vector $A=\begin{bmatrix} x_1 & x_2 & x_3 & . & . & x_{n-1} & x_n \end{bmatrix}$ the Complement of this Vector is given by filling it with the Complement of each element w.r.t all other elements of the vector. That is, the complement of x_i, namely x_i^c is

Case 1: Only Complement

The Weighted Average of all other elements of this Vector except x_i

Case 2: Orthogonal Complement

The Weighted Average of all other elements of this Vector except x_i, with a Sign to be fixed as follows:

If $\sum_{i=1}^{j} x_i x_i^c$ is Positive, then $x_{(j+1)}^c$ is chosen such that $x_{(j+1)} x_{(j+1)}^c$ is Negative. And if, $\sum_{i=1}^{j} x_i x_i^c$ is Negative, then $x_{(j+1)}^c$ is chosen such that $x_{(j+1)} x_{(j+1)}^c$ is Positive. Also, $1 < j \le (n-1)$.

That is, $A^c = \begin{bmatrix} x_1^c & x_2^c & x_3^c & . & . & x_{n-1}^c & x_n^c \end{bmatrix}$.

The weight is given by

$$w_i = \left\{ \frac{x_i}{\sum_{i=1}^{n} x_i} \right\}$$

$$x_{i=p}^c = \frac{\sum_{\substack{i=1 \\ i \ne p}}^{n} w_i x_i}{\sum_{\substack{i=1 \\ i \ne p}}^{n} w_i}$$

Notion of the Complement of A Given n Dimensional Matrix [8]

The Complement of any element $A(p_1, p_2, p_3, ..., p_{n-1}, p_n)$ of an n Dimensional Matrix A with dimension sizes $l_1, l_2, l_3, ..., l_{n-1}, l_n$ is given as follows:

Case 1: Simple Complement

The required value is given by

$$\left\{\frac{\sum_{\substack{i_n=1\\i_n\neq p_n}}^{l_n}\sum_{\substack{i_{n-1}=1\\i_{n-1}\neq p_{n-1}}}^{l_{n-1}}\ldots\ldots\sum_{\substack{i_3=1\\i_3\neq p_3}}^{l_3}\sum_{\substack{i_2=1\\i_2\neq p_2}}^{l_2}\sum_{\substack{i_1=1\\i_1\neq p_1}}^{l_1}\left\{w_{i_1i_2i_3\ldots i_{n-1}i_n}a_{i_1i_2i_3\ldots i_{n-1}i_n}\right\}}{\sum_{\substack{i_n=1\\i_n\neq p_n}}^{l_n}\sum_{\substack{i_{n-1}=1\\i_{n-1}\neq p_{n-1}}}^{l_{n-1}}\ldots\ldots\sum_{\substack{i_3=1\\i_3\neq p_3}}^{l_3}\sum_{\substack{i_2=1\\i_2\neq p_2}}^{l_2}\sum_{\substack{i_1=1\\i_1\neq p_1}}^{l_1}\left\{w_{i_1i_2i_3\ldots i_{n-1}i_n}\right\}}\right\}$$

where the weight term is given by

$$w_{i_1i_2i_3\ldots i_{n-1}i_n}=\left\{\frac{a_{i_1i_2i_3\ldots i_{n-1}i_n}}{\sum_{i_n=1}^{l_n}\sum_{i_{n-1}=1}^{l_{n-1}}\ldots\ldots\sum_{i_3=1}^{l_3}\sum_{i_2=1}^{l_2}\sum_{i_1=1}^{l_1}\left\{a_{i_1i_2i_3\ldots i_{n-1}i_n}\right\}}\right\}$$

