MHD Mixed Convection of Fluids in an Inclined Enclosure Driven by Horizontally and Vertically Moving Walls

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***Abstract* - In this work, MHD mixed convection flow and heat transfer in an inclined lid driven enclosure with different angles (φ=0o to 90o) for two cases has been studied numerically. Simulations are carried out over a range of parameters: Prandtl numbers (Pr=0.7 and 7.2), Hartmann number(0≤Ha≤100), Richardson number (Ri=0.01,1.0,100) and Reynolds number (Re=100). The governing equations are solved by the finite volume method with a SIMPLE algorithm. It has been found that the average Nusselt number decreases with increasing Hartmann number. It is concluded that on increasing the Richardson number, the overall heat transfer is increased. Also, it is found that existence of the magnetic field suppresses the convective heat transfer and fluid flow in enclosure.**

***Index Terms* - mixed convection, inclined enclosure, magnetic field effect.**

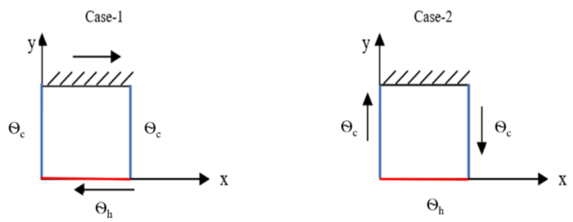
1.INTRODUCTION

Mixed convection in lid driven enclosures has received considerable attention as it is a complex problem due to shear flow caused by the movement of moving wall and buoyancy induced flow. This type of problem is encountered with its novel applications in cooling of electronic devices, food processing, heat transfer in solar passive design, solar receivers, material processing and so on. The fluid flow and heat transfer in a rectangular cavity with moving top wall was investigated by Shankar et al. [1]. Heat transfer on joule heating and magnetic field in a lid driven cavity with heated semicircular source on one wall is studied by Rahmann et al. [2] and he found that magnetic field is a good control parameter for heat transfer and fluid flow. Oztop [3] made a numerical study on laminar MHD mixed convection in a lid driven cavity heated by a corner heater. Nasrin [4] investigated the optimality of heat transfer in a horizontal lid driven enclosure with wavy bottom wall for different aspect ratio of cavity. Sajjadi et al. [5] analyzed the mixed convection heat transfer in a two-sided lid driven cavity with various Richardson's number and it was found that heat transfer declines with the growth of the magnetic field for two cases. Annunziata et al. [6] investigated the mixed convection in inclined lid driven cavity with heat flux boundary conditions and obtained that heat transfer rate increases as inclination angle increases whereas average Nusselt number decreases with increase of Richardson number, because of stratified field configuration. Recently Roy et al. [7] examined the mixed convection in closed cavity with different direction of moving walls and found that fluid flow is highly influenced by the direction of wall's motion. Most recently, mixed convection inside horizontal (top or bottom) wall sliding lid driven 2D enclosure in which vertical walls are at various boundary conditions has been investigated [8-10]. Very recently, MHD mixed convection inside a trapezoidal enclosure with rotating circular solid cylinder filled with Cu-water nanofluid saturated with a porous media has been numerically examined by Ali et al. [11].

In most of the above-mentioned studies, heat transfer analysis is examined only with the isothermal heating or cooling at horizontal walls or vertical walls of the enclosure either with an inclination angle or with a magnetic effect. However, till date, the effect of MHD mixed convection heat transfer due to horizontal or vertical walls moving in the opposite directions in the inclined enclosure is not yet investigated. The main aim of this work is to analyze the heat transfer in an inclined enclosure with both vertical, horizontal walls are moving in the opposite direction in the presence of magnetic field. As the magnetic field controls the heat transfer, the findings of this study will be useful in maintaining the electronic components under effective and safe operational state.

2. FORMULATION OF THE PROBLEM

The physical configuration consisting of a square enclosure filled with steady, laminar, incompressible fluid is shown in the Figure 1. The fluid motion and heat transfer are investigated for two different cases. Case 1. Top wall moves along the positive direction whereas the bottom wall moves in the opposite direction of top wall with uniform velocity U=0.5. Case 2. Left wall moves in the upward direction whereas the right wall moves in the downward direction with uniform velocity V=0.5. The enclosure is isothermally heated at the bottom wall whereas the side walls are isothermally cooled and the top wall is considered perfectly insulated.



