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Partial GARCH Models for Forecasting Volatility

Dr. Ashwini Kumari
Department of Statistics, Alva's College, Moodbidri, D.K-574227, Karnataka, India

***Abstract* - This paper deals with the forecasting of volatility using the partial GARCH model. Volatility is also influenced by some exogenous factors such as climate, scientific development, growth rate of population, political situation of a country, demonetisation etc. The influences of exogenous factors are considered in partial GARCH model.**

In this paper we modify the GARCH model by incorporating the influence of exogenous factors for improving accuracy in forecasting volatility. To study the impact of exogenous factor in volatility, we introduce the nonparametric component in the GARCH model. An improved forecasting model for Volatility is developed by combining GARCH with Nonparametric functional estimate. The performance of the proposed estimator is compared with GARCH estimator through a simulation study. Simulation study reveals that proposed model shows minimum mean square forecasting error compared to the existing model.

***Index Terms* - ARCH, GARCH, Partial linear models, Volatility.**

I.INTRODUCTION

In financial-market volatility is an important variable. Volatility means the conditional standard deviation of the asset return. Volatility plays a crucial role in financial market because it is associated (linked) with risk and uncertainty. Therefore modeling, estimation and forecasting volatility is one of the main aspects of financial market. It is very much required for investors, managers and policy makers because a better prediction of volatility will help them to identify the market movements and also to take risk free decision regarding their investment. There is a strong relationship between volatility and market performance. When the volatility increases, risk also increases but returns decrease. The variations in the market performance are due to exogenous and endogenous factors. The factors which affect the movement of market outside the system are called exogenous factor. For example Demonetization of Rs.500 and Rs.1000 on 8th November 2016 disrupt the Indian economy. Demonetization has a positive and negative impact on various sectors. The sectors which are positively affected by this demonetization are Pharma, Banking Sector etc. because, after the announcement of demonetization their return is increased. But the Indian Stock market is negatively affected by the demonetization. After the demonetization most of the investors have started to withdraw their money from the market. Some of them withdraw their money due to lack of fund and some others waited for further fall so that get an opportunity to buy the shares at a lower price. Therefore, developing a model which incorporates the effect of exogenous factors on market variation will help the traders for better prediction so that wise decision regarding their investment can be made.

Most of the studies carried on volatility are based on the assumption that conditional variance can be best explained by the GARCH model. (GARCH models are very powerful tool to explain the conditional variance). So far the models incorporating the effect of exogenous variables on volatility were not considered. However, there are many articles which hint that exogenous factor affect volatility.

Garner [9] says that the stock market crash which occurred in 1987 reduced consumer spending in the USA. According to Maskus [11] volatility in foreign exchange markets has an impact on trade.

Hamao, Masulis, and Ng [15] examined the daily opening and closing stock price indexes of Tokyo, London and New York and got evidence for spillover effects of price volatility from the U.S and U.K stock markets to the Japanese market.

Engle and Ng [7] attribute the arrival of new and unanticipated information as the key cause for the volatility. Engle R F and Ng V [8] studied the effect of news on volatility by introducing the news impact curve which helps them to summarize the effect of news on volatility. For the empirical analysis, they used daily Japanese stock returns from 1980 to 1988 and find that negative shocks (news) introduce more volatility than the positive shocks.

Bollerslev T and Melvin M [5] have used Deutschemark/dollar quote to study the link between exchange rate volatility and bid-ask spread quoted by Banks. They show that foreign exchange market bid-ask spreads are characterized by strong temporal dependence and also measured the exchange rate volatility as the conditional variance of the ask price estimated by an MA(1)-GARCH(1,1) model and find that there is a strong positive relationship between volatility and spreads. The coefficient estimates of the effect of the conditional variance on the spread are highly statistically significant.

