MHD Visco-Elastic Fluid Flow and Heat Transfer with Variable Thermal Conductivity Embedded in a Porous Medium

Dr.SAYED ANIS FATIMA.H
PG-Department of Mathematics, Govt. Autonomous College Kalaburgi

***Abstract* - Magneto hydrodynamic flows, heat and mass transfer due to combined effect of porosity and visco-elasticity with variable thermal conductivity over a non-isothermal stretching sheet have been investigated numerically and analytically. The effect of various physical parameters like visco-elastic parameter, heat source/sink parameter, thermal conductivity is analyzed on temperature (both PST and PHF CASE) profiles respectively.**

***Index Terms* - Magneto hydrodynamic flow, Thermal conductivity, visco-elasticity, porosity.**

INTRODUCTION

The study of boundary layer flow over a stretching sheet play an important role in many engineering processes, such as aerodynamic extrusion of plastic sheets has many practical applications in industrial manufacturing processes, such as extrusion of polymer sheet, cooling of metallic sheet in cooling bath, manufacturing of plastic films, artificial fibres, and paper production etc. The study of momentum and heat transfer is found to be necessary for determining the quality of final products of such processes which is explained in detail by Karwe and Jaluria and Sakiadis [1961a, 1961b] was the first amongst the others to study such problems by considering the boundary layer viscous fluid flow over a continuous solid surface moving with constant velocity. It was then extended to that of stretching of a boundary sheet with linear velocity by Crane [1970]. This work has subsequently attracted several researchers; Erickson.et.al [1966] extended this problem to the case in which the transverse velocity at the moving surface is non-zero. Tsou-et.al [1967], who investigated heat transfer effect of moving sheet with constant surface velocity and temperature. So many other researchers carried out extensive analysis of heat and mass transfer phenomena associated with such flow, but their investigations are restricted only to the flow of Newtonian fluid. However, in reality most of the liquids used in industrial applications particularly in polymer processing applications are of non-Newtonian in nature. The non-Newtonian fluids are being considered more important and appropriate in technological applications in comparison with Newtonian fluids. A large class of real fluids does not exhibit the linear relationship between stress and rate of strain. Because of non-linear dependence, the analysis and behavior of non- Newtonian fluids tends to be more complicated in comparison to Newtonian fluids.

In view of the importance of these applications, Rajagopal et.al [1984], have studied the flow behavior of visco-elastic fluid over a stretching sheet and gave an approximate solution for the flow. It is more appropriate to consider the non-Newtonian behavior of these fluids in the analysis of the boundary layer flow and heat transfer characteristics, because in industrial applications most of the fluids such as plastic films and artificial fibers are not strictly Newtonian. Considering the survey of literature in non-Newtonian fluid flow, Siddappa and Abel [1985] have presented a similar flow analysis without heat transfer in the flow of non-Newtonian fluids of the type Walter's liquid *B* [1994]. Abel and Veena (1998) studied the visco-elastic fluid flow and heat transfer in a porous medium over a stretching sheet. Gupta and Sridar [1985] analyzed the effect of visco-elastic parameter on non-Newtonian flow through porous medium. Many researchers such as Dandapat and Gupta [1989], Anderson [1992,1995], Chakrabarthi and Gupta [1979], Sarpakaya.T [1961] have done their work on MHD visco-elastic fluid Flows. In above all studies the physical properties of the fluid are assumed to be constant, but for liquid metals, it has been found that the thermal conductivity *k* varies with temperature in an approximately linear manner, which is also true in some polymer solutions in the class of Walter's liquid *B* [1994], and that leads to non-linearity in the boundary value problem of heat transfer. Chiam [1996,1998] has done his work by taking the thermal conductivity as a function of temperature. Prasad et.al [2000] analyzed the effect of momentum and heat transfer of visco elastic fluid flow over a non-isothermal stretching sheet assuming the thermal conductivity varying linearly with temperature. Motivated by all these investigations, we contemplate to study the MHD visco-elastic fluid flow over a stretching sheet in presence of variable thermal conductivity. Heat transfer characteristic is also analyzed. Because of the complexity and non-linearity in the proposed problem, it has been solved numerically by shooting technique with fourth order Runge-kutta integration scheme.

