Extension of Fuzzy M Group Concept to Q Fuzzy Set

M.U.Makandar
Assistant professor, KIT's IMER, Gokul Shirgaon, Kolhapur

***Abstract* - In this paper we introduce the notion of Q-fuzzy lattice ordered m-groups and investigated some of its basic properties. We also study the homomorphic image, pre-image of Q-fuzzy lattice ordered m-groups, arbitrary family of Q-fuzzy lattice ordered m-groups and Q-fuzzy lattice ordered m-groups using T-norms. We introduce the notion of sensible Q-fuzzy lattice ordered m-groups in groups and some related properties of lattices are discussed.**

***Index Terms* - Lattice ordered group, Q-Fuzzy lattice ordered m-group, Sensible fuzzy lattice, pre-image, direct product.**

1.INTRODUCTION

A fuzzy algebra has become an important branch of research. A. Rosenfeld 1971 [9] used the concept of fuzzy set theory due to Zadeh 1965 [5]. Since then, the study of fuzzy algebraic substructures is important when viewed from a Lattice theoretic point of view. N. Ajmal and K.V. Thomas [1] initiated such types of study in the year 1994. It was latter independently established by N. Ajmal [1] that the set of all fuzzy normal subgroups of a group constitute a sub lattice of the lattice of all fuzzy subgroups of a given group and is Modular. Nanda [8] proposed the notion of fuzzy lattice using the concept of fuzzy partial ordering. More recently in the notion of set product is discussed in detail and in the lattice theoretical aspects of fuzzy subgroups and fuzzy normal subgroups are explored. G.S.V. Satya Saibaba [3] initiate the study of L-fuzzy lattice ordered groups and introducing the notice of L-fuzzy sub l- groups. J.A. Goguen [4] replaced the valuation set [0,1] by means of a complete lattice in an attempt to make a generalized study of fuzzy set theory by studying L-fuzzy sets. A Solairaju and R. Nagarajan [11] introduced the concept of lattice valued Q-fuzzy sub-modules over near rings with respect to T-norms. DrM.Marudai & V. Rajendran [6] modified the definition of fuzzy lattice and introduce the notion of fuzzy lattice of groups and investigated some of its basic properties. Gu [12] introduced concept of fuzzy groups with operator. Then S. Subramanian, R Nagrajan & Chellappa [10] extended the concept to m fuzzy groups with operator. In this paper we introduce the notion of Q-fuzzy lattice ordered m-groups and investigated some of its basic properties. We study the homomorphic image, pre-image of Q-fuzzy lattice ordered m-groups, arbitrary family of fuzzy lattice ordered m-groups and fuzzy lattice ordered m normal groups. We introduce the notion of sensible Q- fuzzy lattice ordered m-groups in groups using T-norms and some related properties of lattices are discussed.

2.PRELIMINARIES

Definition 2.1: Let Q and G be any two sets. A mapping A: G x Q → [0,1] is called a Q–fuzzy set in G.

Definition 2.2: Let μ: X x Q → [0, 1] be a fuzzy set & G ϵ þ(X) = Set of all fuzzy sets on X. Q be any set. A fuzzy set μ on G x Q is called a Q fuzzy group if i) $\mu(x\,y, q) \geq \min\{\mu(x, q), \mu(y, q)\}$ ii) $\mu(x^{-1}, q) \geq \mu(x, q)$, for all x, yϵ G . & q ϵ Q.

Definition 2.3: Q-Normal fuzzy subgroup-Let μ: X x Q → [0, 1] be a fuzzy set & G ϵ þ(X). A fuzzy set μ on G x Q is called a Q normal fuzzy subgroup if $\mu(x^{-1} y\, x, q) \geq \mu(y, q)$ for all x, yϵ G & qϵ Q.

Definition 2.4: Lattice ordered group (l- group)- A lattice ordered group is a system (G,.,≤) if i) (G, ·) is a group ii) (G,≤) is a lattice . iii) x ≤ y implies a x b ≤ a y b (compatibility) For a, b, x, yϵ G

Definition 2.5: Q- Fuzzy lattice ordered group- Let μ: X x Q → [0, 1] be a fuzzy set & G is a lattice ordered set, G ϵ þ(X). A function μ on G x Q is said to be a Q-fuzzy lattice ordered group if i) $\mu(x\,y, q) \geq \min\{\mu(x, q), \mu(y, q)\}$ ii) $\mu(x^{-1}, q) \geq \mu(x, q)$ for all x, yϵ G ,qϵ Q

Definition 2.5: M Group-Let G be a group, M be any set if i) m x ϵ G. ii) m (x y) = (m x) y = x m y for all x, yϵ G, m ϵ M. Then G is called a m group.