Andersen and Bollerslev [2] show that largest returns are linked with release of public information particularly certain macroeconomic announcements. In order to show that market activity is closely related to price variability, they have used the Deutsche Market Dollar exchange rate and finds that its volatility increases markedly around the time of announcements of US macroeconomic data, such as the employment report, the Producer price index, or the quarterly GDP.

Robert F Engle and Andrew J Patton [14] used twelve years of daily data on the Dow Jones Industrial Index to study the stylized facts like persistence in volatility, its mean-reverting behavior, the asymmetric impact of negative versus positive return innovations and the possibility that the effect of exogenous or predetermined variables on volatility.

Riman [13] collected the annual data from various issues of the Central Bank of Nigeria Statistical Bulletin over a period of 1970 to 2012 to study the asymmetric effect of oil price shocks on exchange rate volatility in Nigeria and conclude that the public investment, private investment and industrial development in Nigeria are negatively affected by the volatility in crude oil price.(oil price shocks).

Ogundipe et al. [12] studied the impact of oil prices on exchange rate volatility of Nigeria and found that a proportionate change in Oil price leads to more than proportionate change in exchange rate volatility.

Harlod Ngalawa and Adebayo Augustine Kutu [10] develop a model for the variations of Brazil exchange rates. They wanted to study whether the global oil prices and international interest rates (global shocks) affects the exchange rates. According to their study, Brazil exchange rates are highly influenced by global shocks. Then they suggest that while formulating and implementing the policies regarding the exchange rate Brazilian Government should consider the impact of oil prices and global interest rate.

Generalised Autoregressive Conditional Heteroscedasticity (GARCH) models [4] and stochastic volatility models [1] are the two models which have been widely used for analyzing time varying volatility in financial econometrics. The success of the GARCH – models at capturing volatility clustering in financial market is extensively recorded in the literature. But while making a prediction we should identify the various factors that exert a significant impact on current price movements.

Improving the accuracy in forecasting is the main objective of developing the model for volatility. In this paper, we introduce the exogenous factor in the volatility model and examined the accuracy in forecasting volatility and also derive the l-step ahead of the forecast for the proposed model. The proposed model is also compared with the volatility model without exogenous factors.

II.THE MODEL AND ITS ESTIMATION

The investors of financial market are highly interested in the estimation of both risk and return. The return on an asset is given by

$$y_t = \sigma_t \varepsilon_t \qquad (1)$$

Where $\{\varepsilon_t\}$ is a sequence of independent and identically distributed (i.i.d) random variables with mean zero and variance 1. σ_t follows GARCH (p, q) and has the representation

$$\sigma_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2 + \ldots + \alpha_p y_{t-p}^2 + \beta_1 \sigma_{t-1}^2 + \ldots + \beta_q \sigma_{t-q}^2 \qquad (2)$$

The proposed new model incorporating exogenous effect on volatility is of the form

$$\sigma_t^2 = w_1 g(x) + w_2 \left[\alpha_0 + \alpha_1 y_{t-1}^2 + \beta_1 \sigma_{t-1}^2\right] \qquad (3)$$

Where g(x) is the exogenous factor and w_1 and w_2 are the weights to be chosen optimally under mean squared error criteria satisfying $w_1+w_2=1$.

The kurtosis of y_t is defined as

$$K^{(y)} = \frac{3E\left(\sigma_t^4\right)}{E\left[\sigma_t^2\right]^2} \qquad \mathbf{(4)}$$

According to Jensen's inequality if X is a random variable and f is a convex function then f[E(x)] < E[f(x)]. In equation (4), $\left[E\left(\sigma_t^2\right)\right]^2 < E\left(\sigma_t^4\right)$.

Therefore Kurtosis of y_t is > 3.