Mathematical Formulation:

Consider a steady state two-dimensional incompressible visco-elastic laminar flow of a Walter's liquid *B* in porous media over a
semi infinite stretching sheet coinciding with the plane $y=0$. The flow is generated due to stretching of the sheet, caused by the simultaneous application of two equal and opposite forces along *x-axis*. Keeping the origin fixed, the sheet is then stretched with a speed varying linearly with the distance from the origin $x=0$.This flow obeys the modified version (Prasad.et.al [2000]) of the rheological equation of state derived by Beard and Walter's [1964]. The flow field is then exposed under the influence of uniform transverse magnetic field in such way that the effect of the induced magnetic field is negligible (Anderson [1992]). Hence the basic boundary layer equation governing the flow, heat and mass transfer in presence of internal heat generation takes the form

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\gamma\frac{\partial^2 u}{\partial y^2}-k_0\left\{u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right\}-\frac{\sigma B_o^2}{\rho}u-\frac{\gamma}{k'}u \tag{2}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+Q(T-T_\infty) \tag{3}$$

Here, σ is the electrical conductivity, B_0 is the applied magnetic field, k_0 is the visco-elastic parameter of the Walter's liquid *B*. k' permeability of porous medium, *Q* is the volumetric rate of heat generation, *k* is the thermal conductivity. The other quantities have their usual meanings.

The boundary conditions governing the flow are

$u = bx$

$v = 0$

$T = T_\omega = T_\infty + A_1 x^\lambda$ (PST case)

$$-k\frac{\partial T}{\partial y}=Bx^\lambda \quad \text{(PHF case)}$$

$$u\rightarrow 0 \qquad \frac{\partial u}{\partial y}\rightarrow 0$$

$$T\rightarrow T_\infty \quad as \quad y\rightarrow\infty \tag{5}$$

Here *u* and *v* are velocity components along *x* and *y* directions respectively. A_1, *B* and A_2 are arbitrary constants, which depend on the nature of the boundary surface. T_w, T_∞ are temperature of the stretching sheet and temperature of the flow region far away from the sheet respectively.

Flow Analysis:

In order to obtain the mathematical form of the velocity, we introduce the following similarity transformations

$$u=bxf'(\eta),\quad v=-\sqrt{b\nu}\,f(\eta)$$

Where
$$\eta=\sqrt{\frac{b}{\nu}}.y \tag{6}$$

With these changes of variables, equation (1) is identically satisfied and equation (2) is transformed into the following non-linear ordinary differential equation.

$$f'^2-ff''=f'''-k_1\{2f'f'''-ff''''-f''^2\}-Mnf'-k \tag{7}$$

Where

$$k_1=\frac{k_o b}{\gamma},\quad Mn=\frac{\sigma B_o^2}{b\rho},\ k_2=\frac{\gamma}{k'b}$$

are non-dimensional visco-elastic, Magnetic and porosity parameters respectively and the boundary condition takes the form

$$f=0 \quad f'=1 \quad at \quad \eta=0$$

$$f'\rightarrow 0 \quad f''\rightarrow 0 \quad as \quad \eta\rightarrow\infty \tag{8}$$

Where prime denotes differentiation w.r.t η.

