Plane Symmetric Bianchi Type-I Cosmological Model in $f(T)$ Gravity Using Hybrid Expansion Law

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***Abstract* - In this paper, we have investigated Plane Symmetric Bianchi type-I cosmological models with f(T) in the framework of Teleparallel Gravity by using Hybride Expansion Law. in which the energy source represent the CMB radiation and further we have discussed the behavior of model with the physical and kinematical parameters.**

***Index Terms* - CMB radiation energy source, f(T) Gravity, Hybrid Expansion, Plane symmetric Bianchi type-I.**

1.INTRODUCTION

The modified theories of gravitation are $f(R), f(R,T), f(T,\tau)$, $f(R, L_m)$ and so on (Nojiri & Odintsov (2007), Sotiriou & Faraoni (2010), Felice & Tsujikawa (2010), Houndjo (2012), Sadeghi et al. (2013), Harko et al. (2014). Amongst these modified theories interesting results have been found in the $f(T)$ theory of gravity which is the generalization of teleparallel gravity. Bianchi type I universe has been investigated by Sharif and Rani (2011a) and discussed the accelerated expansion of the universe. The cosmic microwave background (CMB) is one of the cornerstone of the homogeneous, isotropic model anisotropies in the CMB are related to small perturbation, superimposed on the perfectly smooth background which are belived to seed formation of galaxies and large scale structure in the universe. Pant and oil [8] have been Investigated two fluid cosmological models using Bianchi type-II Space time. Patil et.al [9] has Investigated magnetized anisotropic bianchi type-XI cosmological model for two fluid in general relativity. Gamal and Nashed [10] investigated anisotropic models with two fluids in linear and quadratic form of $f(T)$ gravitational models. Eckart [11] developed the relativistic theory of non- equilibrium thermodynamic to study the effect of bulk viscosity. Kori and Mukherjee [12] explored the evaluation of bianchi cosmologies with bulk viscosity and particle creation. Desikan [13] studied the effect of bulk viscosity for FRW models. Bamba et.al. [14] studied the cosmological equation of state in exponential, logarithmic and their combined $f(T)$ models. Myrzalulov [15] discussed different $f(T)$ models with scalar field and gave systematic solution for scale factor and scalar field. [16] Sharif and Rani obtained the bianchi type-I universe using different Gravity $f(T)$ models, recently V. M. Raut [17] investigated bulk viscous cosmological model in $f(T)$ Gravity by Hybrid expansion law.

Motivated by above Investigation, we have Investigated of Plane Symmetric Bianchi type-I cosmological model with $f(T)$ Gravity using Hybrid expansion law, in which the energy source represented by CMB radiation and further we have discussed the behaviour of model with physical and kinematical parameters.

2. *F(T)* GRAVITY FORMALISM

In this Paper give brief description of ***f(T)*** gravity model and details of its field equations. we define the action by generalizing teleparallel gravity theory (TG). i.e ***f(T)*** theory as,

$$S = \int \left[T + f(T) + L_{matter} \right] e d^4 x \qquad (1)$$

The modified field equation of the teleparallel theory of gravity is obtained by varying the action with respect to the vierbein vector field h_{μ}^{i} (Setare and Darabi (2018)) which yields,

$$\left[e^{-1}\partial_{\mu}\left(eS_{i}^{\mu\nu}\right)-h_{i}^{\lambda}T^{\rho}{}_{\mu\lambda}S_{\rho}{}^{\nu\mu}\right]$$

$$\left(1+f_{T}\right)+S_{i}^{\mu\nu}\partial_{\mu}\left(T\right)f_{TT}+\frac{1}{4}h_{i}^{\nu}\left[T+f\left(T\right)\right]=\frac{1}{2}k^{2}h_{i}^{\rho}T_{\rho}^{\nu}$$

(2)

Where $S_{i}^{\mu\nu}=h_{i}^{\rho}S_{\rho}{}^{\mu\nu}$, $f_{T}=\dfrac{df}{dT}$, $k^{2}=8\pi G$ and consider Nonlinear $f\left(T\right)$ gravity i.e $f\left(T\right)=T+nT^{2}$ (S. R, Bhoyar et al. 2017).

