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The Brilliance of the Pauli Exclusion Principle

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***Abstract*— The exceptional brilliance of one of the fundamental principles of quantum mechanics, the Pauli Exclusion Principle is discussed here. While there is no doubt about the importance of the exclusion principle in nature and in the stability of matter, the Pauli principle, however, does not often get the appreciation it should have got because of its sheer magnificence in the field of quantum mechanics. This study tries to change that by discussing four key characteristics related to the principle; proposing the principle even before the development of quantum mechanics, introducing a new quantum number, now called 'spin', its importance in the latter developed quantum mechanics, and its implications to the Schrödinger's wave equation by prohibition of arbitrary permutation symmetry, and at the same time presenting Pauli's genius and farsightedness in proposing it.**

***Index Terms*— Pauli Exclusion Principle, quantum mechanics, spin quantum number, Schrödinger wave equation, permutation symmetry**

INTRODUCTION

1. The Pauli Exclusion Principle and how it all started

Pauli's contribution to the field of Quantum Mechanics and to chemistry and physics, in general, is of immense significance. Pauli's exclusion principle was a fundamental principle of Quantum Mechanics, that laid the groundwork to help us understand different properties of matter and bring order to the then seemingly mysterious periodic table. Austrian physicist Wolfgang Pauli was born in 1900, the same year when Planck first introduced the energy quanta. Within just 24 years Pauli went on to proposed the exclusion principle that became one of the most important theories in quantum mechanics. In 1913 Niels Bohr had proposed that electron in an atom can only occupy certain quantized orbitals. It however could not say anything as to why the electron didn't simply crowd the lowest possible energy level.
Pauli had worked on trying to explain the anomalous Zeeman effect by that time. He was convinced that the two problems were somehow related. He went on to establish a connection between the two seemingly separate topics and introduced a new quantum mechanical property, now known as 'spin'. In his paper submitted for publication in January 1925, Pauli formulated his now famous principle as follows [1]:
"In an atom there cannot be two or more equivalent electrons, for which in strong fields the values of all four quantum numbers coincide. If an electron exists in an atom for which all of these numbers have definite values, then this state is 'occupied'.''
In this paper, we will discuss the brilliance of Pauli while proposinghis, apparently 'incomplete', Pauli Exclusion Principle and his farsightedness while doing so. We will do so by exploring four characteristics related to the principle—forming the principle before the development quantum mechanics, introducing a new quantum property now known as the 'spin' and proposing that spin has no classical significance, Heisenberg and Dirac reverifying his claims while applying the Schrödinger's wave function to multi-electron systems, and going one step ahead of Schrödinger by implying the total wave function can have only two permutation symmetry out of all the possible types.

2. Asserting the importance of the Exclusion Principle

Pauli's Principle is a fundamental theory in quantum mechanics, its importance and implications can be seen in a wide range of subjects. It explains the properties of atoms and their classification in the modern periodic table, gives insight into features of complex molecules, and with all said and done it is, responsible for the stability of matter [2].
The stability of electrons in a quantum state is described by the quantum mechanical model of an atom. The stability of large complex molecules with multiple electrons and nucleons is explained with the

help of Pauli's exclusion principle. The principle was able to give a reason as to why the electrons didn't simply occupy the lowest possible energy state. It is the reason why bulk matter exists, is stable, and occupies volume.

Dyson and Lenard considered the balance between attractive forces (electron-nucleus) and repulsive forces (electron-electron, nucleus-nucleus) that exist in matter and showed that ordinary matter would collapse without the presence of the exclusion principle [3,4]. And furthermore, Pauli's principle was able to provide a theoretical basis for the first time for the structure of the periodic table of elements. It is therefore one of the most vital and important theories of nature.

We will now consider the four features related to the exclusion principle to showcase the magnificence of this principle and the far-sightedness of Pauli while proposing it.

3.1 Forming the Exclusion Principle before the development of Quantum Mechanics

Pauli arrived at the formation of his principle while trying to explain the regularities in the anomalous Zeeman effect in the presence of magnetic fields. A normal Zeeman effect is observed when a spectral line of an atom splits into three lines under a weak magnetic field. An anomalous Zeeman effect is observed if the spectral line splits into more than three lines. He was trying to examine critically the simplest case, the doublet structure of alkali spectra.