The exact solution of equation (7) corresponding to the boundary conditions (8) is obtained as

$$f = \frac{1}{\alpha}(1 - e^{-\alpha\eta})$$

Where

$$\alpha = \sqrt{\frac{1 + Mn + k_2}{1 - k_1}} \qquad (9)$$

The solutions for velocity field is obtained as

$$u = bxe^{-\alpha\eta} \quad , \quad v = -\sqrt{b\gamma}\ \frac{1 - e^{-\alpha\eta}}{\alpha} \qquad (10)$$

It is of some interest to note that our result (10) gives the result of Anderson [1995] in the limiting case of *k=0.*

Heat Transfer Analysis:

The energy equation in presence of variable thermal conductivity with internal heat generation /absorption for the two dimensional flow is

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + Q(T - T_\infty) \qquad (11)$$

Where ρ is the density, c_p is the specific heat at constant pressure, k is the thermal conductivity, which is assumed to be variable with temperature and is given by

$$k = k_\infty(1 + \varepsilon\theta),$$

Where $\theta = \dfrac{T - T_\infty}{T_w - T_\infty}$ (PST CASE)

$k = k_\infty(1 + \varepsilon g)$,

Where $g(\eta) = \dfrac{T - T_\infty}{T_w - T_\infty}$, (PHFCASE) (12)

Where $\varepsilon = \dfrac{k_w - k_\infty}{k_\infty}$ is very small parameter which depends on the nature of the fluid and k_∞ is the conductivity of the fluid far away from the sheet and Q is volumetric rate of heat generation. Thermal boundary conditions depend upon the type of the heating process. Here we considered two different type of heating processes namely

(1) Prescribed surface temperature

(2) Prescribed power law heat flux

CASE (1): PRESCRIBED SURFACE TEMPERATURE

In this case we consider the boundary conditions as

$T = T_\omega = T_\infty + A x^\lambda$ at $y = 0$.

$T \to T_\infty$ as $y \to \infty$ (13)

T_w is variable wall temperature, A is a constant and λ is wall temperature parameter. When λ =0 the thermal boundary condition becomes isothermal. Using (9), (10) , (12), equation (11) and corresponding boundary condition (13) takes the form

$(1+ \varepsilon\,\theta)\ \ \theta'' + Pr\, f\theta' - Pr\,(\lambda f' - \beta)\,\theta + \varepsilon\,(\theta')^2 = 0$ (14)

With boundary conditions

$\theta(\eta) = 1$ at $\eta = 0$

$\theta \to 0$ as $\eta \to$ (15)

Where prime denotes differentiation w.r.t η and Pr = $\dfrac{\mu c_p}{k}$, $\beta = \dfrac{Q}{\rho c_p b}$

CASE2: PRESCRIBED POWER LAW HEAT FLUX

For this heating process, the boundary conditions are

$$-k\frac{\partial T}{\partial y} = Bx^\lambda$$

at $y = 0$

$T \to T_\infty$ as $y \to \infty$ (16)

Where λ is the wall heat flux parameter, B is constant. Defining

$$g(\eta) = \frac{T - T_\infty}{T_w - T_\infty} ,$$

Where $T_w - T_\infty = \dfrac{Bx^\lambda}{k}\sqrt{\dfrac{b}{\upsilon}}$ (17) and

$k = k_\infty(1 + \varepsilon g)$,

With this change of variable equation (11) and corresponding boundary conditions (17) takes the form

$(1+ \varepsilon g)\, g'' + \varepsilon\,(g')^2 + Pr\, f\, g' - Pr(\lambda f' - \beta)\, g = 0$ (18)

And the boundary conditions are

$g'(\eta) = -1$ at $\eta = 0$

$g(\eta) \to 0$ as $\eta \to \infty$ (19)

The important physical quantities of our interest is the wall temperature T_w which is defined as

$$T_w = T_\infty + \frac{Bx^\lambda}{k}\sqrt{\frac{b}{\nu}}g(0) \qquad (20)$$

In the next section we solve equations (14) and (18) subject to the boundary conditions (15) and (19) respectively.

METHOD OF SOLUTIONS

Since equation (14) and (18) are non-linear ordinary differential equations and exact solution do not seems to be feasible, therefore we solve equation (14) and 18) numerically by using most efficient numerical shooting technique with fourth order Runge kutta algorithm to solve them.

