Theoretical Estimations of the Temperature - Dependent Mass Spectra of Heavy Quarkonium

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Abstract - **In this work, the time-dependent mass spectra of heavy quarkonia for S, P, and D states have been computed using an alternative potential model based on a combination of Hulthen and Hellmann potentials, studied within the framework of non-relativistic Schrodinger equations. The solution of the radial Schrödinger equation is employed to compute the spectroscopy of heavy quarkonia. The outcome provides satisfying results in comparison with other experimental studies.**

Index Terms - **Heavy Quarkonia spectroscopy, Schrödinger equation, Hulthen and Hellmann Potential.**

INTRODUCTION

Heavy quarkonium consists of heavy quarks and antiquarks and the properties of this heavy quarkonium are well described by the Schrödinger equation, so the solution of this equation with a spherically symmetric potential is of major concern in describing the spectra of quarkonia [1]

A hold type potential, the Cornell potential, is commonly used in modeling the interaction potentials of quark and antiquark systems [2].The exact solution of the Schrödinger equation with this potential is unknown. As a result, the solutions to the Schrödinger equation can only be determined if we know the confining potential for a specific physical system.

Recently, a lot of interest has been directed towards combining two or more potentials in non-relativistic potential regime. [3]. The purpose of combining two or three physical potentials is to broaden the range of applications.

THEORETICAL FRAMEWORK

We use the Nikhiforov–Uvarov (NU) method for heavy quarkonium to analyse the Schrödinger equation with the combination of Hulthen and Hellman potentials. The quark antiquark potential have been formed by combining the Hulthen and Hellman potentials.

$$V_{hhp}(r) = V_h(r) + V_{hp}(r) \qquad (1)$$

Where $V_h(r)$ is the Hulthen potential of the form [4]

$$V_h(r) = -\frac{A_h\, e^{-\alpha r}}{1-e^{-\alpha r}} \qquad (2)$$

where α is a screening parameter and A_h is the strength of the Hulthen potential. The Hellman potential is the superposition of the Yukawa and Coulomb potentials [5].

$$V_{hp}(r) = -\frac{A_c}{r} + \frac{A_y e^{-\alpha r}}{r} \qquad (3)$$

Where A_c and $A_y$are the strengths of the Coulomb and Yukawa potentials, respectively, and r is the distance between two particles, Finally, the potential employed is of the form [6].

$$V_{hhp}(r) = -\frac{A_h\, e^{-\alpha r}}{1-e^{-\alpha r}} - \frac{A_c}{r} + \frac{A_y e^{-\alpha r}}{r} \qquad (4)$$

To make equation temperature dependent The screening parameter is replaced with the Debye mass $m_D(T)$, which is temperature-dependent and vanishes at $T \rightarrow 0$ and we have,

$$V_{hhp}(r,T) = -\frac{A_h\, e^{-m_D(T)r}}{1-e^{-m_D(T)r}} - \frac{A_c}{r} + \frac{A_y e^{-m_D(T)r}}{r} \qquad (5)$$

The exponential term in equation (5) is extended to the third order, making the model potential interact in the quark-antiquark system. By substituting the solution into equation (4), we have

$$V_{hhp}(r,T) = -\frac{\alpha_0}{r} + \alpha_1 r + \alpha_2 r^2 \alpha_3 + \alpha_2 r^2 + \alpha_3 \qquad (6)$$

We have used the approximate solution of the Schrödinger equation [7] with the Hulthen plus Hellman potential in the present study. The energy eigen values are

$$E_{nl} = A_h\left(\frac{1}{2} - \frac{m_D(T)}{4\delta}\right) + A_y m_D\,(T)\left(\frac{3m_D\,(T)}{2\delta} - m_D^2(T) - 1\right) - \frac{\hbar}{8\mu}\left[\frac{\frac{2\mu}{\hbar^2}\left(A_y - A_c + \frac{A_h}{m_D(T)} + \frac{\mu_{m(T)}}{\hbar^2\delta^2}\left(3A_y m_D(T)\right) - \frac{A_h}{2} - \frac{8\mu A_y m_D^3(T)}{3\hbar^2\delta^3}\right)}{n + \frac{1}{2} + \sqrt{\left(l + \frac{1}{2}\right)} + \frac{\mu A_y m_D^2(T)}{\hbar^2\delta^3}\left(1 - \frac{m_D(T)}{\delta}\right) - \frac{\mu A_h m_D(T)}{6\hbar^2\delta^3}}\right] \qquad (7)$$