Steady two-dimensional laminar MHD mixed convection flow of viscous incompressible Boussinesq fluid such as air (Pr=0.7) and water (Pr=7.2) with constant properties is assumed. Under these assumptions, the governing equations in non-dimensional form are as follows:

$$\boldsymbol{\nabla}.\boldsymbol{U} = 0 \qquad (1)$$

$$(\boldsymbol{U}.\boldsymbol{\nabla})U = -\frac{\partial P}{\partial X} + \frac{1}{Re}\boldsymbol{\nabla}^2 U + Ri\Theta sin\varphi \qquad (2)$$

$$(\boldsymbol{U}.\boldsymbol{\nabla})V = -\frac{\partial P}{\partial Y} + \frac{1}{Re}\boldsymbol{\nabla}^2 V + Ri\Theta cos\varphi - \left(\frac{Ha^2}{Re}\right)V \qquad (3)$$

$$(\boldsymbol{U}.\boldsymbol{\nabla})T = \frac{1}{RePr}\boldsymbol{\nabla}^2 T \qquad (4)$$

The boundary conditions are:

$U(X,Y) = 0 \ or \ \pm 0.5, \quad V(X,Y) = 0, \qquad \Theta = 1 \quad \forall Y = 0, \quad 0 \le X \le 1$

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$U(X,Y) = 0 \ or \ \pm 0.5, \quad V(X,Y) = 0, \qquad \frac{\partial\Theta}{\partial y} = 0 \quad \forall Y = 1, \quad 0 \le X \le 1$ (5)

The rate of heat transfer across the enclosure is calculated as Nusselt number. The heat transfer coefficient in terms of the local Nusselt number is defined as $Nu = -\left(\frac{\partial\theta}{\partial Y}\right)_{Y=0}$. The average Nusselt number at the bottom wall is $\overline{Nu_b} = \int_0^1 Nu_b \, dX$. The numerical scheme is carried out for accurate solution in the average Nusselt number for 161×161 uniform grid size throughout the present computations.

3. CODE VALIDATION

Numerical Technique and Computer Code Validation

Numerical Technique

The SIMPLE algorithm [12] were applied to solve the dimensionless governing equations (Eqs. (1) ~ (4)). A power law scheme and central difference scheme were used to treat the convection terms and diffusion terms, respectively. A line-by-line procedure using the tri-diagonal matrix algorithm (TDMA) were utilized to solve for the resulting sets of discretized algebraic equations for each variable.

The following convergence criterion is employed:

$$\left|\frac{\varphi_{n+1}(i,j) - \varphi_n(i,j)}{\varphi_{n+1}(i,j)}\right| \le 10^{-6} \qquad (6)$$

where φ represents the variables U, V and T. Furthermore, the iteration index n and n+1 denote respectively the previous and current calculation. Finally, (i, j) are the spatial indices in the computational domain.

Computer program validation

The simulation model described above was programmed in FORTRAN and used to compute the average Nusselt number at the enclosure side - walls shown in Fig. 1 for various grid sizes ranging from 21x21 to 141x141. Consequently, for computational expediency, the grid size was set as 121x121 in all of the remaining simulations as the Nusselt number remains unchanged. The comparison shown in Table 1 reveals a good agreement between the current results for the average Nusselt number within the enclosure with those presented by Pirmohammadi et al. [13] for Pr=0.733 in the presence of magnetic field effect.

4. RESULTS AND DISCUSSION

The resulting flow of fluid is shown in Figure 2 (for case-I) and Figure 3(for case-II) where the top and bottom rows give the streamlines for Pr=0.7 and Pr=7.2 respectively, for increasing the values of Ha=0,10,20,50,100 with Ri=1, Re=100, φ=60°. As shown in Figure 2 (case-I) the maximum absolute value of the stream function decreases as Ha increases, which means mixed convection is weaken for the top and bottom wall opposite direction movement with same velocity. In addition, for Ha=100 the maximum stream function values are nearly zero and therefore, there is no movement of fluid in the enclosure. Figure 3 depict the effect of mechanically driven left end right walls in the opposite direction (case-II) dominates the entire cavity and generates recirculating vortex. As Ha increases (for Ha=100), two counter recirculating vortices appeared in the enclosure with low stream function values for fluid air (Pr=0.7). But for the fluid water (Pr=7.2) the small eddy appears in the middle of the enclosure whereas the streamlines at the right wall is elongated and split into two small vortices and also the streamlines at left wall aligned to the direction of wall moving.

