Perfect fluid coupled with electromagnetic field Cosmological Model in Modified Theory of Gravity

R. V. Mapari

Department of Mathematics, Government Vidarbha Institute of Science and Humanities, Amravati-444604, (India)

***Abstract*— In the present study, we have described the solution of the Friedmann-Lemaȋtre-Robertson-Walker (FLRW) space-time with perfect fluid coupled with electromagnetic field in the frame of $\mathbf{f(R,T)}$ gravity. To find the physical parameters we have considered exponential cosmic scale factor. We have described the Hubble parameter and deceleration parameter and we found the universe is expanding and accelerating with increasing rate of expansion. Also, we have discussed the profile of pressure, density and equation of state (EoS) parameter ω.**

***Index Terms*— $\mathbf{f(R,T)}$ gravity, perfect fluid, electromagnetic field, Hubble Parameter.**

I. INTRODUCTION

Because of the concept of accelerated universe expansion, modern cosmology has taken a new turn. This idea was observed by type-Ia supernovae experiments and suggests that the universe is expanding at a faster rate than previously thought [1-6]. There are numerous theoretical models available for observing the behaviour of accelerated expansion and dark energy. The quintessence scalar field models [7, 8], phantom field [9-11], K-essence [12, 13], tachyon field [14, 15], quintom [16, 17], and Chaplygin gas [18, 19] are some examples. We have studied one of the modified theory of gravitation proposed by Harko [20] is $\mathrm{f(R,T)}$ theory. Where R is the Ricci scalar and T is a trace of the stress-energy tensor. $f(\mathrm{R,T})$ theory have been discussed by many researchers in which, N Godani et al. [21] have proposed a different type of function for $f(R,T)$ gravity in the form of $f(\mathrm{R,T}) = \mathrm{R} + \xi \mathrm{T}^{1/2}$, where ξ, R, T are constant, scalar curvature and trace of stress-energy tensor respectively. In which they have studied FRW model and analyzed energy conditions. Some of the authors studied $f(R,T)$ theory for the higher dimensional cosmological model by taking same form of function as we have chosen in this paper [22]. Recently, MSQM solutions in $f(R,T)$ gravity with a cosmological constant discussed in [23, 24]. Bianchi-V model in the presence of $f(R,T)$ gravity using modified holographic Ricci dark energy discussed in [25] and they have found negative value of the deceleration parameter. Like-wise many researchers have analyzed $f(R,T)$ theory with a different energy sources and in different cosmological models [26, 27]. Samanta [28] have studied $f(R,T)$ gravity theory for the Bianchi universe filled with Wet dark fluid. The $f(R,T)$ theory is very useful to explane the late time acceleration and nature of the dark energy and hence discussion is going on from many researchers in $f(R,T)$ theory because of its ability to explain mysterious things in cosmology and astrophysics [29-37].

FRW Universe is homogenous and isotropic for the false vacuum model, Zel'dovich fluid and radiation dominated fluid [38]. The solutions of field equations of FRW with dark energy in the form of modified Chaplygin gas investigated in [39]. In teleparallel gravity, researchers have explored the barotropic bulk viscous FRW Universe [40]. The higher dimensional FRW model is expanding and free of initial singularity explored in[41]. The FRW model can be used to describe the dark energy-dominated universe discussed in [42]. Some authors have noticed the universe has expansion and anisotropic nature [43]. Sahoo et al. [44] have investigated the background cosmology of an isotropic flat Universe. In FRW model, the dust and dark energy exhibited a deceleration to acceleration transition [45]. Some researchers have studied the flat FRW universe in the frame of fractal cosmology [46]. The FRW model accurately depicts the Universe's current accelerating and expanding situation and when filled with ordinary matter that obeys the energy constraints discussed in [47]. We have arrange this paper as: Section I is about the introduction, section II for the metric and field equations of $f(R,T)$ theory. In section III, we have found solution of the field equations. Section IV is about the observations and discussion from graphs and in section V, we have concluded our work.

II. FIELD EQUATIONS OF $f(R,T)$ THEORY

We considered the spatially homogeneous and flat FLRW metric given in the form,

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2) \quad (1)$$

Where, $a(t)$ is cosmic scale factor.

The field equations of $f(R,T)$ gravity are the formalism of Hilbert-Einstein variational principle as follows,

$$S = \frac{1}{2\kappa}\int f(R,T)\sqrt{-g}d^4x + \int \mathcal{L}_m\sqrt{-g}d^4x \quad (2)$$

The gravitational field equations for $f(R,T)$ gravity is given by,

$$f_R(R,T)R_{ij} - \frac{1}{2}f(R,T)g_{ij} + (g_{ij}\nabla^i\nabla_j - \nabla_i\nabla_j)f_R(R,T) = \kappa\overleftrightarrow{T_{ij}} - f_T(R,T)\overleftrightarrow{T_{ij}} - f_T(R,T)\theta_{ij} \quad (3)$$

Where,

$$\theta_{ij} = g^{\alpha\beta}\frac{\partial T_{\alpha\beta}}{\partial g^{ij}},\ f_R(R,T) = \frac{\partial f(R,T)}{\partial R}\ and\ f_T(R,T) = \frac{\partial f(R,T)}{\partial T} \quad (4)$$

∇_i is the covariant derivative. We choose a system for $\kappa = \frac{8\pi G}{c^4} = 1$,

Where G is the Newtonian Gravitational constant and c is the speed of light in vacuum. $\overleftrightarrow{T_{ij}}$ is the standard matter energy-momentum tensor derived from the Lagrangian $\mathcal{L}_m$. We choose matter Lagrangian only for perfect fluid distribution as $\mathcal{L}_m = -p$. The functional $f(R,T)$ have many choices like,

$$f(R,T) = \begin{cases} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R)f_3(T) \end{cases} \quad (5)$$