Case 2: Orthogonal Complement

The required value is given by

$$\left\{\frac{\sum_{\substack{i_n=1\\i_n\neq p_n}}^{l_n}\sum_{\substack{i_{n-1}=1\\i_{n-1}\neq p_{n-1}}}^{l_{n-1}}\ldots\ldots\sum_{\substack{i_3=1\\i_3\neq p_3}}^{l_3}\sum_{\substack{i_2=1\\i_2\neq p_2}}^{l_2}\sum_{\substack{i_1=1\\i_1\neq p_1}}^{l_1}\left\{w_{i_1i_2i_3\ldots i_{n-1}i_n}a_{i_1i_2i_3\ldots i_{n-1}i_n}\right\}}{\sum_{\substack{i_n=1\\i_n\neq p_n}}^{l_n}\sum_{\substack{i_{n-1}=1\\i_{n-1}\neq p_{n-1}}}^{l_{n-1}}\ldots\ldots\sum_{\substack{i_3=1\\i_3\neq p_3}}^{l_3}\sum_{\substack{i_2=1\\i_2\neq p_2}}^{l_2}\sum_{\substack{i_1=1\\i_1\neq p_1}}^{l_1}\left\{w_{i_1i_2i_3\ldots i_{n-1}i_n}\right\}}\right\}$$

where the weight term is given by

$$w_{i_1i_2i_3\ldots i_{n-1}i_n}=\left\{\frac{a_{i_1i_2i_3\ldots i_{n-1}i_n}}{\sum_{i_n=1}^{l_n}\sum_{i_{n-1}=1}^{l_{n-1}}\ldots\ldots\sum_{i_3=1}^{l_3}\sum_{i_2=1}^{l_2}\sum_{i_1=1}^{l_1}\left\{a_{i_1i_2i_3\ldots i_{n-1}i_n}\right\}}\right\}$$

And the sign of the term

$$\left\{\frac{\sum_{\substack{i_n=1\\i_n\neq p_n}}^{l_n}\sum_{\substack{i_{n-1}=1\\i_{n-1}\neq p_{n-1}}}^{l_{n-1}}\ldots\ldots\sum_{\substack{i_3=1\\i_3\neq p_3}}^{l_3}\sum_{\substack{i_2=1\\i_2\neq p_2}}^{l_2}\sum_{\substack{i_1=1\\i_1\neq p_1}}^{l_1}\left\{w_{i_1i_2i_3\ldots i_{n-1}i_n}a_{i_1i_2i_3\ldots i_{n-1}i_n}\right\}}{\sum_{\substack{i_n=1\\i_n\neq p_n}}^{l_n}\sum_{\substack{i_{n-1}=1\\i_{n-1}\neq p_{n-1}}}^{l_{n-1}}\ldots\ldots\sum_{\substack{i_3=1\\i_3\neq p_3}}^{l_3}\sum_{\substack{i_2=1\\i_2\neq p_2}}^{l_2}\sum_{\substack{i_1=1\\i_1\neq p_1}}^{l_1}\left\{w_{i_1i_2i_3\ldots i_{n-1}i_n}\right\}}\right\}$$ is given as follows:

If $\sum_{i_n=1}^{j_n}\sum_{i_{n-1}=1}^{j_{n-1}}\ldots\ldots\sum_{i_3=1}^{j_3}\sum_{i_2=1}^{j_2}\sum_{i_1=1}^{j_1}\left\{a_{i_1i_2i_3\ldots i_{n-1}i_n}b_{i_1i_2i_3\ldots i_{n-1}i_n}\right\}$ is Positive, then the sign of $b_{(j_1+1)(j_2+1)(j_3+1)\ldots\ldots(j_{n-1}+1)(j_n+1)}$ is chosen such that $a_{(j_1+1)(j_2+1)(j_3+1)\ldots\ldots(j_{n-1}+1)(j_n+1)}b_{(j_1+1)(j_2+1)(j_3+1)\ldots\ldots(j_{n-1}+1)(j_n+1)}$ is Negative.