Table 1. Comparison of present numerical results with those obtained by Pirmohammadi et al. [12] for Pr=0.733 in the presence of magnetic field.

Ra	Ha	$\overline{Nu}$	
		Pirmohammadi et al. [13]	Present Study
10^4	0	2.29	2.243192
	10	1.97	1.920023
	50	1.06	1.027829
	100	1.02	0.990508
10^5	0	4.62	4.570618
	25	3.51	3.461675
	100	1.37	1.317014
	200	1.16	1.108523

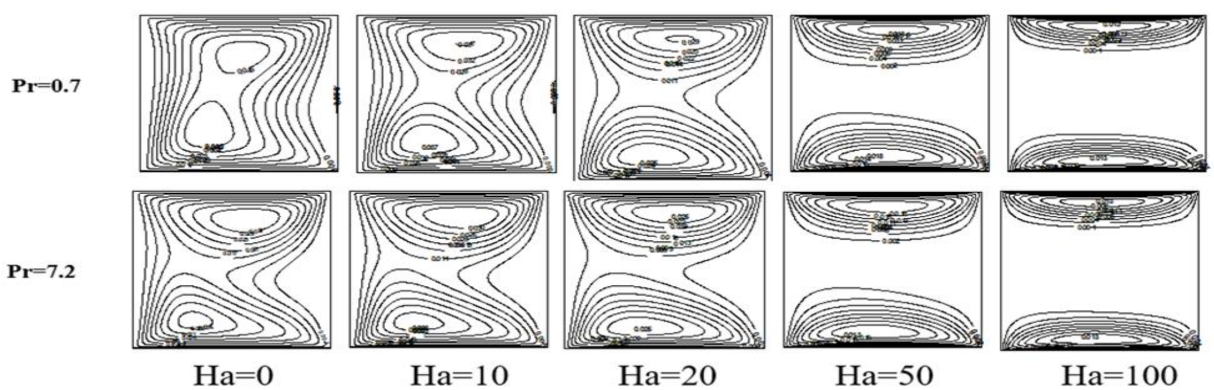

Figure 2: Streamlines for different Ha, Ri=1, Re=100, φ=60° in Case-I

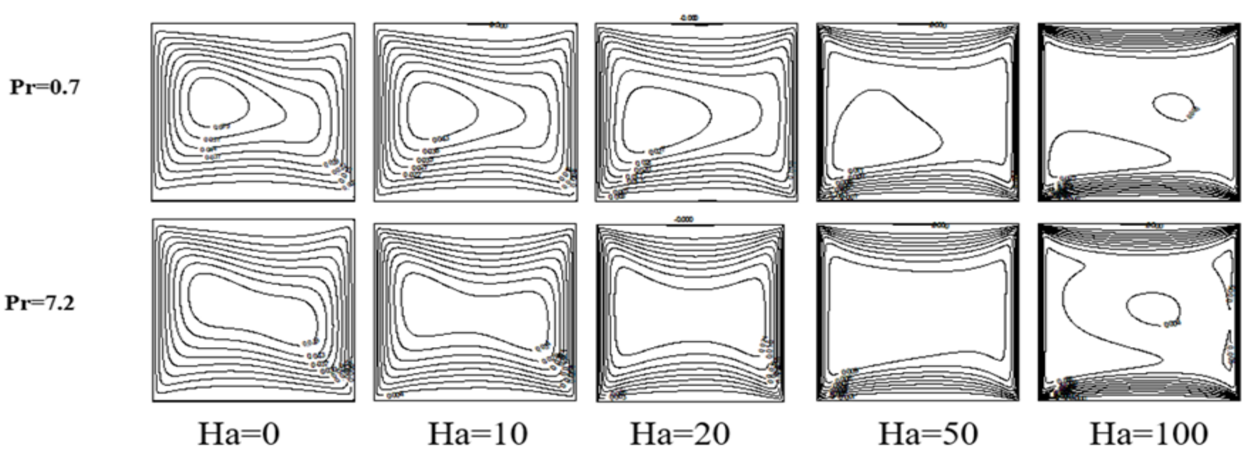

Figure 3: Streamlines for different Ha, Ri=1, Re=100, φ=60° in Case-II.