Definition 2.6: Q-Fuzzy m group- Let μ: X x Q→ [0, 1] be a fuzzy set & G be a M group.
A fuzzy set on G x Q, G ϵ þ(X) is called a Q-fuzzy m group if i) μ (m (x y), q) ≥ min.
{μ (m x, q), μ (my, q)} ii) μ (mx^{-1}, q) ≥ μ (m x, q) for all x, yϵ G, m ϵ M & q ϵ Q.

Definition 2.7: A t-norm T, we mean a function T: [0, 1] x [0, 1] [0, 1] satisfying the following conditions:
(T1) T (0, x) = 0
(T2) T (x, y) = T (x, z) if y= z
(T3) T (x, y) = T (y, x)
(T4) T (x, T (y, z)) = T (T (x, y), z), for all x, y, z ϵ [0, 1].

Definition 2.8: Let T be a t-norm. A fuzzy set A is said to be sensible under T if Im (A) CΔ T.
Where Δ T= {α ϵ [0, 1] / T (α, α) = α}

Definition 2.9: For any Q fuzzy m group G and tϵ [0, 1], We define the set
U (μ: t) = {(x, q) ϵ G x Q | μ(m x ,q) ≥ t} which is called an upper cut off μ and can be used to the characterization of μ .

Definition 2.10: Q -fuzzy lattice ordered m-group- μ: X x Q → [0, 1], G ϵ þ(X),
M C X. A function μ on G x Q is said to be a Q- fuzzy lattice ordered m-group if

1. (G, ·) is a M-group.
2. (G,·, ≤) is a lattice ordered group.
3. μ (m (x y), q) ≥ min {μ (m x, q), μ (m y, q)}
4. μ ($(m x)^{-1}$, q) ≥ μ (m x, q)
5. μ (m x v my, q) ≥ min {μ (m x, q), μ (m y, q)}
6. μ (m x ʌ my, q) ≥ min {μ (m x, q), μ (my, q)}
   For all x, y ϵ G & q ϵ Q

Definition 2.11: Let ɵ: X x Q→ Y x Q be a map. A and B are Q fuzzy lattice ordered m groups in X and Y respectively. Then the inverse image of B under ɵ is a fuzzy set defined by
$ɵ^{-1}(B) = \mu_{ɵ^{-1}(B)}$ (x, q) = μ_B ɵ (x, q)

Definition 2.12: Let μ_A be a fuzzy set of G x Q. Let ɵ: G x Q → G' x Q be a map.
Define the map $\mu_{A^ɵ}$: G x Q→ [0,1] by $\mu_{A^ɵ}$ (x, q) = μ_A ɵ (x, q)

Definition 2.13: Let f: G x Q→ G' x Q be a lattice group Q homomorphism and A be a Q fuzzy lattice ordered m group of G' then A f (x, q) = (A o f) (x ,q) = $f^{-1}(A)$ (x ,q).

3.PROPERTIES OF Q-FUZZY LATTICE ORDERED M-GROUP

Proposition 3.1: Let G x Q and G' x Q be two Q fuzzy lattice ordered m-groups and
ɵ: G x Q→ G' x Q be a m-homomorphism defined by ɵ (m x, q) = m ɵ (x, q). If B is a Q fuzzy lattice ordered m-group of G' then the pre-image $ɵ^{-1}(B)$ is a Q fuzzy lattice ordered m-group of G.