We calculated the mass spectra of heavy quarkonium systems with the same flavour quark and antiquark, such as charmonium and bottomonium, and used the following relation[8],[9].

$$M = 2m + E_{nl} \quad (8)$$

Thus using eq.(7) into eq.(8),the mass spectra of heavy quarkonia for the potential as

$$M = 2m + A_h\left(\frac{1}{2} - \frac{m_D(T)}{4\delta}\right) + A_y m_D(T)\left(\frac{3m_D(T)}{2\delta} - m_D^2(T) - 1\right)$$
$$- \frac{\hbar}{8\mu}\left[\frac{\frac{2\mu}{\hbar^2}\left(A_y - A_c + \frac{A_h}{m_D(T)} + \frac{\mu_{m(T)}}{\hbar^2\delta^2}\left(3A_y m_D(T)\right) - \frac{A_h}{2} - \frac{8\mu A_y m_D^3(T)}{3\hbar^2\delta^3}\right)}{n + \frac{1}{2} + \sqrt{\left(l + \frac{1}{2}\right)} + \frac{\mu A_y m_D^2(T)}{\hbar^2\delta^3}\left(1 - \frac{m_D(T)}{\delta}\right) - \frac{\mu A_h m_D(T)}{6\hbar^2\delta^3}}\right] \quad (9)$$

Table 1: Charmonium $c\bar{c}$ spectra in GeV

m_c=1.209GeV, $\delta = 0.245$GeV, $\mu = 0.6135$

$A_h = -1.593$GeV $A_y = 20.456$GeV

$A_c = 0.017$GeV $m_D(T) = 1.49$GeV $\hbar = 1$

State	Present	[10]	[11]	[12]
1S	3.097	3.096	3.096	3.096
2S	3.678	3.686	3.686	3.686
1P	3.525	3.433	3.255	3.525
2P	3.769	3.910	3.779	3.773
3S	4.041	3.984	4.040	4.040
4S	4.239	4.150	4.269	4.263
1D	3.504	3.767	3.504	3.770
2D	4.147	-	-	4.159

Table 2: Bottomonium $b\bar{b}$ spectra in GeV

m_b=4.823GeV, $\delta = 0.255$GeV, $\mu = 2.5115$

$A_h = -1.591$GeV $A_y = 9.456$GeV

$A_c = 0.026$Ge, $m_D(T) = 1.54$GeV , $\hbar = 1$

State	Present	[10]	[11]	[12]
1S	9.462	9.460	9.460	9.460
2S	10.024	10.023	10.023	10.023
1P	9.799	9.840	9.619	9.899
2P	10.179	10.160	10.114	10.260
3S	10.355	10.280	10.355	10.355
4S	10.577	10.420	10.567	10.580
1D	9.996	10.140	9.864	10.164
2D	10.297	-	-	-

RESULTS AND DISCUSSION

In this paper, we used the Hulthen and Hillmann potentials to calculate the wave functions and energy eigenvalues of the Schrodinger equation. The potential was made temperature dependent by replacing the screening parameter with Debye mass. We used approximate solutions to calculate heavy meson masses such as charmonium and bottomonium for states ranging from 1S to 2D. Tables 1 and 2 show the computed values of the cc¯ and bb¯. systems. Our results clearly show that the potential model of Hulthen and Hellmann provides a better agreement with experimental data [12] as well as other researchers' work [10],[11] on the masses of cc¯and bb¯. states. Finally, our findings could be useful in future research.

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