The effect of magnetic field on the average Nusselt number for various Ri, Pr for case-I, II are shown in Figure 4 and 5, respectively. It is evident from the Figures 4-5 that by increasing Ha(Hartmann number), the decreasing average Nusselt number indicates the reducing heat transfer rate in mixed convection of both the fluids Pr=0.7 and Pr=7.2. Also, it is clear from these figures that heat transfer reduction for Pr=7.2 (water) is higher than Pr=0.7(air) in two cases I and II. Moreover, on comparing case I and II, heat transfer decrease for Pr=7.2 is more higher in case I than in case II.

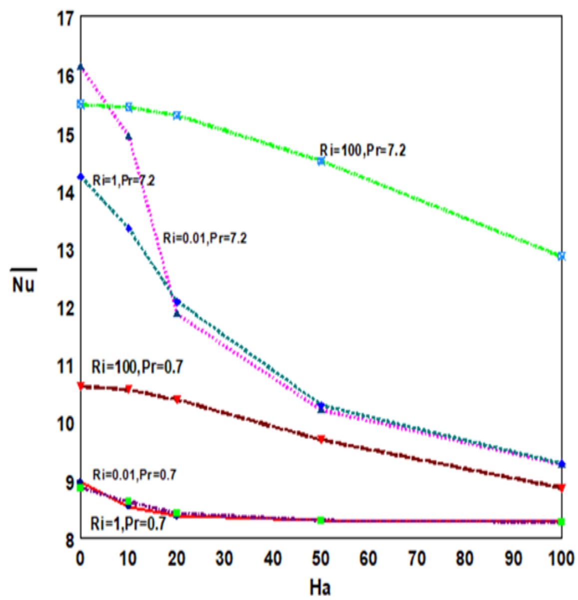

Figure 4: Effect of Hartmann Number on the average Nusselt number for Ri=0.01,1.0,100, Pr=0.7,7.2 for Case-I

As can be seen from Figure 6 for the range of inclination angle φ=0° to 30° , on seeing case I and case II, the heat transfer rate increases for Pr=7.2 is greater than Pr=0.7. But between φ=30° to 50° the heat transfer rate gets decrease for Pr=7.2. It is interesting to see that for Pr=7.2, increase in heat transfer again at φ=60° and further decrease at φ=90° . So, the required heat transfer (either decrease or increase) rate can be obtained by tilting the enclosure in the application of equipment cooling, material processing and solar ponds etc., To illustrate the effect of Richardson number on the flow characteristics the Nusselt number graph is presented in Figure 7.

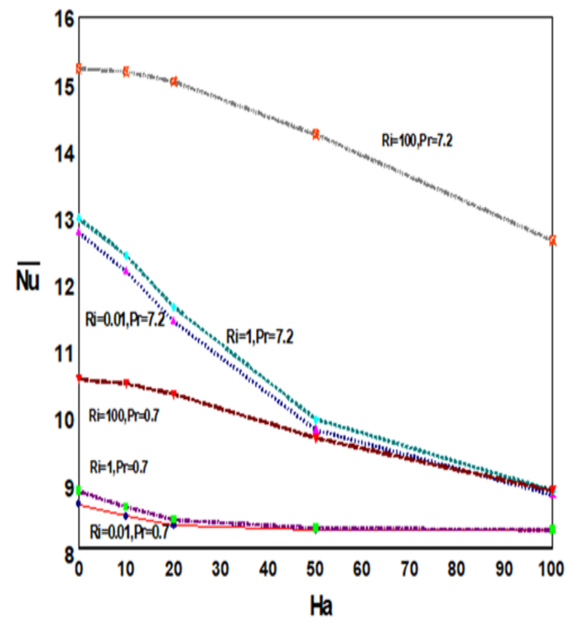

Figure 5: Effect of Hartmann Number on the average Nusselt number for Ri=0.01,1.0,100, Pr=0.7,7.2 for Case-II

The monotonic increase in heat transfer rate is clearly observed for the two fluids air and water for the Richardson number between 0.01 and 1.0. But, when Ri increases to 100 the rate of heat transfer attains the maximum for Pr=7.2 in case-I when compared with case-II and also for Pr=0.7 in case-I, II.