And if $\sum_{i_n=1}^{j_n}\sum_{i_{n-1}=1}^{j_{n-1}}\ldots\ldots\sum_{i_3=1}^{j_3}\sum_{i_2=1}^{j_2}\sum_{i_1=1}^{j_1}\left\{a_{i_1i_2i_3\ldots i_{n-1}i_n}b_{i_1i_2i_3\ldots i_{n-1}i_n}\right\}$ is Negative, then the sign of $b_{(j_1+1)(j_2+1)(j_3+1)\ldots\ldots(j_{n-1}+1)(j_n+1)}$ is chosen such that $a_{(j_1+1)(j_2+1)(j_3+1)\ldots\ldots(j_{n-1}+1)(j_n+1)}b_{(j_1+1)(j_2+1)(j_3+1)\ldots\ldots(j_{n-1}+1)(j_n+1)}$ is Positive. It should be noted that here, $b_{i_1i_2i_3\ldots i_{n-1}i_n}$ represents the Complement of the Matrix Element of A, namely $a_{i_1i_2i_3\ldots i_{n-1}i_n}$ in the Complement Matrix B which is the complement of Matrix A.

Special Type of Distance Based Clustering Using Distance to Complement of the Feature Point –Type I

Let there be *m* number of feature points each of *n* dimensions. Let them be represented by $\overline{x}_p$, where $p=1\ to\ m$. Also, let the elements of the feature points be represented by x_{pq}, where $p=1\ to\ m$ and $q=1\ to\ n$. We now find the weighted average of all these feature points which is just the feature point gotten by taking the weighted averages element-wise as follows

$${}^{r}x_q=\frac{\sum_{p=1}^{m}w_{pq}x_{pq}}{\sum_{p=1}^{m}w_{pq}}$$

With

$$w_{pq}=\frac{x_{pq}}{\sum_{p=1}^{m}x_{pq}}$$

This weighted average point is represented by ${}^{r}\overline{x}$ indicating that it is the most representative point for all the given feature points. Its elements are represented by ${}^{r}x_q$ where $q=1\ to\ n$.

We now find the distances between this most representative point ${}^{r}\overline{x}$ and each of all other feature points. Let these be represented by $d\left(\overline{x}_p,{}^{r}\overline{x}\right)$ for $p=1\ to\ m$. We now arrange these distances in increasing order. Let this order be a function f_1 given by a map from the Set $\{p\}_{p=1\ to\ m}$ to the same

Set $\{p\}_{p=1\ to\ m}$ but with the possibility that the map need not be necessarily congruent but such that the increasing order of distances aspect is satisfactorily met. Let these distances be denoted by $d_1\left(\bar{x}_{p=f_1^{-1}(1)},{}^r\bar{x}\right), d_2\left(\bar{x}_{p=f_1^{-1}(2)},{}^r\bar{x}\right), d_3\left(\bar{x}_{p=f_1^{-1}(3)},{}^r\bar{x}\right), \ldots\ldots, d_m\left(\bar{x}_{p=f_1^{-1}(m)},{}^r\bar{x}\right)$

Now, if we need K number of clusters, we find the first K Number of points that are closest to the most representative point ${}^r\bar{x}$. That is, we consider the points $\bar{x}_{p=f_1^{-1}(1)}$, $\bar{x}_{p=f_1^{-1}(2)}$, $\bar{x}_{p=f_1^{-1}(3)}, \ldots\ldots, \bar{x}_{p=f_1^{-1}(K-1)}, \bar{x}_{p=f_1^{-1}(K)}$. Now, we consider each of these K points and find the distances to their respective *Complement* points.

Here, we can take a *Complement*

1. Along as the Vector as detailed in the section on Notion of the Complement of a given Vector and
2. Along the given all feature points element wise.

That is

$$x^c_{(j=p)q} = \frac{\sum_{\substack{j=1\\ j\neq p}}^{m} w_{jq} x_{jq}}{\sum_{\substack{j=1\\ j\neq p}}^{m} w_{jq}}$$

