Bayesian Inference of Volume Fraction of Vapor Formed During Nucleate Boiling Phenomenon of Water Undergoing Phase Change over Two Heated Cylinders in Tandem Arrangement

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***Abstract*—Fluid flow and heat transfer phenomena through cylinders are common in fluid dynamics. Fluid flow interaction, vortex dynamics, interferences in flow and engineering applications such as compact heat exchangers, nuclear reactor fuel rods and hot-wire anemometry are of major research interest . The main objective is to determine the conditional probability of VF forming around a heated cylinder's surface while it is submerged in water. The flow is calculated using the RANS equations. In the dataset of VF, Bayesian inference was utilized to calculate the prior, likelihood, and posterior probabilities for a given condition. A new value of probability is substituted for the VF at a given region around the heated cylinder. As a result, it's fascinating to observe how Bayesian inference performs on a multivariate dataset. The validity of the findings is made against the literature. The nucleate boiling phenomenon that originates at the surface of a heated cylinder is significantly influenced by cylinder rotation. Using a sample collected using Markov Chain Monte Carlo and Variational Inference methods; the inference methodology was able to detect a statistically significant difference of 0.4 between the means of VFs when the cylinder is at rest and when the same cylinder is rotating.**

***Index Terms*—Bayesian Inference, Film Boiling, k-ε Turbulence Model, RANS, Volume Fraction.**

I. INTRODUCTION

A plethora of industrial applications involving both fluid and heat movement around a cylinder or array of cylinders, as well as a plethora of fluid dynamics difficulties, have provided cause and incentive for the suggested research. Inter-fluid interactions, interference, and vortex involvement are some of the issues that reduce the productivity of a fluid dynamics-intensive sector. Compact heat exchangers, cooling of electronic equipment, turbulent boundary layer generation, hot-wire anemometry, chimney flows, cooling towers, and nuclear reactor fuel rods are some of the major engineering applications where both fluid flow and heat transmission take centre stage. Flow-induced forces and heat-induced effects on flow are both present in these structures. The way the difficulties of phase transition across a heated cylinder or a set of cylinders were explored and the techniques of forecasting fluid dynamics and heat transmission were discovered were truly inspiring.

The findings of M. M. Zdravkovich[1] in 1997 on flow around circular cylinders with various arrangements, the work of S. Mittal et al.[2] on incompressible flow over two-cylinders in tandem and side-by-side arrangement, and the numerical work of Meneghini J. R. et al.[3] on flow interference study in a two-cylinder arrangement are examples of early work emphasizing the importance of cylinder arrangement. J. Li et al.[4] presented their findings on laminar flow past one and two cylinders. In 2003, M. M. Zdravkovich [5] focused on the application side of flow around a circular cylinder, whereas Zhang H. et al.[6] focused on turbulent flow interference in a two-cylinder system. Liu et al.[7] investigated the wake pattern around a pair of circular cylinders in tandem, while Ryu et al.[8] calculated hydrodynamic coefficients for the two cylinders in tandem with a gap between them. Mahir and Altac's[9] work on convective heat transfer around two-cylinder arrangements, Singha and Sinhamahapatra's[10] contribution to low Reynolds number flow around two-cylinder arrangements, Ding[11], Kitagawa and Ohta [12], Deng et al.[13], Ljungkrona et al.[14] in two–cylinder arrangements are some of the truly eye-opening works.

The goal of their research was to see what happens when the space between the cylinders and the flow Reynolds number are varied. Circular cylinders have been used to study both side-by-side and cluster layouts [15]. The literature is replete with computational [16] and experimental [17] investigations on heat and mass transfer during phase change processes from heated surfaces of cylinders held at various spatial arrangements when operated under a varied operational circumstances depending on the engineering applications.