RESULTS AND DISCUSSION

The heat transfer phenomena are usually analyzed from the numerical values of the two physical parameters namely:

(i) wall temperature gradient and wall concentration gradient in PST case, and

(ii) wall temperature and wall concentration in PHF case. Numerical results for both the cases is recorded in Table-I, II and III.

In order to have a clear insight of the physical problem, numerical results are displayed with the help of graphical illustrations.

* Table-I shows a comparison of the present results for visco-elastic fluid in presence of variable thermal conductivity for the temperature gradient $-\theta'(0)$ with T.C.Chiam [1970], our results are qualitatively good in agreement with T.C.Chiam, but quantitively less in magnitude.

* Table-II shows the values of temperature gradient $-\theta'(0)$ (PSTcase) and temperature at the wall $g(0)$ (PHFcase).

*Table-III shows the value of wall concentration gradient $\phi'(0)$ for different values of Sc and k_1 .

* Results for prescribed surface temperature (PST) are drawn in Fig.1-3 and for prescribed power law heat flux (PHF) are drawn in Fig.4-6 .

Fig-1 shows the variation of temperature profile against the space variable η for different values of visco-elastic parameter k_1, it can be seen from Fig-1 that temperature profile increases when k_1 increase. This is due to the fact that the thickening of thermal boundary layer occurs due to the increase of Visco-elastic normal stress.

The effect of heat source/sink parameter (β) on temperature profile $\theta(\eta)$ in the boundary layer is shown in Fig-2. It is observed that the effect of heat source (β > 0) in the boundary layer generates the energy, which causes the temperature to increase, while the presence of heat sink (β < 0) in the boundary layer absorbs the energy, which causes the temperature to decrease. These behaviors are even true in the absence of porosity, which is represented in Fig-2.

Fig-3 shows the effect of thermal conductivity (ε) on temperature profile $\theta(\eta)$ in the boundary layer, it is observed that temperature profile $\theta(\eta)$ increases as thermal conductivity parameter (ε) increase, because there is more heat flow through the stretching surface. It is also true in absence of porosity. This result is qualitatively good in agreement with Chiam [1970].

The graphs for the situation when the boundary has been prescribed with heat flux (PHF) are shown in Fig.4-6. It is noticed from these figures that the wall temperature is not unity, it is changed at the wall with the change of physical parameters like Visco-elastic parameter (k_1), heat source/sink (β) and thermal conductivity (ε) and have same qualitative effects as those we found in PST case but quantitatively wall temperature is more in PHF case.

SUMMARY AND CONCLUSION

Magneto hydrodynamic flows, heat and mass transfer due to combined effect of porosity and visco-elasticity with variable thermal conductivity over a non-isothermal stretching sheet have been investigated numerically and analytically. The effect of various physical parameters like visco-elastic parameter, heat source/sink parameter, thermal conductivity parameter is analyzed on temperature.

The specific conclusions derived from our study are summarized as follows

1. The increase of thermal conductivity **ε** leads to decease the temperature gradient $-\theta'(0)$ in PST Case and to increase the wall temperature g (0) in PHF Case. This observation is even true in presence of porosity parameter but with reduced magnitude.

2. The effect of visco-elastic parameter is to increase the temperature profile in both PST and PHF cases.
3. The effect of heat source/sink parameter (β) and visco-elastic parameter k_1 is seen to increase the temperature distribution in the flow region.
4. Temperature profile $\theta(\eta)$ in (PST case) and $g(\eta)$ (PHF case) are quantitatively more in porous media

TABLE I: Comparison of the values of - θ′(0) for different Values of ε in presence of variable T_ω

ε	Chaim [1998]	Present result
-0.5	2.057411	0.629242
-0.4	1.796036	0.560520
-0.3	1.606109	0.485970
-0.2	1.461201	0.441656
-0.1	1.346566	0.404823
0.0	1.253314	0.373853
0.1	1.175756	0.347452
0.2	1.110079	0.324425
0.3	1.053628	0.304106
0.4	1.004495	0.286025
0.5	0.961272	0.269801