In December 1924 Pauli published a paper trying to explain the Zeeman effect [5].

In the paper, he showed that Bohr's theory of doublet structure that was based on the non-vanishing angular momentum of electrons in closed stationary states was wrong. It was so because a closed shell has no angular or magnetic moments. Pauli concluded that to explain the analogous Zeeman effect a new quantum mechanical property has to be introduced. In his paper he wrote with great intuition for that time:

"According to this point of view, the doublet structure of alkali spectra … is due to a particular two-valuednessof the quantum theoretic properties of the electron, which cannot be described from the classical point of view."

This non-classical property is now called 'spin'. It was some remarkable intuition from Pauli to predict the existence of spin even before the development of modern quantum mechanics. He began with looking to explain something and while doing so he ended up giving a theory of something entirely different and the theory turned out to be one of the most important theories in the development of quantum mechanics.

3.2 Introducing spin as a new quantum mechanical property and proposing that spin has no classical significance

In his paper Pauli described four quantum numbers – n, l, j, and m_j – n and l were commonly known by that time as principal quantum number and angular momentum quantum number. Pauli introduced $j = l \pm ½$, – the total angular momentum quantum number, and its projection m_j. For this new quantum number, Pauli didn't give any physical sig and was sure that quantum number 'j' couldn't be described physically.

As noted by Pauli in his Nobel Prize lecture [6], physicist found it hard to fully grasp the concept of spin, since no meaning was given to the fourth degree of freedom of an electron in terms of a model. Studies [7] have shown that the concept of spin is a consequence of relativistic quantum mechanics. However, researchers like Muradlijar [8] have tried to assert that electron spin has a classical origin, basing their claims on the so-called Stochastic Electrodynamics.

The point is that the authors of Stochastic Electrodynamics, Marshall and Boyer, have in a series of papers [9-14], inserted in classical electrodynamics the Zero-Point Field (ZPF) or the Zero-Point Radiation (ZPR) based on Planck's constant ħ and connected it with *Zitterbewegung* (rapid oscillatory motion of the massless charge with the velocity *c* around a center of mass). The creators of stochastic electrodynamics then have stressed that ZPF has a classical nature.

In his recent paper in 2018, Boyer [14] tried to show that ħ can be inserted in quantum mechanics and be used as a scaling factor. He said that as $ħ \to 0$ it losses its quantum mechanical properties, but classical physics remains the same. But, as was pointed out by Kaplan [15] in his book, this view of Boyer was not correct. It is not true that if some quantum mechanical properties can be used in classical physics it loses its quantum mechanical properties. Rather, on the contrary, the ability of ZPR

to be used in quantum mechanics indicates that it has some quantum property. The zero-point radiation is a quantum phenomenon, its energy equal to ½ $\hbar\omega_o$. In the classical limit when $\hbar \rightarrow 0$, it does not exist.
This shows that Pauli was indeed completely right when he said that electron spin is a quantum property and that it can't be explained classically.

3.3 Heisenberg and Dirac reverifying Pauli's claims while applying the Schrödinger's wave function to multi-electron systems
After 2 years of the announcement of Pauli's exclusion principle quantum mechanics was formed, with Heisenberg's formulation of matrix mechanics [16], and Schrödinger's wave mechanics [17,18], which was based on de Broglie's [19] idea that matter can have wavelike properties. When the new-born quantum mechanics was first formed, its first applications to multi-electron systems were carried out independently by Heisenberg [20] and Dirac [21]. In both of these studies, when the anti-symmetric wave function was constructed and solved it was found that multiple electrons cannot occupy the same quantum mechanical state. This shows that the Pauli Exclusion Principle (which limits multiple electrons from occupying the same state) was a consequence of the antisymmetric nature of Schrödinger's wave equation.
Dirac represented a multi-electron anti-symmetric wave function as a determinant with a one-electron wave function say, ψn_i

$$\Psi n_1 n_2 ... n_r(1,2,\ldots,r) = \begin{vmatrix} \Psi n_1(1) & \Psi n_1(2) & \ldots & \Psi n_1(r) \\ \Psi n_2(1) & \Psi n_2(2) & \ldots & \Psi n_2(r) \\ \ldots & \ldots & \ldots & \ldots \\ \Psi n_r(1) & \Psi n_r(2) & \ldots & \Psi n_r(r) \end{vmatrix}$$