The teleparallel Lagrangian density is describe by the torsion scalar T is define as

$$T=S_{\rho}{}^{\mu\nu}T^{\rho}{}_{\mu\nu} \quad (3)$$

where $S_{\rho}{}^{\mu\nu}$ antisymmetric tensor; $T^{\rho}{}_{\mu\nu}$ is the torsion tensor which are respectively defined

$$S_{\rho}{}^{\mu\nu}=\frac{1}{2}\left(K^{\mu\nu}{}_{\rho}+\delta_{\rho}^{\mu}T^{\theta\nu}{}_{\theta}-\delta_{\rho}^{\nu}T^{\theta\mu}{}_{\theta}\right) \quad (4)$$

$$T^{\lambda}{}_{\mu\nu}=\Gamma^{\rho}{}_{\nu\mu}-\Gamma^{\rho}{}_{\mu\nu}=h_{i}^{\rho}\left(\partial_{\mu}h_{\nu}^{i}-\partial_{\nu}h_{\mu}^{i}\right) \quad (5)$$

where $\Gamma^{\rho}{}_{\mu\nu}$ is the Weitzenböck connection and the contorsion tensor $K^{\mu\nu}{}_{\rho}$ has the following form,

$$K^{\mu\nu}{}_{\rho}=-\frac{1}{2}\left(T^{\mu\nu}{}_{\rho}-T^{\nu\mu}{}_{\rho}-T_{\rho}{}^{\mu\nu}\right) \quad (6)$$

3. METRIC AND ITS FIELD EQUATION

Take into account the plane symmetric Space-time as,

$$ds^{2}=dt^{2}-A^{2}\left(t\right)\left[dx^{2}+dy^{2}\right]-B^{2}\left(t\right)dz^{2} \quad (7)$$

Where A and B are the cosmic scale factors

The corresponding Torsion scalar yields,

$$T=-2\left(2\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}^{2}}{A^{2}}\right) \quad (8)$$

Let us assume that the energy momentum tensor for radiation field with density ρ_{r}, Pressure $P_{r}=\frac{1}{3}\rho_{r}$ and four velocity $\left(u_{i}\right)^{r}=\left(1,0,0,0\right)$ Where $g^{ij}\left(u_{i}\right)^{r}\left(u_{j}\right)^{r}=1$, so in the co-moving co-ordinate system which yields,

$$T_{1}^{1}=T_{2}^{2}=T_{3}^{3}=-\frac{\rho_{r}}{3},T_{4}^{4}=\rho_{r} \quad (9)$$

Using equation (7) and (9) the field equation of teleparallel gravity can be written as,

$$\left(T+f\left(T\right)\right)-4\left(2\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}^{2}}{A^{2}}\right)\left(1+f_{T}\right)=2k^{2}\rho_{r}$$

(10)

$$4\left(\frac{\dot{A}\dot{B}}{AB}+\frac{\ddot{A}}{A}+\frac{\dot{A}^{2}}{A^{2}}\right)\left(1+f_{T}\right)-16\frac{\dot{A}}{A}\left(\frac{\dot{A}}{A}\left(\frac{\ddot{B}}{B}-\frac{\dot{B}^{2}}{B^{2}}\right)+\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)\left(\frac{\ddot{A}}{A}-\frac{\dot{A}^{2}}{A^{2}}\right)\right)f_{TT}-\left(T+f\left(T\right)\right)=-\frac{2}{3}k^{2}\rho_{r}$$

(11)

4. SOLUTION OF FIELD EQUATION

In this paper we have obtained two highly non-linear differential equation with six unknowns viz. $A,B,f\left(T\right),\rho_{r},P_{r},T$, initially the system undetermined according to solve these systems of equation we have used extra physical conditions,