Early scholars [18] performed some of the most noteworthy work on multiphase and two-phase (water to vapour) applications with cylinders as considerations and scenarios. Rotation is one of the operating conditions used by numerous researchers in their studies of cylinder flow [19]. Rotation has been shown to affect flow dynamics, wake characteristics, heat, momentum, and mass transfer. The foundations for advanced research on conjugate heat transfer have been laid further by early but more fundamental activities on vortex dynamics [20], interference effect between cylinders [21], influence of rotation on flow [22], flow behaviour around cylinder [23], and understanding of computational challenges [24].

Researchers have made significant contributions to the examination of a multivariate flow in which essential characteristics such as drag, lift, and moment are necessary [25]. Many published works [26] have used statistical modelling and fundamental applications of statistics to anticipate the behavioral features of flow and the methods of statistical computations are provided. Previous research on the effect of the rate of rotation of a cylinder or two on the phase change process due to nucleate boiling when water flows over (and around) two heated cylinders in a tandem arrangement is sparse, and some gaps (new methods of prediction, correlation of flow variables and their effect) have been identified during the literature survey, and these lacunae in the earlier works have sparked interest to investigate the effect.

The Reynolds-averaged Navier-Stokes equations were used to compute the flow, which was followed by a validation of the key computed variables. The spacing between two cylinders in a tandem arrangement is twice the diameter of the cylinder. When a cylinder is at rest and in rotation, the phase change of water into vapour has been seen. Many essential variables, including the volume proportion of both water and vapour, were created by numerically simulating the phase change process. The dataset obtained from the aforementioned variables was subjected to Bayesian inference.

II. GOVERNING FLOW EQUATIONS

The governing flow equations such as mass, momentum and energy conservation equations of the time-averaged equations can be written in a conservative form as follows:

Mass conservation equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} = 0 \tag{1}$$

Momentum conservation equation:

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial (\rho u_i u_j + p\,\delta_{ij})}{\partial x_j} = \frac{\partial (\tau_{ij} - \rho \overline{u_i u_j})}{\partial x_j} \tag{2}$$

Energy conservation equation:

$$\frac{\partial (\rho e_o)}{\partial t} + \frac{\partial (\rho e_o u_i + p u_i)}{\partial x_i} = \frac{\partial (\tau_{ij} u_j - \rho \overline{u_i u_j} u_j)}{\partial x_i} - \frac{\partial (q_i + C_p \rho \overline{u_i \theta})}{\partial x_i} + \frac{\partial}{\partial x_i}\left[\left(\mu_l + \frac{\mu_i}{\overline{\sigma_k}}\right)\frac{\partial k}{\partial x_i}\right] \tag{3}$$

Turbulence Models

In the case of the Reynolds-averaged Navier-Stokes (RANS) equations, a Standard Turbulence Model (STM) such as the two-equation (k-ε) turbulence model is used:

$$\frac{\partial \rho k}{\partial t} + \frac{\partial \rho u_j k}{\partial x_j} = -\rho \overline{u_j u_i}\frac{\partial u_i}{\partial x_j} + \frac{\partial}{\partial x_j}\left[\left(\mu_l + \frac{C_\mu k^2}{\overline{\sigma_k}\varepsilon}\right)\frac{\partial k}{\partial x_j}\right] - \rho\varepsilon(1 + M_\tau^2) \tag{4}$$

$$\frac{\partial \rho \varepsilon}{\partial t} + \frac{\partial \rho u_j \varepsilon}{\partial x_j} = -C_{\varepsilon l}\rho \overline{u_j u_i}\frac{\partial u_i}{\partial x_j}\frac{\varepsilon}{k} + \frac{\partial}{\partial x_j}\left[\left(\mu_l + \frac{C_\mu k^2}{\overline{\sigma_k}\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right] - f_2 \rho \widetilde{C_{\varepsilon 2}}\frac{\varepsilon}{k}\left[\varepsilon - \nu_l\left(\frac{\partial\sqrt{k}}{\partial n}\right)^2\right] \tag{5}$$

