Check-matrix H of a (-1,+1) Double Error Correctable Integer Codes

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***Abstract* - During the transformation of signal, there are few occurrences of the errors that might take place due to several externalities like extra noise or bad weather. In this paper, we focused on finding the Check matrix of a (-1,+1) Double Error Correctable Integer Code, which will help us detect the errors and eventually correct them. You will also find the mathematical approach I used to get the check matrix, which can be further used to find the check matrix through computational Optimization, and the applications of the research pursued.**

***Index Terms* - Quadrature Amplitude Modulation, Error Correcting Codes, Check matrix, Double Error Correctable Integer Codes.**

INTRODUCTION

The 21st-century era differs from the 19th and 20th because in the 21st century the communication isn't physical but through the internet. Due to the lots of technological advancements, economists analyze that the 4th Industrial revolution would be carried out by the Internet of Things(IoT). Modern technologies like Alexa and our space ambitions make the internet more important, and hence a small error in the transmitted information needs to be resolved at the earliest.

Integer Codes being flexible helps us resolve the issues of error for any modulation schemes in the Digital Communication System, For example, MQAM and MPSK. Integer Codes, defined over finite rings and are efficient in opting for the type of error and constructing codes capable of correcting the errors. In the paper, we will find the check matrix for these kinds of Double Error Correctable Integer codes.

Check Matrix is a matrix when multiplied by the received message calculates syndrome. Syndrome has nothing to do with the actual message. In other words, we can consider syndrome as the error that occurred while transmitting the information. If the result of the product of Check Matrix and Message received is zero then it means that there is no error in the message received.

PROBLEM

The focus of the research paper is to give a mathematical approach to solving the problem by the study of Za, which includes the set of all the errors, and the vector W. The set Za contains all the positive numbers until A, for example Za = {1,2,3,.....,A-1}. For the (+1,-1) Double error correctable Integer Code, the value of A is A ≥ 2n2 + 1.

Let Za = {1, 2, 3, 4, …., A -1}. For a given A, find a vector W = (w1, w2, w3, …, wn) of a
maximum possible length n (A ≥ 2n2 +1), such that:
0 ≠ wi ≠ wj ≠ -wi ≠ -wj ≠ wi + wj ≠ wi - wj ≠ -wi + wj ≠ -wi - wj,
where wi , wj, Za , 1 ≤ i ≠ j ≤ A-1 and addition and subtraction are modulo 'A' operations.
For example: Let k=9. If we take w1 = 1 and w2 = 3 we obtain
0 ≠ 1 ≠ 3 ≠ 8 ≠ 6 ≠ 4 ≠ 7 ≠ 2 ≠ 5.
The length of the vector w is 2 and it is the maximum possible, A = 2n2 +1.

RESULTS

For n = 2,
If we take $w_1 = 1$ and $w_2 = 3$
W = (1,3)
0 ≠ 1 ≠ 3 ≠ 8 ≠ 6 ≠ 4 ≠ 7 ≠ 2 ≠ 5.
The length of the vector W is 2 and it is the maximum possible, $A = 2n^2 +1$.
Few More W vectors for A = 9 and n = 2: (1,3) (2,3) (6,8) (3,4) (6,7) (5,6) (3,5) (4,6)

For n = 3,
If we take $w_1 = 1$, $w_2 = 3$, and $w_3 = 8$

W = (1,3,8)
$1 \neq 20 \neq 3 \neq 18 \neq 8 \neq 13 \neq 4 \neq 17 \neq 2 \neq 19 \neq 11 \neq 5 \neq 10 \neq 16 \neq 9 \neq 12 \neq 7 \neq 14$
The length of vector W is 3 and it is the maximum possible, $A = 21 = 2n^2 + 2$
Few More W vectors for A = 21 and n = 3: (2,5,6) & (6,7,9)

For n = 4,
If we take $w_1 = 1$, $w_2 = 3$, $w_3 = 9$, and $w_4 = 14$
W = (1,3,9,14)
$1 \neq 3 \neq 9 \neq 14 \neq 38 \neq 25 \neq 36 \neq 30 \neq 2 \neq 4 \neq 37 \neq 35 \neq 10 \neq 8 \neq 29 \neq 31 \neq 25 \neq 13 \neq 24 \neq 26 \neq 12 \neq 6 \neq 33 \neq 27 \neq 17 \neq 11 \neq 22 \neq 28 \neq 23 \neq 5 \neq 34 \neq 16$
The length of vector W is 4 and it is the maximum possible, $A = 39 = 2n^2 + 6$

For n = 5,
If we take $w_1 = 1$, $w_2 = 3$, $w_3 = 9$, $w_4 = 16$, and $w_5 = 27$
W = (1,3,9,16,27)
$1 \neq 64 \neq 3 \neq 62 \neq 9 \neq 56 \neq 16 \neq 49 \neq 27 \neq 38 \neq 2 \neq 4 \neq 63 \neq 61 \neq 10 \neq 8 \neq 55 \neq 57 \neq 15 \neq 17 \neq 50 \neq 48 \neq 28 \neq 26 \neq 39 \neq 37 \neq 12 \neq 6 \neq 59 \neq 53 \neq 19 \neq 13 \neq 52 \neq 46 \neq 30 \neq 24 \neq 35 \neq 41 \neq 25 \neq 7 \neq 58 \neq 40 \neq 36 \neq 18 \neq 47 \neq 29 \neq 11 \neq 43 \neq 54 \neq 22$
The length of vector W is 5 and it is the maximum possible, $A = 65 = 2n^2 + 14$

Given below are the results in the form of table:

N (Cardinality)	A(Quasi Perfect)	Formula for Quasi Perfect A	W Vector/ H matrix
2	9	$2n^2 + 1$	(1,3) (2,3) (6,8) (3,4) (6,7) (5,6) (3,5) (4,6)
3	21	$2n^2 + 2$	(1,3,8) (2,5,6) (6,7,9)
4	39	$2n^2 + 6$	(1,3,9,14)
5	65	$2n^2 + 14$	(1,3,9,16,27)

CONCLUSION

This paper is focused on developing a mathematical model to find the H matrix for n<5. The only problem with the mathematical approach is that it becomes tough to find the W vector as the n grows because it is exponential growth. For n = 6, due to a big amount of possibilities it becomes humanly impossible to calculate the W vector. But in the future, we can analyze a trend between different W vectors for a particular 'n' at Quasi Perfect A and create a computational Algorithm, or at least a generalized approach, to find the W vectors/ H matrix for n > 5. This paper has solved the biggest problem, which was to develop a mathematical model and find results to analyze and create a generalized approach for finding W vectors/ H matrix for (+1, -1) double error correctable integer codes.

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