For GARCH (1, 1) unconditional variance of y_t is

$$Var\left(y_t\right) = E\left(\sigma_t^2\right)$$

$$= w_1 g(x) + w_2\left[\alpha_0 + \alpha_1 E\left(y_{t-1}^2\right) + \beta_1 E\left(y_{t-1}^2\right)\right]$$

Since y_t is a stationary process, the V(y_t) = V(y_{t-1}) =E(y_{t-1}^2)

$$Var\left(y_t\right) = \frac{w_1 g(x) + w_2\alpha_0}{1 - w_2\left(\alpha_1 + \beta_1\right)} = \sigma_*^2 \quad (5)$$

A forecast of a proposed model is obtained using methods similar to those of GARCH model. Consider the proposed model for volatility in equation (3). By assuming that forecast origin is t, we can write the l-step-ahead forecast for σ_{t+l}^2 as

$$\sigma_{t+1}^2 = w_1 g(x) + w_2\left[\alpha_0 + \alpha_1 y_t^2 + \beta_1\sigma_t^2\right]$$

$$\sigma_t^2(1) = w_1 g(x) + w_2\alpha_0 + w_2\left(\alpha_1 + \beta_1\right)\sigma_t^2$$

The above equation can be written as

$$\sigma_t^2(1) = \sigma_*^2 + w_2\left(\alpha_1 + \beta_1\right)\left(\sigma_t^2 - \sigma_*^2\right) \text{ Where } \sigma_*^2 = \frac{w_1 g(x) + w_2\alpha_0}{1 - w_2\left(\alpha_1 + \beta_1\right)}$$

$$\sigma_{t+2}^2 = w_1 g(x) + w_2\left[\alpha_0 + \alpha_1 y_{t+1}^2 + \beta_1\sigma_{t+1}^2\right]$$

$$\sigma_t^2(2) = w_1 g(x) + w_2\alpha_0 + w_2\left(\alpha_1 + \beta_1\right)\sigma_{t+1}^2$$

The above equation can be written as

$$\sigma_t^2(2) = \sigma_*^2 + \left(\alpha_1 + \beta_1\right)^2\left(\hat{\sigma}_t^2 - \sigma_*^2\right)$$

$$\vdots$$

$$\vdots$$

$$\sigma_t^2(l) = \sigma_*^2 + w_2\left(\alpha_1 + \beta_1\right)^l\left(\hat{\sigma}_t^2 - \sigma_*^2\right) \quad (6)$$

For GARCH(1,2):

$$\sigma_t^2(l) = \sigma_{**}^2 + w_2\left(\alpha_1 + \beta_1 + \beta_2\right)^l\left(\hat{\sigma}_t^2 - \sigma_*^2\right) \quad (7)$$

Where

$$\sigma_{**}^2 = Var\left(y_t\right) = \frac{w_1 g(x) + w_2\alpha_0}{1 - w_2\left(\alpha_1 + \beta_1 + \beta_2\right)}$$

The additional term g(x) in the proposed model for volatility in equation (3) indicates the effect of exogenous factors on the volatility. The exogenous factor g(x) is estimated using the local linear method and it is given by

$$\hat{g}(x) = \frac{\sum_{t=1}^{T} w_t\sigma_t}{\sum_{t=1}^{T} w_t + \frac{1}{T^2}} \quad (8)$$

III. DISCUSSION

The empirical analysis is carried out for the simulated random IBM price series which contains 7955 observations, by considering 939 observations. Simulating stock market prices and returns can be accomplished using a number of techniques. Most commonly, Geometric Brownian Motion (Volatility is assumed to be constant) is used to simulate stock prices. This technique generates price returns which are normally distributed. But return series are generally non-normal. As an alternative technique, in this paper the function fractal is used to generate these non-normal price series. The advantage of this technique is that price returns tend to have volatility that clusters, similar to actual returns. Descriptive statistics for the generated series and log return series are presented in Table 1.