Table-II wall temperature gradient -θ′(0) in PST case and wall temperature g(0) in PHF case for different values k_1, β and ε

k_1	β	ε	-θ′(0)		g(0)	
			k_2=0.0	k_2=0.2	k_2=0.0	k_2=0.2
0.1	0.05	0.1	0.311677	0.278478	3.659318	4.104178
0.2			0.283316	0.244083	3.959827	4.504343
0.3			0.247677	0.199024	4.381710	5.077729
0.1	-0.1	0.1	0.486861	0.471930	2.231337	2.313248
	0.0		0.384581	0.362416	2.919283	3.185716
	0.1		0.117757	0.12234	4.955648	4.960275
0.1	0.05	-0.1	0.370659	0.333395	2.308513	2.451500
		0.0	0.337616	0.302868	2.858875	3.113565
		0.1	0.307821	0.272346	3.839057	4.451992
		0.2	0.284021	0.249032	4.565487	4.718392

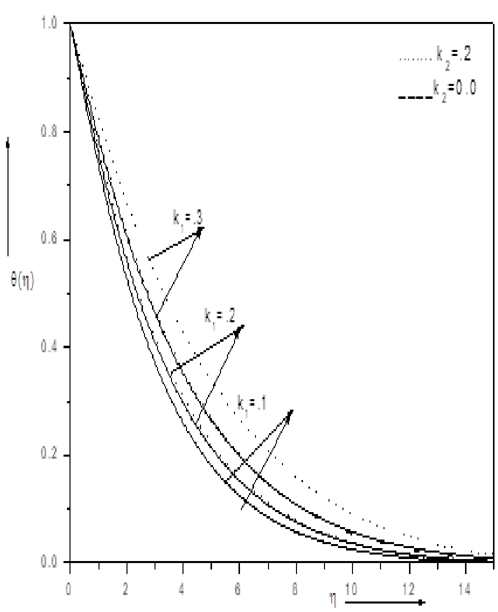

fig1:variation of temperature profile θ(η) Vs η for different values of visco-elastic parameter k_1 when Mn=λ=ε=.1,β=.05 and Pr=1(PST Case)

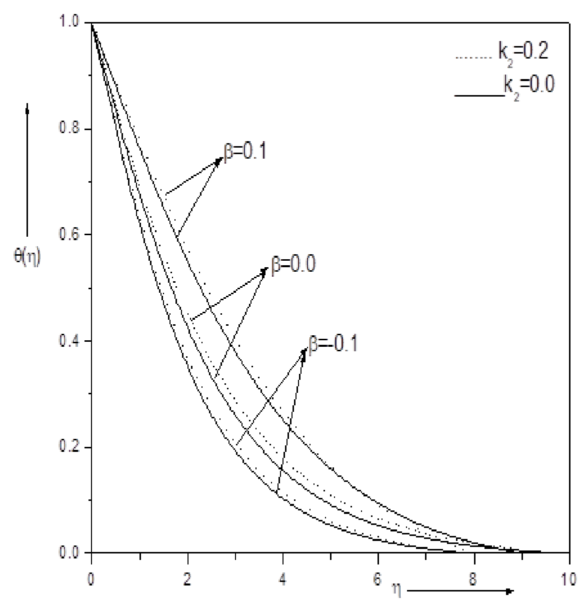

fig2:variation of temperature profile θ(η) Vs η for different values of β when k_1=Mn=λ=ε=.1 and Pr=1(PST case)