Proof- Assume B is a Q-fuzzy lattice ordered m-group of G'. Let x, y ϵ G
1. $\mu_{ɵ^{-1}(B)}$ (m (x y), q) = μ_B ɵ (m (x y), q)
= μ_B (m ɵ (x y, q))
= μ_B(m ɵ (x, q) ɵ (y, q))
≥ min { μ_B(m ɵ (x, q)), μ_B(m ɵ (y, q))}
≥ min { μ_B (ɵ (m x, q)), μ_B(ɵ (m y, q))}
≥ min {$\mu_{ɵ^{-1}(B)}$ (m x, q), $\mu_{ɵ^{-1}(B)}$ (m y, q)}
2. $\mu_{ɵ^{-1}(B)}$ $(m x)^{-1}$, q) = μ_B ɵ ($(m x)^{-1}$, q)
= μ_B (ɵ $(m x))^{-1}$, q)
= μ_B (m ɵ $(x))^{-1}$, q)
≥ μ_B (m ɵ (x), q)
≥ μ_B (ɵ (m x), q)
≥ $\mu_{ɵ^{-1}(B)}$ (m x, q)
3. $\mu_{ɵ^{-1}(B)}$ (m x v my, q) = μ_B ɵ (m x v m y, q)
= μ_B ɵ (m x, q) v ɵ (my, q)
≥ min { μ_B ɵ (m x, q), μ_B ɵ (m y, q))
≥ min {$\mu_{ɵ^{-1}(B)}$ (m x, q), $\mu_{ɵ^{-1}(B)}$ (m y, q)}
4. $\mu_{ɵ^{-1}(B)}$ (m x ʌ my, q) = μ_B ɵ (m x ʌ m y, q)
= μ_B ɵ (m x, q) ʌ ɵ (my, q)
≥ min { μ_B ɵ (m x, q), μ_B ɵ (m y, q))
≥ min {$\mu_{ɵ^{-1}(B)}$ (m x, q), $\mu_{ɵ^{-1}(B)}$ (m y, q)}
Therefore $ɵ^{-1}(B)$ is a Q- fuzzy lattice ordered m-group of G.

Proposition 3.2: Let G x Q and G' x Q be two Q- fuzzy lattice ordered m-groups and
ɵ: G x Q→G' x Q be a m-epimorphism. B is a fuzzy set in G'x Q. If $ɵ^{-1}(B)$ is a Q-fuzzy lattice ordered m-

group of G x Q then B is a Q-fuzzy lattice ordered m group of G'x Q.

Proof-Let (x, q), (y, q) ϵ G' x Q, therefore there exist an element a, b ϵG such that ɵ (a ,q) = (x ,q) and ɵ(b,q) = (y,q)

1. μ_B (mxy, q) $=\mu_B$ m (xy, q) $=\mu_B$ m (x, q) (y, q)
= μ_B (m(ɵ(a,q) ɵ(b,q)))
= μ_B (m (ɵ (a b), q)
= μ_B ɵ (m (a b), q)
= $\mu_{\theta^{-1}(B)}$ (m (a b), q)
≥ min { $\mu_{\theta^{-1}(B)}$ (ma, q)), $\mu_{\theta^{-1}(B)}$ (mb, q))}
≥ min { μ_B ɵ (m a, q), μ_B ɵ (m b, q)}
≥ min { μ_B m ɵ (a, q), μ_B m ɵ (b, q)}
≥ min { μ_B m(x, q), μ_B m (y, q)}
≥ min { μ_B (m x, q), μ_B (my, q)}

2. μ_B $((m\ x)^{-1}, q) = \mu_B\ (m\ x, q)^{-1}$
= μ_B $(m\ (x, q))^{-1}$
= μ_B $(m\ ɵ\ (a, q))^{-1}$
= μ_B $(ɵ\ (m\ a), q)^{-1}$
= μ_B ɵ $((m\ a)^{-1}, q)$
= $\mu_{\theta^{-1}(B)}$ $((m\ a)^{-1}, q)$
≥ $\mu_{\theta^{-1}(B)}$ (m a, q)
≥ μ_B ɵ (m a, q)
≥ μ_B m ɵ (a, q)
≥ μ_B m (x, q)
≥ μ_B (m x, q)

3. μ_B (m x v m y, q) $=\mu_B$ ((m x, q) v (m y, q))
$=\mu_B$ (m (x, q) v m (y, q))
= μ_B (m ɵ (a, q) v m ɵ (b,q))
= μ_B (ɵ (m a, q) v ɵ (m b, q))
= μ_B ɵ ((m a v m b), q)
$=\mu_{\theta^{-1}(B)}$ ((m a v m b), q)
≥ min {$\mu_{\theta^{-1}(B)}$ (ma, q), $\mu_{\theta^{-1}(B)}$ (m b, q)}
≥ min { μ_B ɵ (m a, q), μ_B ɵ (m b, q))
≥ min { μ_B m ɵ (a, q), μ_B m ɵ(b,q))
≥ min { μ_B m (x, q), μ_B m (y, q)}