Table 1:Descriptive Statistics for the generated price series and log return series

	Price series	Return Series
Number of observations	939	938
Mean	84.20591	-4.599451e-07
Median	84.20353	0
Minimum	84.19046	-3.11116e-05
Maximum	84.22723	2.4818e-05
Variance	0.0001325601	1.856593e-11
Standard deviation	0.01151348	1.856593e-11
Skewness	0.2388996	-1.224667
Kurtosis	1.638872	11.07642
Jarque Bera	81.418,	2783.8
p-value	< 2.2e-16	< 2.2e-16

Jarque Bera test suggests that the generated series are not normally distributed (p-value is less than 0.000001).The return series are leptokurtic and negatively skewed. ACF and PACF of squared returns and squared log returns suggests GARCH (1, 2) model as an appropriate model for Volatility.

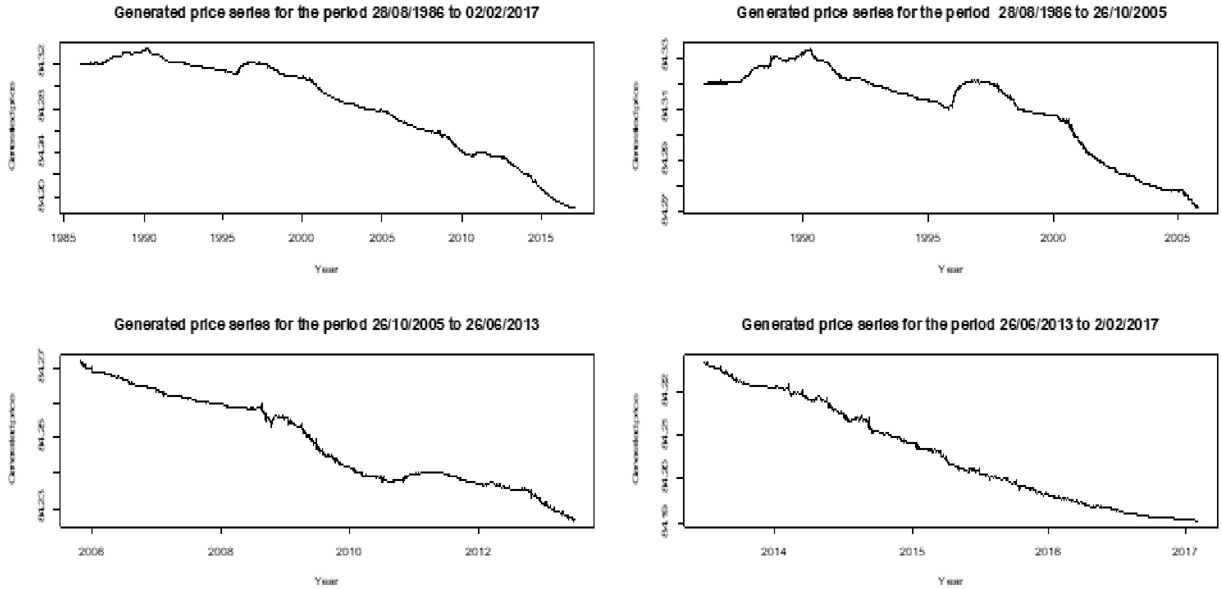

Figure1: Representing the generated price series

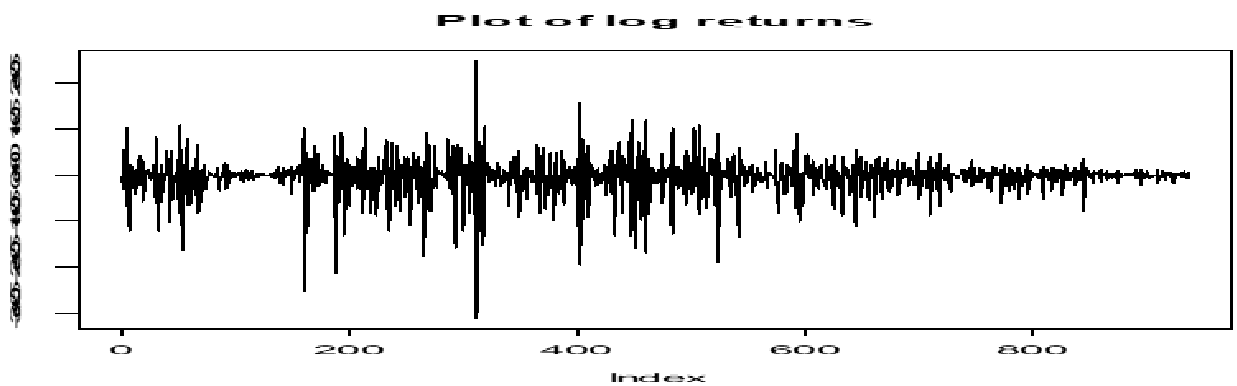

Figure2: Plotting log return series

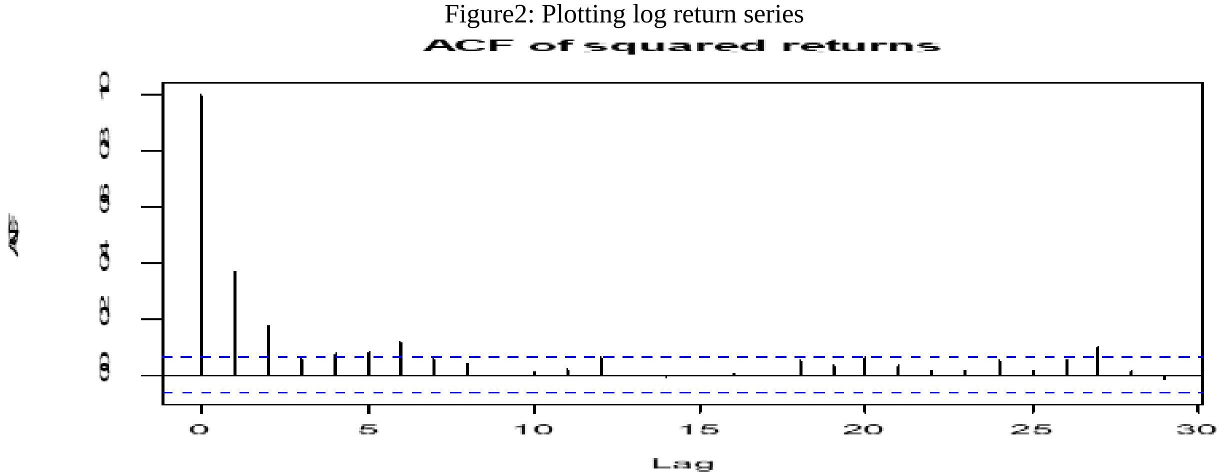

Figure3: Plotting autocorrelation function of squared returns

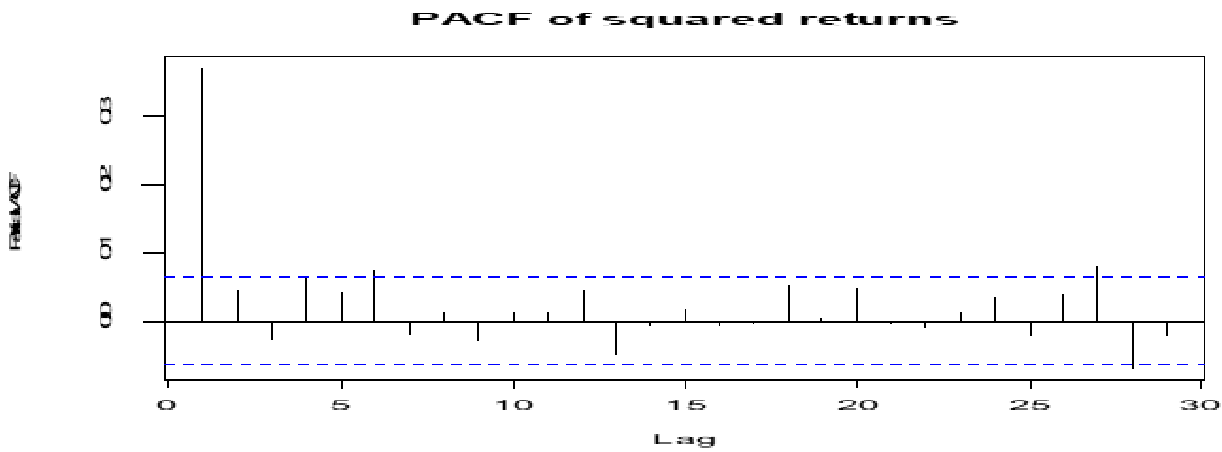

Figure 4: Plotting partial autocorrelation function of squared returns