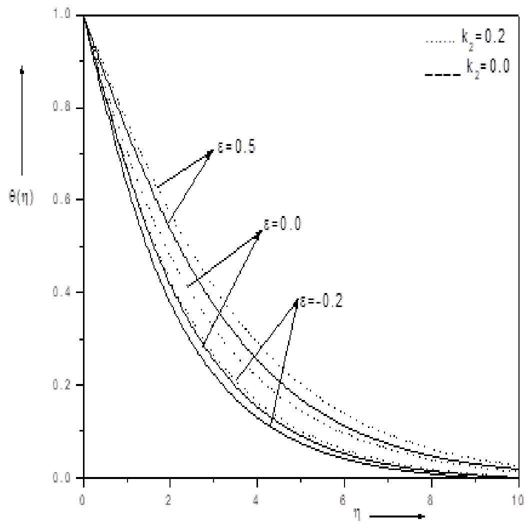

Fig3:variation of temperature profile $\theta(\eta)$ Vs η for different values of ε when k_1=Mn=λ=.1,β=.01,and Pr=1(PST Case)

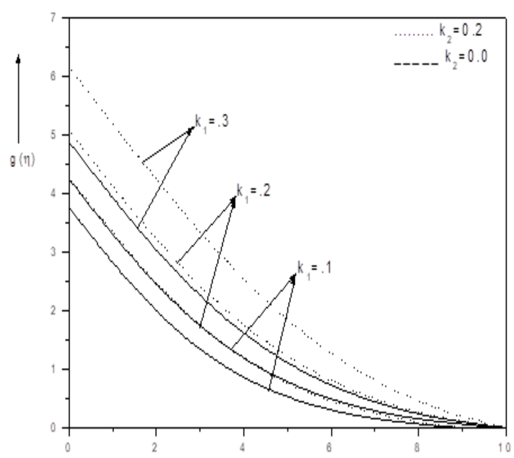

Fig4:variation of temperature profile $g(\eta)$ Vs η for different values of visco-elastic parameter k_1 when Mn=λ=ε=.1,β=.05 and Pr=1(PHFCase)

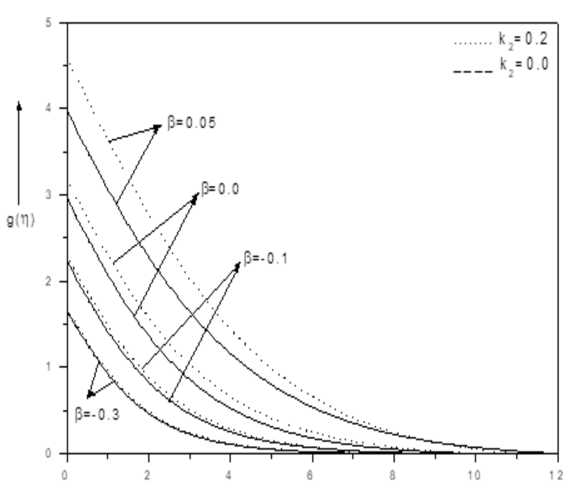

Fig5:variation of temperature profile $g(\eta)$ Vs η for different values of β when k_1=Mn=λ=ε=.1 and Pr=1(PHFcase)

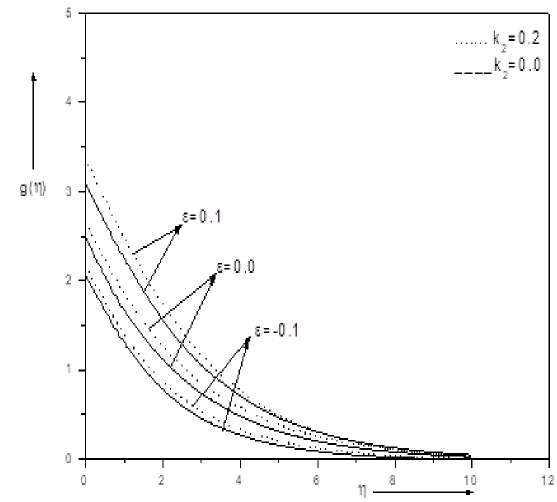

Fig6:variation of temperature profile $g(\eta)$ Vs η for different values of ε when k_1=Mn=λ=.1,β=.01,and Pr=1(PHF Case)

REFERENCES

[1] Anderson.H.I: MHD flow of visco-elastic fluid past a stretching surface Acta, Mechanica 95(1992) 227-230.