Where, $C_\mu = 0.09$, $C_{\varepsilon 1} = 1.44$, $\overline{\sigma_k} = \sigma_k = 1.4$, $\overline{\sigma_\varepsilon} = \sigma_\varepsilon = 1$ and $\overline{C_{\varepsilon 2}} = C_{\varepsilon 2} = 1.92$

$$f_\mu = exp\left[\frac{-3.41}{\left(1+\frac{R_T}{50}\right)^2}\right];\ R_T = \frac{k^2}{\mu_t \varepsilon};\ f_s = 1 - 0.3\ exp\,(R_T^2)$$

Boundary conditions for epsilon (ε) and k at the wall are

$$\varepsilon_{wall} = \nu_l\left(\frac{\partial\sqrt{k}}{\partial n}\right)^2;\ k_{wall} = 0$$

The turbulent stress components are

$$\rho\overline{u_j u_i} = 2\rho v_t S_{ji} - \frac{2}{3}\delta_{ji}\rho k$$

$$S_{ji} = \frac{1}{2}\left[\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j}\right] - \frac{1}{3}\delta_{ji}\frac{\partial u_j}{\partial x_i}$$

III. METHODOLOGY

The specific problem is to determine how nucleate boiling affects the phase change process when cylinder(s) rotate at the same time, and what could be the possible causes of fluctuation in volume fraction of water (liquid phase) and steam (vapour phase) while a flow is stagnant and in motion. The Reynolds-averaged Navier-Stokes equations were used to compute the flow, which was followed by a validation of the key computed variables. The dataset obtained from the aforementioned variables, computed through numerical simulation, was subjected to Bayesian inference. Fig. 1 shows a basic specification of numerical set-up (in the x-y plane) for two circular cylinders in tandem with boundary conditions. The required dimensions of the fluid domain are given in terms of a cylinder's diameter (D). Both of the cylinders have the same diameter. In the x- and y-direction, the components of velocity (U) are u and v, respectively. T is the temperature at the inlet of the flow (from left to right in the flow domain) that has a free stream value and is equal to the wall temperature stated at the boundary of the solid surfaces of both cylinders.

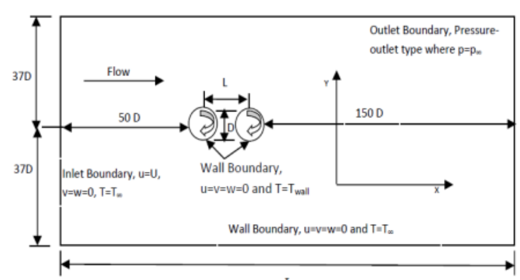

Fig. 1 Numerical model setup in x-y Plane

The validation of a flow over a single circular cylinder (with and without heat flux) is substantial, and the validation of a flow over two circular cylinders is carried out with experimental results for rotating and non-rotating cylinders with varying heat flux published in the literature. Academic version of the Ansys® Fluent was utilized to solve the problem mentioned.

A. Study of Mesh Independence and Convergence

The residuals of the values of variables are used to determine the convergence of the numerical solutions produced from the above-mentioned issue. Convergence occurs in this study when the total residual values are less than 10^{-5}. During the simulation, all of these parameters reached their acceptable steady-state solutions. The mesh resolution has little effect (less than 10%) on the solutions.

B. Boundary Conditions

Boundary conditions for the above set up are as follows:

1 Inlet to the domain: Velocity inlet, $U\infty=1$ m/s ($U\infty=0$ when inlet is a wall)
2 Outlet from the domain: Gauge pressure outlet, P= 0 N/m^2
3 Wall of the domain: No slip wall boundary (top and bottom surface)
4 Cylinder wall surface: Temperature, T=120ºC (for both the cylinders)
5 Cylinder 1: Stationary/rotational wall
6 Cylinder 2: Stationary/rotational wall

IV. RESULTS AND DISCUSSIONS

First, the isolated circular cylinder's mean drag coefficient, mean lift coefficient, and Strouhal numbers were compared to those of other researchers, as shown in Table 1. The flow's mean drag coefficient vs. Re is presented in Fig. 2. The current figures of the mean drag coefficient are reasonably comparable to previously reported statistics. Fig. 3 depicts the path lines and contours of mean flow around an isolated cylinder, whereas Fig. 4 depicts the Strouhal number of vortex shedding from the isolated cylinder.