ARCH Lagrange Multiplier test provides Chi-squared value as 143.36 with12 degrees of freedom and p-value is less than 2.2e-16 which indicates that ARCH effect is significant. To decide the appropriate model we compare the Akaike and MSE for different GARCH (p,q) model.

Table 2: Estimates, Akaike and MSE of GARCH(1, 0), GARCH(1,1) and GARCH(1,2) model parameters for the generated series.

	α_0	α_1	β_1	β_2	Akaike	MSE
GARCH(1,0)	8.423700e-11	5.005332e-02	-	-	-18.94048	2.138835e-10
GARCH(1,1)	1.855853e-14	5.000000e-02	9.000000e-01	-	-21.88145	1.853875e-11
GARCH(1,2)	1.855853e-14	5.000000e-02	4.500000e-01	4.500000e-01	-21.97412	1.853875e-11

GARCH (1, 2) model is appropriate as it corresponds to minimum Akaike and MSE.

Hence the proposed model:

$$\sigma_t^2 = W_1 g(x) + W_2\left(\alpha_0 + \alpha_1 y_{t-1}^2 + \beta_1\sigma_{t-1}^2 + \beta_2\sigma_{t-2}^2\right) \quad (9)$$

The additional term g(x) in the proposed model (9) is used to represent the exogenous factors influences on volatility. By using conditional standard deviation corresponding to residual, the additional term g(x) is estimated by nonparametric local linear Kernel smoothing technique.

Accordingly, we get

$$\hat{g}(x) = \frac{\sum_{i=1}^{T} w_i\,\sigma_i}{\sum_{i=1}^{T} w_i + \frac{1}{T^2}}$$