[2] Anderson.H.I: An exact solution of the Navier-stokes equation for MHD flow, Acta, Mechanica, 113(1995) 241-244

[3] Abel.M.S , Veena.P: Visco-elastic fluid flow and heat transfer in a porous medium over a stretching sheet,Int.J.Non-Lin. Mech,33(1998) 531-538.

[4] Abel.M.S, Sonth. R.M, Khan.S.K, Parasad.K.V: Heat and Mass transfer in a visco-elastic fluid flow over an accelerating surface with heat source/sink and viscous dissipation. Heat Mass transfer. 32(2002) 213-220

[5] Abromowitz.A, Stegun.I.A: Handbook of Mathematical functions 55,providence National Bureau of standards.Amer.Math soc. (1972).

[6] Chiam.T.C: Heat transfer with variable thermal conductivity in a stagnation point flow towards a stretching shee,Int.Comm.Heat Mass Tansfer,23(1996a) 239-248.

[7] Chiam.T.C: Heat transfer in a fluid with a variable thermal conductivity over a linearly stretching sheet,Acta Mechanica129).(1998)63-72

[8] Chakrabarthi.A, Gupta.A.S: Hydro magnetic flow and heat transfer over a stretching sheet, Quart. Appl.Math.37 (1979) 73-78

[9] Crane.L.j: Flow past a stretching plate. ZAMP, 21(1970) 645-647

[10] Dandapat.B.S,Gupta.A.S: Flow and heat transfer in a visco-elastic fluid over a stretching sheet, Int, J.Non-Lin.Mech, 24(1989) 215-219.

[11] Erickson.L.E, Fan.L.T, Fox.F.G: Heat and mass transfer on a moving continuous flat plate with suction or injection. Ind.engg.chem.fundamentals 5(1966) 19-25.

[12] Gupta.R.K, Shridhar.T: Visco-elastic effects in non -Newtonian flows through porous media, Rheol.Acta, 24(1985)148-151.

[13] Karve.M.V,Jaluria.Y: Fluid flow and mixed convection transport from a moving plate in rolling and extrusion processes. ASME J.Heat Transfer, 110(1988) 655-661.

[14] Karve.M.V,Jaluria.Y: Numerical solution of thermal transport associated with a continuously moving flat sheet in materials processing. ASME J.Heat Transfer 113(1991) 612-619

[15] Prasad.K.V ,Abel.M.S: Momentum and Heat transfer in visco-elastic fluid flow in a porous medium over a non-isothermal stretching sheet Int.J.of numerical method for heat fluid flow.10 No.8 (2000) 786-801

[16] Rajgopal.K.R, T.Y.Na, Gupta.A.S : Flow of visco-elastic fluid over a stretching sheet, Rheol..Acta, 23(1984) 213-215

[17] Sakiadis.B.C: Boundary layers behavior on continuous solid surface, A.I.Ch.E.J7.(1961a) 26-28.

[18] Sakiadis.B.C: Boundary layer behavior on continuous flat surfaces. A.I.Ch. E.J.7(1961b) 221-225

[19] Siddappa.B, Abel.M.S: Non-Newtonian flow past a stretching plate,Z.A.M.P 36, (1985)890-892.

[20] Sarpakaya.T : Flow of non-Newtonian fluids in magnetic field.A.I.Ch.E.J 7(1961) 324-328.

[21] Savvas,T.A, Markatos.N.C and Papaspyride.C.D: On the flow of Non-Newtonian polymer solutions. , Appl.Math. Modelling, 18(1994) 14

[22] Tsou.F.K, Sparrow.F.K, Goldstein.R.J : Flow and Heat transfer in the boundary layer on a continuous moving surface, Int . J, Heat Mass transfer, 10 (1967) 21