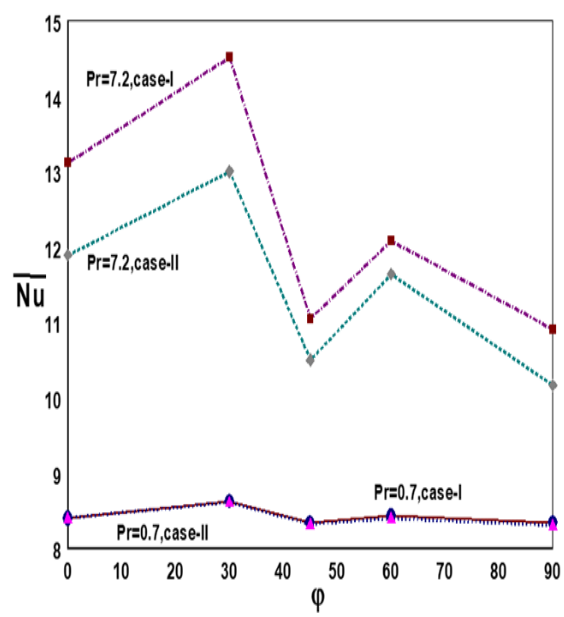

Figure 6: Effect of Inclination angle in Case-I, II

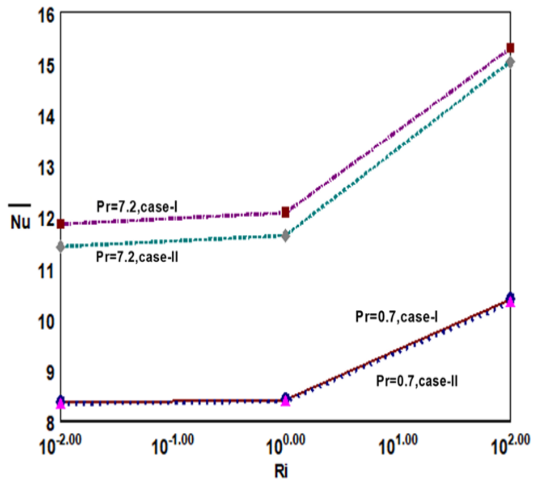

Figure 7: Effect of Richardson Number in Case-I, II

5. CONCLUSION

The MHD mixed convection inside the square enclosure filled with fluid either air (Pr=0.7) or water (Pr=7.2) has been studied. Finite volume method is utilized to stimulate the problem. This study has been performed for the pertinent parameters: Richardson number Ri = 0.01, 1, 100, Hartmann number Ha = 0, 10, 20,50, 100 with Ri=1, Re=100, φ=60o. The results obtained for the above mentioned parameters leads to the following conclusions.

- The heat transfer mechanism and characteristics of the flow of fluid in the inclined lid-driven enclosure depends strongly upon both the strength of the magnetic field and Richardson number.
- The heat transfer rate is intensified in Case-I for Pr=7.2 when compared with Case-II and Pr=0.7 in two cases.
- At Ri=1, the values of Nusselt number strongly depends upon the inclination angle for relatively small value of Hartmann number (Ha=20).
- However, heat transfer decreased with increasing Hartmann number. Hence, Hartmann number can be the control parameter for heat transfer and fluid flow.
- This study can be extended to analyze the aspect ratio of the enclosure and thermodynamic relation.

Nomenclature

B0	Strength of magnetic field
cp	Specific heat, $Jkg^{-1}K^{-1}$
g	gravitational acceleration, ms^{-2}
h	heat transfer coefficient, $W\ m^{-2}\ K$
Ha	Hartmann number, $Ha = B_0 L\sqrt{\frac{\sigma_e}{\mu}}$
L	enclosure height and width, m
K	thermal conductivity, $W\ m^{-1}\ K^{-1}$
Nu	local Nusselt number
$\overline{Nu}$	average Nusselt number
p	dimensional pressure, $N\ m^{-2}$
P	dimensionless pressure
Pr	Prandtl number, $Pr = \frac{\nu_f}{\alpha_f}$
Re	Reynolds number, $Re = \frac{uL}{\nu}$
Ri	Richardson number, $Ri = \frac{g\nabla\rho}{\rho(\nabla u)^2}$
u, v	dimensional velocity components
U, V	dimensionless velocity components, ms^{-1}
x, y	dimensional cartesian coordinates, m
X, Y	dimensionless coordinates

Greek Symbols

α	thermal diffusivity, m^2s^{-1}
β	thermal expansion coefficient, K^{-2} L
θ	dimensionless temperature
ν	kinematic viscosity, m^2s^{-1}
μ	dynamic viscosity, $Kgm^{-1}s^{-1}$
ρ	density, kgm^{-3}
σ	electrical conductivity, S/m

Subscripts

c	cold wall
f	fluid
h	hot wall

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