These are the results for Re = 200. The authors were inspired by the comparison and validation of the approach to investigate the flow variables when there is a flow over two heated circular cylinders. The mean drag coefficient C_D^M, mean lift coefficient C_L^M and Strouhal number S_t are compared with some of the leading references.

Table 1 Comparison of the mean drag coefficient C_D^M, C_L^M and S_t

	C_D^M	C_L^M	S_t
Present Work	1.41	0.692	0.1902
M-M Lui et al (2014)	1.337	0.685	0.1955
B.N. Rajani et al	1.3380	0.4276	0.1936

(2009)			
Wang et al(2009)	-	0.71	0.1950
Zhang et al (2008)	1.34	0.66	0.1970
Linnick and Fascl(2005)	1.34-1.37	0.71	-
Farrani et al (2001)	1.36-1.39	0.71	-
He et al (2000)	1.36	-	0.1978
Data compiled by Zdravkovich (1997)	1.43	-	-
Henderson (1995)	1.34-1.37	-	0.1971

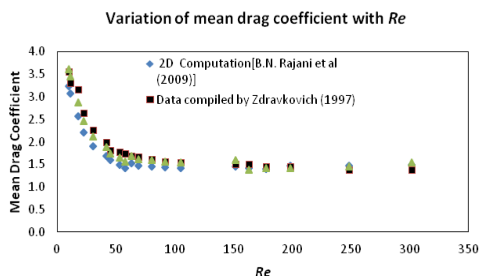

Fig. 2 Comparison of C_D^M for various *Re* for a single cylinder

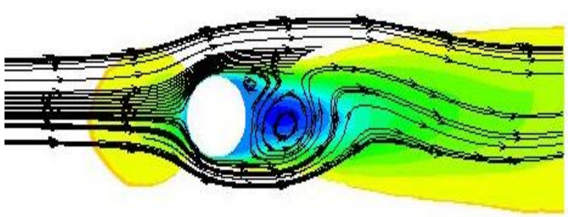

Fig. 3 Instantaneous path lines and mean velocity of flow over an isolated circular cylinder

The impact of the cylinder's rotational effect on the phase change due to nucleate boiling is investigated numerically. The nucleate boiling range is maintained between 100ºC and 130ºC, with the saturation temperature of water at atmospheric pressure being 100ºC. The Eulerian mixing model is used to solve the two-phase issue. Water and vapour are the two phases. For the solution of flow variables, the viscous model (RANS) was combined with a typical (k-ε) turbulence model. The solution approach is based on the SIMPLE algorithm. For all variables involved in the computation, the convergence criterion is absolute and fixed at 10^{-5}. The convergence plot of solution variables in Fig. 4 is based on one of the cases that were conducted.

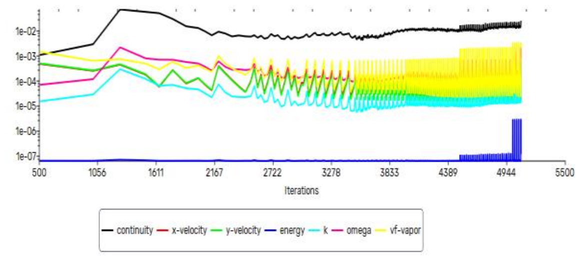

Fig. 4 Convergence plot of solution variables

The impact of rotating rate on the volume fraction (VF) of vapour during the phase transition due to nucleate boiling has been investigated using the scenarios listed below.