4. μ_B (m x ʌ m y, q) $=\mu_B$ ((m x, q) ʌ (m y, q))
$=\mu_B$ (m (x, q) ʌ m (y, q))
= μ_B (m ɵ (a, q) ʌ m ɵ (b,q))
= μ_B (ɵ (m a, q) ʌ ɵ (m b, q))
= μ_B ɵ ((m a ʌ m b), q)
$=\mu_{\theta^{-1}(B)}$ ((m a ʌ m b), q)
≥ min {$\mu_{\theta^{-1}(B)}$ (ma, q), $\mu_{\theta^{-1}(B)}$ (m b, q)}
≥ min { μ_B ɵ (m a, q), μ_B ɵ (m b, q))
≥ min { μ_B m ɵ (a, q), μ_B m ɵ(b,q))
≥ min { μ_B m (x, q), μ_B m (y, q)}

B is a Q-fuzzy lattice ordered m group of G' x Q.

Proposition 3.3: If {A_i} is a family of Q-fuzzy lattice ordered m-group of G x Q then ∩A_i is a Q- fuzzy lattice ordered m-group of G x Q where ∩A_i = {(x, q), ʌ μ_{Ai} (x, q) / x ϵ G, qϵ Q}

Proof- x, y ϵ G

1. (∩ μ_{Ai}) (m (x y), q) = ʌ μ_{Ai} (m (x y), q)
≥ ʌ min { μ_{Ai} (m x, q), μ_{Ai} (m y, q)}
≥ min { (∩ μ_{Ai}) (m x, q), (∩ μ_{Ai}) (m y, q)}

2. (∩ μ_{Ai}) $((m\ x)^{-1}, q)$ = ʌ μ_{Ai} $((m\ x)^{-1}, q)$
≥ ʌ μ_{Ai} ((m x), q)
≥ (∩ μ_{Ai}) ((m x), q)

3. (∩ μ_{Ai})(m x v m y, q) = ʌ μ_{Ai} (m x v m y, q)
≥ ʌ min { μ_{Ai} (m x, q), μ_{Ai} (m y, q)}
≥ min { (∩ μ_{Ai}) (m x, q), (∩ μ_{Ai}) (m y, q)}

4. (∩ μ_{Ai}) (m x ʌ m y, q) = ʌ μ_{Ai}(m x ʌ m y, q)
≥ ʌ min { μ_{Ai} (m x, q), μ_{Ai} (m y, q)}
≥ min { (∩ μ_{Ai}) (m x, q), (∩ μ_{Ai}) (m y, q)}

Applications: Lattice structure has been found to be extremely important in the areas of quantum logic, Erogodic theory, Reynold's operations, Soft Computing, Communication system, Information analysis system, artificial intelligences, and physical sciences.

ACKNOWLEDGEMENT

The author is highly grateful to the referees for their valuable comments and suggestions for improving the paper.

REFERENCES

[1] Ajmal N and K.V.Thomas, The Lattice of Fuzzy subgroups and fuzzy normal subgroups, Inform. sci. 76 (1994), 1 – 11.
[2] Birkoff, G: Lattice theory 3rd edn. Amer.Math. Soc. Colloquium pub.25 (1984).
[3] G.S.V. Satya Saibaba. Fuzzy lattice ordered groups, South east Asian Bulletin of Mathematics 32,749-766 (2008).
[4] J.A. Goguen: L – Fuzzy Sets, J. Math Anal.Appl. 18, 145-174 (1967).
[5] L. A.Zadeh : Fuzzy sets, Inform and Control, 8, 338-353 (1965).
[6] M.Marudai & V. Rajendran: Characterization of Fuzzy Lattices on a Group International Journal of Computer Applications with Respect to T-Norms, 8(8),0975 – 8887 (2010)

[7] Mordeson and D.S. Malik: Fuzzy Commutative Algebra, World Scientific Publishing Co. Pvt. Ltd.
[8] Nanda, S: Fuzzy Lattices, Bulletin Calcutta Math. Soc. 81 (1989) 1 – 2.
[9] Rosenfeld: Fuzzy groups, J. Math. Anal. Appl. 35, 512 – 517 (1971).
[10] S. Subramanian, R Nagrajan & Chellappa, Structure Properties of M-Fuzzy Groups Applied Mathematical Sciences,6(11),545-552(2012)
[11] Solairaju and R. Nagarajan: Lattice Valued Q-fuzzy left R – Submodules of Neat Rings with respect to T-Norms, Advances in fuzzy mathematics 4(2), 137 – 145 (2009).
[12] W.X.Gu. S.Y.Li and D.G.Chen, fuzzy groups with operators, fuzzy sets and system, 66 (1994), 363-371.