The weights are computed using

$$W_i = \left(\sum_{i=1}^{n} e_{it}^2\right)^{-1}\left[\sum_{j=1}^{2}\left(\sum_{i=1}^{n} e_{jt}^2\right)^{-1}\right]^{-1} = 1, \quad i = 1,2$$

Where $e_{1t} = s_t^2 - \hat{g}(x)$ and

$$e_{2t} = s_t^2 - \left(\hat{\alpha}_0 + \hat{\alpha}_1 y_{t-1}^2 + \beta_1\sigma_{t-1}^2 + \beta_2\sigma_{t-2}^2\right)$$

Here s_t is the standard deviation computed from the return series.

For estimating volatility 939 observations are used. We forecast Volatility using the GARCH model and the proposed model. It is observed that the proposed model gives a better approximation of the estimated value. The results are presented in Table 3.

Table 3: Estimated and forecasted values using GARCH model and proposed model

SI. N0	GARCH Model		Proposed Model	
	Estimated	forecasted	Estimated	forecasted
1	1.33425e-06	1.255568e-06	1.368637e-06	1.362246e-06
2	1.35675e-06	1.192789e-06	1.377259e-06	1.362025e-06
3	1.38150e-06	1.133150e-06	1.380575e-06	1.361815e-06
4	1.59615e-06	1.076492e-06	1.462829e-06	1.361615e-06
5	1.66455e-06	1.022668e-06	1.482872e-06	1.361425e-06
6	1.48860e-06	9.715343e-07	1.415448e-06	1.361245e-06