A. Two heated (T_s=120ºC) circular cylinders and the water is stagnant (Scenario 1)

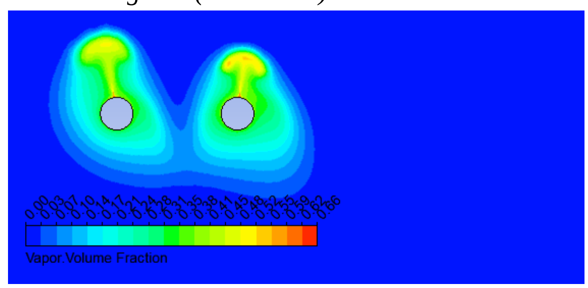

Fig. 5 Contours of volume fraction of vapor for cylinders in stagnant water

The two heated circular cylinders, depicted in Fig. 5, are at rest in stagnant water. The continual exchange of mass, momentum, and energy between the heated surface of each cylinder and the bulk mass of water cause phase change. The highest recorded value of VF is 0.66. The highest VF values are observed at 180 degrees on a cylinder surface, with a difference of around 10% between the highest VF values.

B. Two heated (T_s=120ºC) rotating circular cylinders and the water is stagnant (Scenario 2)

The contour map for a case where both heated cylinders were permitted to rotate at the same rate (314 rad/sec) is given in Fig. 6. Both cylinders needed about the same amount of time to reach their own maximum values of VFs, 0.65 seconds. Both cylinders took nearly the same amount of time to reach their unique maximum values of volume fractions in the same amount of time, 0.65 seconds, as shown in Fig. 7.

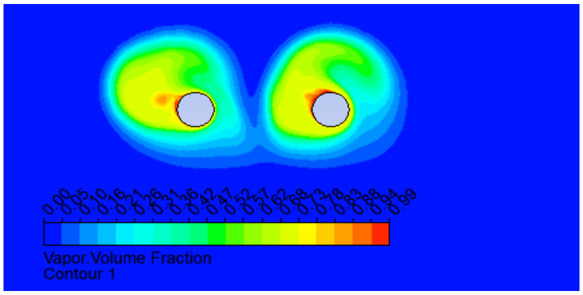

Fig. 6 Contours of VF of vapor round two cylinders (rotating at 314 rad/sec)

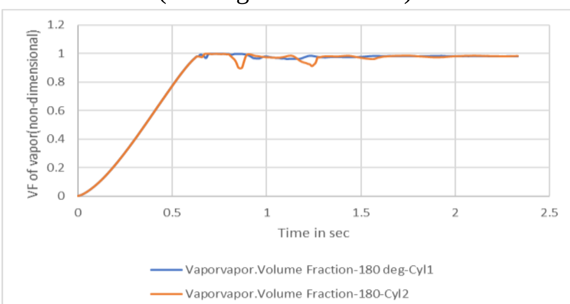

Fig. 7 Comparison of VFs of vapour around the rotating cylinders at location 180 deg

The impact on vapour volume fraction as a result of changing phases from water to vapour is evident in the above studies, and in Fig.8, it is shown that the volume fraction has accelerated from 0.5 when the cylinder was at rest to 0.90 when the rpm is 1000, and then the volume fraction has remained stable in a range of 0.93 to 0.98 for each 1000 rpm increase in rotational speed. The goal of this section of the research has been met because it has been established that rotation has a considerable impact on the total phase change phenomenon and, in particular, on the volume fraction of vapour.

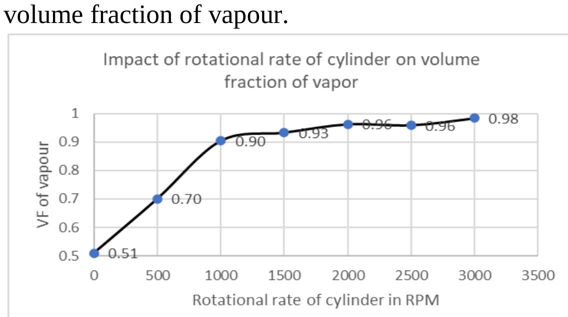

Fig. 8 Impact of rotational rate of cylinder on volume fraction

It was decided to explore conditional probability of volume fraction of vapour using the Bayesian inference method after the numerical simulation, confirmation of the numerical results, and presentation of the study's important conclusions as described above. This is important because the formation of the volume fraction along the cylinder's surface is affected by a number of previously reported dimensions or variables. Because the conditions (in terms of parameters) change, there is a good likelihood that the volume fraction at a certain position of the cylinder will change.