With

$$w_{jq} = \frac{x_{jq}}{\sum_{\substack{j=1\\ j\neq p}}^{m} x_{pq}}$$

Let these be represented by $g_1 = d_1\left(\bar{x}_{p=f_1^{-1}(1)}, \bar{x}^c_{p=f_1^{-1}(1)}\right), g_2 = d_2\left(\bar{x}_{p=f_1^{-1}(2)}, \bar{x}^c_{p=f_1^{-1}(2)}\right), g_3 = \left(\bar{x}_{p=f_1^{-1}(3)}, \bar{x}^c_{p=f_1^{-1}(3)}\right), \ldots$ $\ldots, g_{K-1} = d_{K-1}\left(\bar{x}_{p=f_1^{-1}(K-1)}, \bar{x}^c_{p=f_1^{-1}(K-1)}\right), g_K = d_K\left(\bar{x}_{p=f_1^{-1}(K)}, \bar{x}^c_{p=f_1^{-1}(K)}\right)$

We now arrange these distances in increasing order. Let this order be a function f_2 given by a map from the Set $\{g_h\}_{h=1\ to\ K}$ to the same Set $\{g_h\}_{h=1\ to\ K}$ but with the possibility that the map need not be necessarily congruent but such that the increasing order of distances aspect is satisfactorily met.

Let these distances be represented by $g_{h=f_2^{-1}(1)}, g_{h=f_2^{-1}(2)}, g_{h=f_2^{-1}(3)}, \ldots\ldots\ldots, g_{h=f_2^{-1}(K-1)}, g_{h=f_2^{-1}(K)}$

We now consider the distance $g_{h=f_2^{-1}(1)}$ and the point corresponding to it, namely $\bar{x}_{p=f_1^{-1}\left(h=f_2^{-1}(1)\right)}$ and find all points that bear distance less than or equal to the distance $g_{h=f_2^{-1}(1)}$. Now these points along with the point $\bar{x}_{p=f_1^{-1}\left(h=f_2^{-1}(1)\right)}$ comprise the First Cluster.

We now consider the distance $g_{h=f_2^{-1}(2)}$ and the point corresponding to it, namely $\bar{x}_{p=f_1^{-1}\left(h=f_2^{-1}(2)\right)}$ and find all points that bear distance greater than or equal to the distance $g_{h=f_2^{-1}(1)}$ and less than or equal to the distance $g_{h=f_2^{-1}(2)}$. Now these points along with the point $\bar{x}_{p=f_1^{-1}\left(h=f_2^{-1}(2)\right)}$ comprise the Second Cluster.

We now consider the distance $g_{h=f_2^{-1}(3)}$ and the point corresponding to it, namely $\bar{x}_{p=f_1^{-1}\left(h=f_2^{-1}(3)\right)}$ and find all points that bear distance greater than or equal to the distance $g_{h=f_2^{-1}(2)}$ and less than or equal to the distance $g_{h=f_2^{-1}(3)}$. Now these points along with the point $\bar{x}_{p=f_1^{-1}\left(h=f_2^{-1}(3)\right)}$ comprise the Third Cluster. In this fashion, we find all K number of Clusters. It should be noted that these Clusters may be Overlapping in nature.

In this fashion, we can even find *m* number of Clusters for the given *m* number of feature points.

Note: It can be noted that the aforesaid type of clustering can be implemented separately for both types of complements, namely,

A *Complement*

1. Along as the Vector as detailed in the section on Notion of the Complement of a given Vector and
2. Along the given all feature points element wise.

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This weighted average point is represented by ${}^{r}\overline{x}$ indicating that it is the most representative point for all the given feature points. Its elements are represented by ${}^{r}x_q$ where $q = 1\ to\ n$.

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Now, if we need K number of clusters, we find the first K Number of points that are closest to the most representative point ${}^{r}\overline{x}$. That is, we consider the points $\overline{x}_{p=f_1^{-1}(1)}, \overline{x}_{p=f_1^{-1}(2)}, \overline{x}_{p=f_1^{-1}(3)}, \ldots\ldots, \overline{x}_{p=f_1^{-1}(K-1)}, \overline{x}_{p=f_1^{-1}(K)}$

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$\ldots, g_{K-1} = d_{K-1}\left(\overline{x}_{p=f_1^{-1}(K-1)}, \overline{x}^{c}_{p=f_1^{-1}(K-1)}\right), g_K = d_K\left(\overline{x}_{p=f_1^{-1}(K)}, \overline{x}^{c}_{p=f_1^{-1}(K)}\right)$

We now arrange these distances in increasing order. Let this order be a function f_2 given by a map from the Set $\{g_h\}_{h=1\ to\ K}$ to the same Set $\{g_h\}_{h=1\ to\ K}$ but with the possibility that the map need not be necessarily congruent but such that the increasing order of distances aspect is satisfactorily met.