The spatial volume is,

$$V=a^{3}=A^{2}B=a_{1}^{3}t^{3\alpha}e^{3\beta t} \quad (12)$$

Where α,β are non-negative constant and a_{1} is the present value of the scale factor, which is known as the hybrid expansion law, it is a combination of a power law and an exponential function it can be seen that $\alpha=0$ provide power law cosmology. While $\beta=0$ gives exponential law cosmology. To solve above set of highly non-linear equation the relation between the metric coefficient is consider as

$$B=A^{m} \quad (13)$$

The expansion Scalar,

$$\theta=3H \quad (14)$$

The mean anisotropy Parameter,

$$A_{m}=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_{i}}{H}\right)^{2} \quad (15)$$

The shear Scalar,

$$\sigma^{2}=\frac{1}{2}\left(\sum_{i=1}^{3}H_{i}^{2}-3H^{2}\right)=\frac{3}{2}A_{m}H^{2} \quad (16)$$

Using equation (12) and (13) which yields,

$$A = \alpha_1 t^{\frac{3\alpha}{m+2}} e^{\frac{3\beta t}{m+2}} \,. \quad \text{where} \quad \alpha_1 = a_1^{\frac{3}{m+2}}$$
(17)

$$B = \alpha_2 t^{\frac{3m\alpha}{m+2}} e^{\frac{3m\beta t}{m+2}} \,, \quad \text{where} \quad \alpha_1 = a_1^{\frac{3m}{m+2}} \qquad (18)$$

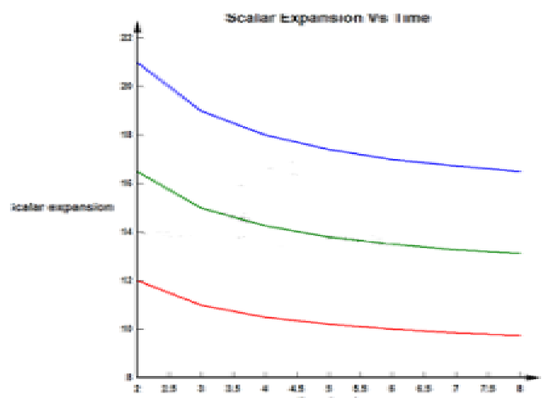
5. PHYSICAL AND KINEMATICAL PROPERTIES

The graphical representation shows Scalar Expansion which yields,

$$\theta = 3\left(\frac{\alpha}{t} + \beta\right) \qquad (19)$$

In Figure: 1 graph lies between Scalar expansion with time so that Scalar expansion along Y-axis and time is plot along X-axis. It shows that the Scalar expansion decrease with increment of time. If time tends to zero then the Scalar expansion tends to Infinity (i.e., $t \to 0$ then $\theta \to \infty$).

Figure:1



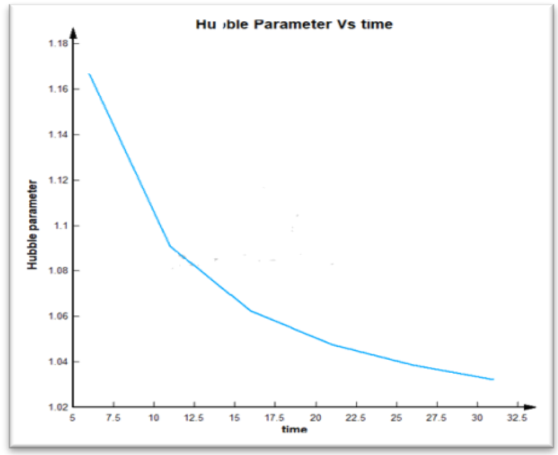
The graphical representation shows mean Hubble parameter which yields,

$$H = \left(\frac{\alpha}{t} + \beta\right) \qquad (20)$$

In Figure: 2 graph lies between Hubble Parameter with time so that Hubble Parameter along Y-axis and time is plot along X-axis. It shows that the Hubble Parameter decrease with increment of time. In initial epoch the value Hubble Parameter tends to Infinity (i.e., $t \to 0$ then $H \to \infty$).