7	1.40805e-06	9.229576e-07	1.378413e-06	1.361074e-06
8	1.36305e-06	8.768098e-07	1.355001e-06	1.360911e-06
9	1.31985e-06	8.329693e-07	1.338447e-06	1.360757e-06
10	1.29825e-06	7.913209e-07	1.324001e-06	1.360610e-06
11	1.31355e-06	7.517548e-07	1.323696e-06	1.360471e-06
12	1.33425e-06	7.141671e-07	1.331629e-06	1.360338e-06
13	1.32300e-06	6.784588e-07	1.321150e-06	1.360212e-06
14	1.35810e-06	6.445358e-07	1.334600e-06	1.360093e-06
15	1.38420e-06	6.123091e-07	1.338433e-06	1.359979e-06
16	1.40850e-06	5.816936e-07	1.347745e-06	1.359871e-06
17		5.526090e-07		1.359769e-06
18		5.249785e-07		1.359671e-06
19		4.987296e-07		1.359579e-06
20		4.737932e-07		1.359491e-06
21		4.501035e-07		1.359408e-06
22		4.275984e-07		1.359328e-06
23		4.062185e-07		1.359253e-06
24		3.859076e-07		1.359181e-06
25		3.666122e-07		1.359113e-06
26		3.482816e-07		1.359049e-06
27		3.308676e-07		1.358987e-06
28		3.143242e-07		1.358929e-06
29		2.986080e-07		1.358874e-06
30		2.836776e-07		1.358821e-06

Table 4: Mean square error and Root mean squared error for existing and proposed model

	MSE	RMSE
GARCH model	9.703346e-15	9.850556e-08
Proposed model	9.277941e-15	9.632207e-08

As the Mean squared error for the proposed model is less compared to the existing model, the proposed model gives minimum forecast error compared to the existing models. The Box plot also supports this.

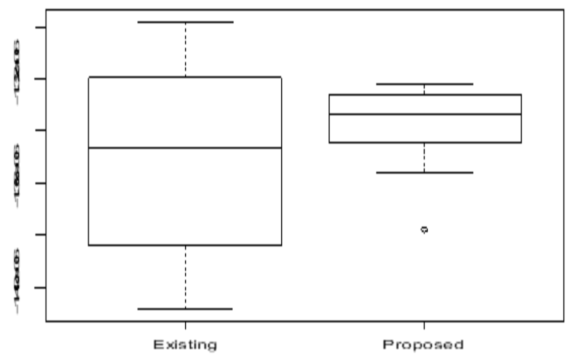

Figure 5: Box Plot for a) Existing (GARCH) b) Proposed

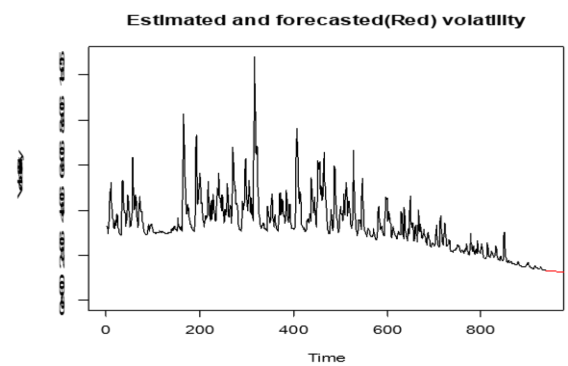

Figure 6: Plotting the estimated and forecasted values from the proposed model

IV. CONCLUSION

The expression for l-step ahead forecast is derived based upon the proposed model which is a weighted average of regression component and GARCH component. We also derived the variance and kurtosis of the proposed model. A simulation study is carried out to examine the performance of a proposed model. It is observed that proposed model gives minimum forecast error compare to the existing GARCH model.

LIST OF ABBREVIATIONS

ACF: Autocorrelation Function
PACF: Partial autocorrelation function

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