Let these distances be represented by $g_{h=f_2^{-1}(1)}, g_{h=f_2^{-1}(2)}, g_{h=f_2^{-1}(3)}, \ldots\ldots, g_{h=f_2^{-1}(K-1)}, g_{h=f_2^{-1}(K)}$

We now consider the distance $g_{h=f_2^{-1}(1)}$ and the point corresponding to it, namely $\overline{x}_{p=f_1^{-1}\left(h=f_2^{-1}(1)\right)}$ and find all points that bear distance less than or equal to the distance $g_{h=f_2^{-1}(1)}$. Now these points along with the point $\overline{x}_{p=f_1^{-1}\left(h=f_2^{-1}(1)\right)}$ comprise the First Cluster.

We now consider the distance $g_{h=f_2^{-1}(2)}$ and the point corresponding to it, namely $\overline{x}_{p=f_1^{-1}\left(h=f_2^{-1}(2)\right)}$ and find all points that bear distance less than or equal to the distance $g_{h=f_2^{-1}(2)}$. Now these points along with the point $\overline{x}_{p=f_1^{-1}\left(h=f_2^{-1}(2)\right)}$ comprise the Second Cluster.

We now consider the distance $g_{h=f_2^{-1}(3)}$ and the point corresponding to it, namely $\overline{x}_{p=f_1^{-1}\left(h=f_2^{-1}(3)\right)}$ and find all points that bear distance less than or equal to the distance $g_{h=f_2^{-1}(3)}$. Now these points along with the point $\bar{x}_{p=f_1^{-1}\left(h=f_2^{-1}(3)\right)}$ comprise the Third Cluster. In this fashion, we find all K number of Clusters. It should be noted that these Clusters may be Overlapping in nature.

In this fashion, we can even find *m* number of Clusters for the given *m* number of feature points.

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$$^{r}x_q = \frac{\sum_{p=1}^{m} w_{pq}x_{pq}}{\sum_{p=1}^{m} w_{pq}}$$

With

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This weighted average point is represented by $^{r}\bar{x}$ indicating that it is the most representative point for all the given feature points. Its elements are represented by $^{r}x_q$ where $q=1 \text{ to } n$.

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Now, if we need K number of clusters, we find the first K Number of points that are closest to the most representative point $^{r}\bar{x}$. That is, we consider the point $\bar{x}_{p=f_1^{-1}(1)}, \bar{x}_{p=f_1^{-1}(2)}, \bar{x}_{p=f_1^{-1}(3)}, \ldots\ldots, \bar{x}_{p=f_1^{-1}(K-1)}, \bar{x}_{p=f_1^{-1}(K)}$.

Now, we consider each of these K points and find the distances to their respective *Orthogonal Complement* points. Let these be represented by $g_1 = d_1\left(\bar{x}_{p=f_1^{-1}(1)}, \bar{x}^{oc}_{p=f_1^{-1}(1)}\right), g_2 = d_2\left(\bar{x}_{p=f_1^{-1}(2)}, \bar{x}^{oc}_{p=f_1^{-1}(2)}\right), g_3 = \left(\bar{x}_{p=f_1^{-1}(3)}, \bar{x}^{oc}_{p=f_1^{-1}(3)}\right), \ldots \ldots, g_{K-1} = d_{K-1}\left(\bar{x}_{p=f_1^{-1}(K-1)}, \bar{x}^{oc}_{p=f_1^{-1}(K-1)}\right), g_K = d_K\left(\bar{x}_{p=f_1^{-1}(K)}, \bar{x}^{oc}_{p=f_1^{-1}(K)}\right)$

We now arrange these distances in increasing order. Let this order be a function f_2 given by a map from the Set $\{g_h\}_{h=1 \text{ to } K}$ to the same Set $\{g_h\}_{h=1 \text{ to } K}$ but with the possibility that the map need not be necessarily congruent but such that the increasing order of distances aspect is satisfactorily met.