V. BAYESIAN INFERENCE

The foundation of Bayesian inference is conditional probability. An important aspect of Bayesian inference is the establishing models and parameters. Models are the mathematical formulation of the events observed and parameters are the influential factors in the model affecting the data observed[27-29]. The Bayes theorem is

$$P(\theta/D) = \frac{[P(D/\theta)*P(\theta)]}{P(D)} \qquad (8)$$

Where, *P(θ)* is the 'prior' and that is the strength of our belief in *θ* without the data *D*. The 'posterior', *P(θ/D)* is the strength of our belief in θ when the data D have been taken into account. *P(D)* is the evidence and *P(D/θ)* is the likelihood function while *P(θ/D)* is the posterior belief. 'Likelihood', is the probability that the data could be generated through the model with parameter values *θ*. 'Evidence' is the probability of the data according to the model, estimated by summing across all possible values of parameter weighted by the strength of belief in those values of parameter. The computation of posterior of a variable requires a prior, likelihood and evidence as per the Bayes' theorem. This is given below

$$P(x) = \int_{\theta} P(x/\theta)\, P(\theta)\, d\theta \qquad (9)$$

When the dataset has lower dimension the above integration could be computed easily whereas for a higher dimensional dataset the computation is cumbersome. Sometime it is impracticable and hence it is recommended to resort to certain approximation methods which could produce posterior for the variable. When the variable is discrete concluding Bayesian inference of the discrete data is difficult through computation. To overcome such difficulties methods such as Markov Chain Monte Carlo (MCMC) and Variational Inference are used for the sample collected from a population. The sampling method used for MCMC involves collection of different samples from a specific sample distribution and compute basic statistical parameters such as mean and variance to approximate Kernel Density Estimation for samples and then inference is drawn for a variable involved. The basic approach of Bayesian inference is used to get the essential information from the dataset obtained from the numerical simulations. This is achieved by the Bayesian Estimation Supersedes the t-test (BEST) with Markov Chain Monte Carlo (MCMC) method. Fig. 9 shows prior, likelihood and posterior probability density function distribution for a sample dataset of the study. The parameter ($Vf = \theta$) is shown along the horizontal axis and the density is shown along the vertical axis.

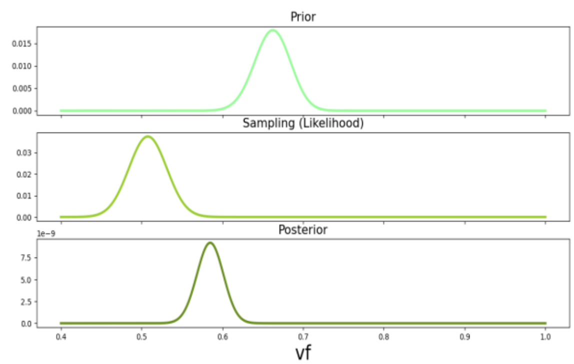

Fig. 9 Bayesian inference: prior, likelihood and posterior plots for a sample dataset

This method could be accessed through the inbuilt package and library of BEST of Rstudio®. Rstudio® is an integrated development environment (IDE) for R, a programming language for statistical computing and graphics. Two important outcomes of the BESTMCMC are the difference of means (0.411) and standard deviation (-0.0046) of two datasets of volume fractions obtained from cylinder at rest and that in motion are shown in the Fig. 10 and Fig. 11, respectively.