Figure:2

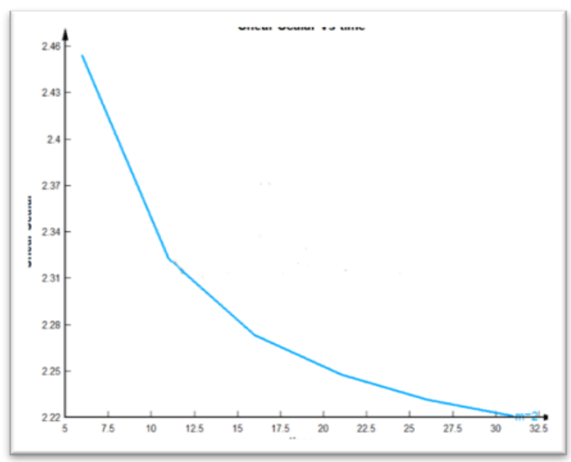
The graphical representation shows mean Shear Scalar which yields,



$$\sigma^2 = \frac{3(m-1)^2}{(m+2)^2}\left(\frac{\alpha}{t} + \beta\right)^2 \qquad (21)$$

In Figure: 3 graph lies between Shear scalar with time so that Shear Scalar along Y-axis and time is plot along X-axis. It shows that the Shear scalar decrease with increment of time. In initial epoch the value shear scalar tends to Infinity (i.e., $t \to 0$ then $\sigma \to \infty$).

Figure:3



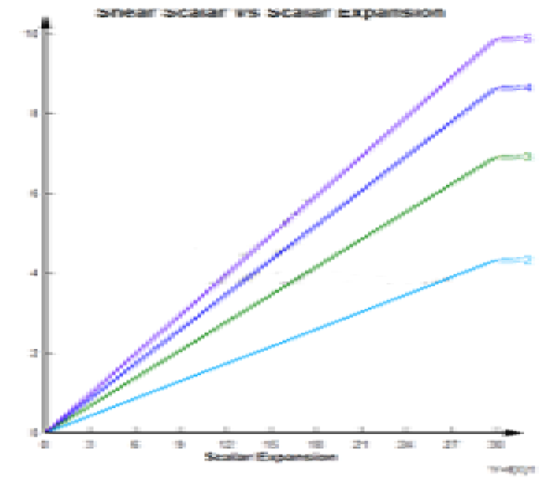
The mean anisotropic Parameter which yields,

$$A_m = \frac{2(m-1)^2}{(m+2)^2} \qquad (22)$$

The graphical representation shows Shear Scalar is considered to be proportional to the expansion scalar which yields,

$$\left(\frac{\sigma}{\theta}\right)^2 = \frac{1}{3}\left(\frac{m-1}{m+2}\right)^2 \qquad (23)$$

Figure: 4



In Figure: 4 graph lies between Shear scalar with Scalar expansion so that Shear Scalar along Y-axis and Scalar expansion is plot along X-axis. It shows that for different value of m we got straight line which passing through origin.

Energy density of Radiation and Pressure of the universe which yields,

$$\rho_r = \frac{1}{2k^2}\left\{\frac{324n(2m+1)(4m^2+10m+5)}{(m+2)^4}\left[\frac{\alpha}{t}+\beta\right]^4 - \frac{36(4m+3)}{(m+2)^2}\left[\frac{\alpha}{t}+\beta\right]^2\right\} \quad (24)$$

$$P_r = \frac{1}{6k^2}\left\{\frac{324n(2m+1)(4m^2+10m+5)}{(m+2)^4}\left[\frac{\alpha}{t}+\beta\right]^4 - \frac{36(4m+3)}{(m+2)^2}\left[\frac{\alpha}{t}+\beta\right]^2\right\} \quad (25)$$

CONCLUSION

In this research paper we have explained f(T) universe is Expanding, an exact solution of the plane symmetric model using CMB radiation field with the help of hybrid Expansion law has been Investigated, in all above cases, the ratio $\left(\frac{\sigma}{\theta}\right)^2 = \frac{1}{3}\left(\frac{m-1}{m+2}\right)^2 \neq 0$ it implies that, these models do not approach isotropy for large value of t. These models come out to be rotating as well as expanding ones, the rate of expansion decreases with time, which can be thought of as realistic models.

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