Let these be represented by $g_{h=f_2^{-1}(1)}, g_{h=f_2^{-1}(2)}, g_{h=f_2^{-1}(3)}, \ldots\ldots\ldots, g_{h=f_2^{-1}(K-1)}, g_{h=f_2^{-1}(K)}$

We now consider the distance $g_{h=f_2^{-1}(1)}$ and the point corresponding to it, namely $\bar{x}_{p=f_1^{-1}\left(h=f_2^{-1}(1)\right)}$ and find all points that bear distance less than or equal to the distance $g_{h=f_2^{-1}(1)}$. Now these points along with the point $\bar{x}_{p=f_1^{-1}\left(h=f_2^{-1}(1)\right)}$ comprise the First Cluster.

We now consider the distance $g_{h=f_2^{-1}(2)}$ and the point corresponding to it, namely $\bar{x}_{p=f_1^{-1}\left(h=f_2^{-1}(2)\right)}$ and find all points that bear distance greater than or equal to the distance $g_{h=f_2^{-1}(1)}$ and less than or equal to the distance $g_{h=f_2^{-1}(2)}$. Now these points along with the point $\bar{x}_{p=f_1^{-1}\left(h=f_2^{-1}(2)\right)}$ comprise the Second Cluster.

We now consider the distance $g_{h=f_2^{-1}(3)}$ and the point corresponding to it, namely $\bar{x}_{p=f_1^{-1}\left(h=f_2^{-1}(3)\right)}$ and find all points that bear distance greater than or equal to the distance $g_{h=f_2^{-1}(2)}$ and less than or equal to the distance $g_{h=f_2^{-1}(3)}$. Now these points along with the point $\bar{x}_{p=f_1^{-1}\left(h=f_2^{-1}(3)\right)}$ comprise the Third Cluster. In this fashion, we find all K number of Clusters. It should be noted that these Clusters may be Overlapping in nature.

In this fashion, we can even find *m* number of Clusters for the given *m* number of feature points.

Special Type of Distance Based Clustering Using Distance to Orthogonal Complement of the Feature Point –Type II

Let there be *m* number of feature points each of *n* dimensions. Let them be represented by $\bar{x}_p$, where $p=1 \text{ to } m$. Also, let the elements of the feature points be represented by x_{pq}, where $p=1 \text{ to } m$ and $q=1 \text{ to } n$. We now find the weighted average of all these feature points which is just the feature point gotten by taking the weighted averages element-wise as follows

$$^{r}x_q = \frac{\sum_{p=1}^{m} w_{pq} x_{pq}}{\sum_{p=1}^{m} w_{pq}}$$

With

$$w_{pq} = \frac{x_{pq}}{\sum_{p=1}^{m} x_{pq}}$$

This weighted average point is represented by $^{r}\bar{x}$ indicating that it is the most representative point for all the given feature points. Its elements are represented by $^{r}x_q$ where $q=1 \text{ to } n$.

We now find the distances between this most representative point $^{r}\bar{x}$ and each of all other feature points. Let these be represented by $d\left(\bar{x}_p, {}^{r}\bar{x}\right)$ for $p=1 \text{ to } m$. We now arrange these distances in increasing order. Let this order be a function f_1 given by a map from the Set $\{p\}_{p=1 \text{ to } m}$ to the same Set $\{p\}_{p=1 \text{ to } m}$ but with the possibility that the map need not be necessarily congruent but such that the increasing order of distances aspect is satisfactorily met. Let these distances be denoted by

$$d_1\left(\bar{x}_{p=f_1^{-1}(1)}, {}^{r}\bar{x}\right), d_2\left(\bar{x}_{p=f_1^{-1}(2)}, {}^{r}\bar{x}\right), d_3\left(\bar{x}_{p=f_1^{-1}(3)}, {}^{r}\bar{x}\right), \ldots\ldots, d_m\left(\bar{x}_{p=f_1^{-1}(m)}, {}^{r}\bar{x}\right)$$