Dirac [21] wrote in his paper after writing the wave function in determinant form:
"An antisymmetric eigenfunction vanishes identically when two of the electrons are in the same orbit. This means that in the solution of the problem with antisymmetric eigenfunctions there can be no stationary states with two or more electrons in the same orbit, which is just Pauli's exclusion principle."
Thus, with the creation of quantum mechanics, exclusion of electron to simultaneously occupy an orbital was necessary, because only that allowed the anti-symmetric nature of the wave function to remain intact. Again, proving the far-sightedness of Pauli while creating his principle, and showing how the exclusion principle was so ahead of its time.

3.4 Introducing specificity in Schrödinger's equation by implying the total wave function can have only two permutation symmetry out of all the possible types
Another brilliance of Pauli's principle is that it goes one step ahead of Schrödinger's wave equation to limit the permutation symmetry of a wave function to only symmetric and anti-symmetric type. This implication of the principle is truly remarkable when it is taken into account that the Schrödinger's wave equation is allowed to have any number of permutation symmetry.
Kaplan [15] in his book (2017) has shown that even though, many well-known textbooks in quantum mechanics say that the idea that only symmetric and anti-symmetric permutation symmetry to be allowed is wrong. This scenario, however, may not be realized. Kaplan points out that if arbitrary permutation symmetry is allowed, it will lead to a contradiction of the concepts of particle identity and independence. And therefore, only degenerate permutation states, namely symmetric and anti-symmetric states, can be allowed (bosons have symmetric wave function and fermions have anti-symmetric wave function).
The Pauli Exclusion Principle (which was first introduced only for electrons but was latter extended to both bosons and fermions) therefore very aptly, introduces specificity in the Schrödinger wave equation.

CONCLUSION

In section 3.1, we established that Pauli was able to assert the incorrectness of Bohr's theory which proposed that closed shells have a non-vanishing angular moment. Moreover, his genius lies in proposing the necessity of a new quantum mechanical property to be correctly able to account for the anomalous Zeeman effect.
It was shown in section 3.2, that Pauli went ahead to support his argument for the necessity of a new quantum property by introducing the quantum number 'j' and going on to state that 'j' does not have and classical significance. His brilliance was again proved when theories that tried to establish a classical

connection of spin, was later found out to be incorrect and incoherent.

Section 3.3 saw Pauli's claims in his Exclusion principle being reverified about two years later, independently by Dirac and Heisenberg, when they tried to construct an anti-symmetric wave function using Schrödinger's equation. It was one of the first applications of the newly formed quantum mechanics, and they found out that the exclusion principle was a consequence of the anti-symmetric nature of the Schrödinger equation and that the wavefunction vanishes when two electrons are in the same state. This was possibly one of the most well-thought insights of Pauli, that he was able to predict the necessity of the principle in quantum mechanics—two years before it was even developed, showing how the principle was so ahead of its time.

The section 3.4 showcases another brilliance of Pauli's principle in which he was able to go a step ahead of the Schrödinger's equation in predicting the prohibition of arbitrary permutation symmetry. It was very correctly shown later that the permutation symmetry of the total wave function can only symmetric or anti-symmetric.

Pauli's principle was way ahead of its time, implications of which are still being found and verified. The tremendous brilliance of Pauli in the Pauli Exclusion Principle is however not always greatly understood and appreciated. Max Born once commented that "I knew he was a genius, comparable only to Einstein himself. But he was a completely different type of man, who in my eyes, did not attain Einstein's greatness."

In concluding remarks, we can all greatly appreciate Pauli's extraordinary brilliance while proposing a fundamental theory of Quantum Mechanics, the Pauli Exclusion Principle, while still being in his 20s.

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