Below in Fig. 12, the comparison of two data groups with their posterior predictions when Group 1 data is for cylinder 1 when it is rotating and Group 2 is for the same cylinder when it is at rest. The total number of observations of volume fraction is $N_1=N_2=200$. Two datasets (the parameter is volume fraction) are given as input to the BESTMCMC algorithm to compute the posterior probability of volume fraction. The 'red' bars are indicating data and the 'blue' curves are indicating the model fitted by Bayesian inference into the datasets. The horizontal axis is the parameter value (in this case volume fraction) and the vertical axis is probability density.

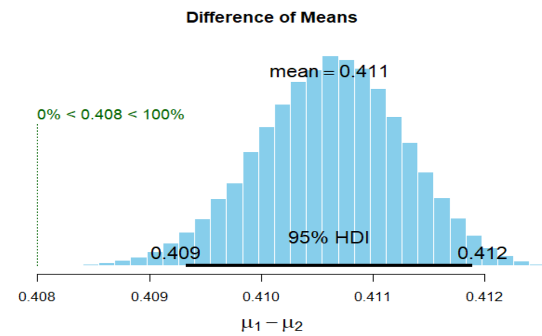

Fig. 10 Posterior probability of difference of means of VF when cylinder is at rest and in rotation

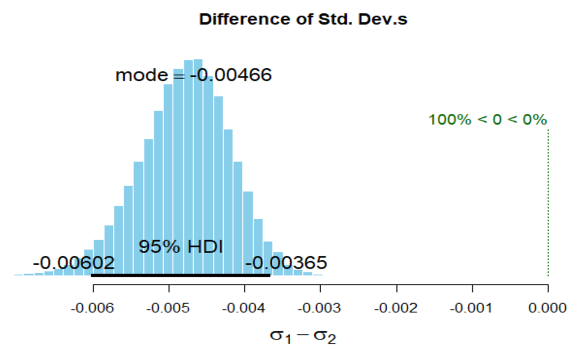

Fig. 11 Posterior probability of difference of standard deviations of VF when cylinder is at rest and in rotation

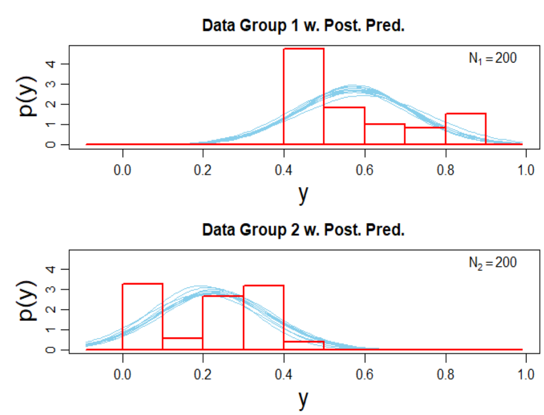

Fig. 12 Comparison of two data groups with their posterior predictions: Cylinder 1 is at rotating (Group 1) and is at rest (Group 2)

VI. CONCLUSION

The Reynolds-averaged Navier-Stokes (RANS) equations were used to calculate the flow. The specific task was to figure out how nucleate boiling impacts the phase change process when cylinders spin at the same time, and what could be the reasons of fluctuations in the volume fraction of water (liquid phase) and steam (vapour phase) while a flow is static and in motion. The validation of a flow over a single circular cylinder (with and without heat flux) was extensive, and the validation of a flow over two circular cylinders was based on known experimental data for rotating and non-rotating cylinders with varied heat flux. The highest VF value ever recorded is 0.66. At 180 degrees on a cylinder surface, the highest VF values are seen, with a difference of roughly 10% between the maximum VF values. For each 1000 rpm increase in rotational speed, the volume fraction increased from 0.5 while the cylinder was at rest to 0.90 when the rpm was 1000, and thereafter stayed steady in a range of 0.93 to 0.98. The phenomenon of nucleate boiling, which begins near the surface of a heated cylinder, has been considerably altered by cylinder rotation.

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