Now, if we need K number of clusters, we find the first K Number of points that are closest to the most representative point $^{r}\bar{x}$ That is, we consider the points $\bar{x}_{p=f_1^{-1}(1)}, \bar{x}_{p=f_1^{-1}(2)}, \bar{x}_{p=f_1^{-1}(3)}, \ldots\ldots, \bar{x}_{p=f_1^{-1}(K-1)}, \bar{x}_{p=f_1^{-1}(K)}$.

Now, we consider each of these K points and find the distances to their respective *Orthogonal Complement* points. Let these be represented by

$$g_1 = d_1\left(\bar{x}_{p=f_1^{-1}(1)}, \bar{x}^{oc}_{p=f_1^{-1}(1)}\right), g_2 = d_2\left(\bar{x}_{p=f_1^{-1}(2)}, \bar{x}^{oc}_{p=f_1^{-1}(2)}\right), g_3 = \left(\bar{x}_{p=f_1^{-1}(3)}, \bar{x}^{oc}_{p=f_1^{-1}(3)}\right), \ldots$$
$$\ldots, g_{K-1} = d_{K-1}\left(\bar{x}_{p=f_1^{-1}(K-1)}, \bar{x}^{oc}_{p=f_1^{-1}(K-1)}\right), g_K = d_K\left(\bar{x}_{p=f_1^{-1}(K)}, \bar{x}^{oc}_{p=f_1^{-1}(K)}\right)$$

We now arrange these distances in increasing order. Let this order be a function f_2 given by a map from the Set $\{g_h\}_{h=1 \text{ to } K}$ to the same Set $\{g_h\}_{h=1 \text{ to } K}$ but with the possibility that the map need not be necessarily congruent but such that the increasing order of distances aspect is satisfactorily met.

Let these be represented by

$$g_{h=f_2^{-1}(1)}, g_{h=f_2^{-1}(2)}, g_{h=f_2^{-1}(3)}, \ldots\ldots, g_{h=f_2^{-1}(K-1)}, g_{h=f_2^{-1}(K)}$$

We now consider the distance $g_{h=f_2^{-1}(1)}$ and the point corresponding to it, namely $\bar{x}_{p=f_1^{-1}\left(h=f_2^{-1}(1)\right)}$ and find all points that bear distance less than or equal to the distance $g_{h=f_2^{-1}(1)}$. Now these points along with the point $\bar{x}_{p=f_1^{-1}\left(h=f_2^{-1}(1)\right)}$ comprise the First Cluster.

We now consider the distance $g_{h=f_2^{-1}(2)}$ and the point corresponding to it, namely $\bar{x}_{p=f_1^{-1}\left(h=f_2^{-1}(2)\right)}$ and find all points that bear distance less than or equal to the distance $g_{h=f_2^{-1}(2)}$. Now these points along with the point $\bar{x}_{p=f_1^{-1}\left(h=f_2^{-1}(2)\right)}$ comprise the Second Cluster.

We now consider the distance $g_{h=f_2^{-1}(3)}$ and the point corresponding to it, namely $\bar{x}_{p=f_1^{-1}\left(h=f_2^{-1}(3)\right)}$ and find all points that bear distance less than or equal to the distance $g_{h=f_2^{-1}(3)}$. Now these points along with the point $\bar{x}_{p=f_1^{-1}\left(h=f_2^{-1}(3)\right)}$ comprise the Third Cluster. In this fashion, we find all K number of Clusters. It should be noted that these Clusters may be Overlapping in nature.

In this fashion, we can even find m number of Clusters for the given m number of feature points.

Cluster Validation

For the aforesaid type of Clusters, it is recommended that Silhouette Score [9], [10] be computed for Clusters Validation.

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