We assumed the model $f(R,T) = R + 2f(T)$. Where $f(T)$ is an arbitrary function of the trace of the energy-momentum tensor and we choose $f(T) = \lambda T$, λ be a constant. Now the relativistic field equations of $f(R,T)$ gravity theory for perfect fluid coupled with electromagnetic fields are,

$$R_{ij} - \frac{1}{2}Rg_{ij} = (\kappa + 2\lambda)\overleftrightarrow{T_{ij}} + \lambda(\overleftrightarrow{T} + 2p)g_{ij} \quad (6)$$

Where,

$$\overleftrightarrow{T_{ij}} = T_{ij} + E_{ij} \quad (7)$$

In the Eqn. (7),

T_{ij} is the energy-momentum tensor for the perfect fluid distribution and it is given by,

$$T_{ij} = (p + \rho)u_iu_j - g_{ij}p \quad (8)$$

Together with

$$g^{ij}u_iu_j = 1 \quad (9)$$

Where, p, ρ, u^i are internal pressure, rest mass density and four-velocity vectors of the distribution respectively.

E_{ij} is the electromagnetic energy-momentum tensor, given by,

$$E_{ij} = \frac{1}{4\pi}[F_{i\alpha}F_j^\alpha - \frac{1}{4}g_{ij}F_{\alpha\beta}F^{\alpha\beta}] \quad (10)$$

Here F_{ij} is the electromagnetic field tensor obtained from the four potential ϕ_i,

$$F_{ij} = \phi_{i,j} - \phi_{j,i} \quad (11)$$

$$F^{ij}_{;j} = -4\pi\rho_c u^i \quad (12)$$

In the co-moving transformation system the magnetic field is considered along z-axis only, therefore non-vanishing components of electromagnetic fields F_{ij} are only F_{12} and F_{21}. Also, we have electromagnetic field tensor is anti-symmetric.

The first set of Maxwell's equation,

$$F_{ij,k} + F_{jk,i} + F_{ki,j} = 0 \quad (13)$$

Gives,

$$F_{12} = constant = M \quad (14)$$

Now from Eqn. (7), (8), (10) for the metric (1), we have

$$E_1^1 = E_2^2 = -E_3^3 = -E_4^4 = \frac{M^2}{8\pi a^4} \quad (15)$$

$$T_1^1 = T_2^2 = T_3^3 = -p\ ;\ T_4^4 = \rho \quad (16)$$

By using Eqn. Eqn. (15) and (16), the field Eqns.(6) for the Eqn.(1) can be reduced as,

$$2\dot{H} + 3H^2 = (1+2\lambda)\left(p - \frac{M^2}{8\pi a^4}\right) - (p+\rho)\lambda \quad (17)$$

$$3H^2 = (1+2\lambda)\left(\frac{M^2}{8\pi a^4} - \rho\right) - (p+\rho)\lambda \quad (18)$$

Here dot for differentiation w.r.t. time t.

From Eqn.(17) and Eqn.(18), pressure p, density ρ and EoS parameter $\omega = p/\rho$ reads as,

$$p = \frac{2}{(1+2\lambda)}\left[\frac{(1+3\lambda)}{(1+2\lambda)}\dot{H} + \frac{3H^2}{2} + \frac{(1+3\lambda)M^2}{8\pi a^4}\right] \quad (19)$$

$$\rho = \frac{1}{(1+2\lambda)}\left[-2\lambda\dot{H} + 3H^2 + \frac{M^2}{8\pi a^4}\right] \quad (20)$$

$$\omega = 2\frac{\frac{(1+3\lambda)}{(1+2\lambda)}\dot{H} + \frac{3H^2}{2} + \frac{(1+3\lambda)M^2}{8\pi a^4}}{-2\lambda\dot{H} + 3H^2 + \frac{M^2}{8\pi a^4}} \quad (21)$$

The nature of the above physical parameters p, ρ and ω is depend on Hubble parameter and λ.

III. SOLUTION OF FIELD EQUATIONS:

We are interested to check dynamical behavior of the universe, hence we have found solution of field equations by considering cosmic scale factor with an exponential evolution which was discussed in [48, 49] and given by,

$$a(t) = D\, exp\left(\alpha \frac{t^2}{{t_*}^2}\right) \qquad (22)$$

Where, t_* is arbitrary time, $D > 0$ and $\alpha > 0$ are constants.

It is noted that the scale factor $a(t) = D$ at $t = 0$. It is also observed that $\dot{a} < 0$ for $t < 0$, and $\dot{a} > 0$ for $t > 0$ and $\dot{a} = 0$ for $t = 0$.

Hubble parameter is given by,

$$H = \frac{\dot{a}}{a} \qquad (23)$$

From Eqn.(22) and Eqn(23) we have,

$$H = \frac{2\alpha t}{{t_*}^2} \qquad (24)$$

Scalar expansion,

$$\theta = 3H \qquad (25)$$

Eqn.(24) and (25) gives,

$$\theta = \frac{6\alpha t}{{t_*}^2} \qquad (26)$$

Spatial Volume,

$$V = a^3(t) \qquad (27)$$

Eqn.(22) and (27) gives,

$$V = D^3 exp\left(3\alpha \frac{t^2}{{t_*}^2}\right) \qquad (28)$$

The deceleration parameter obtained from,

$$q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right) \qquad (29)$$

From Eqn.(24) and (29) we have,

$$q = -1 + \frac{{t_*}^2}{2\alpha} logt \qquad (30)$$

By using Eqn.(22) and (24) in Eqn. (19), (20) and (21), the pressure, density and EoS parameter obtained respectively as,

$$p = \frac{12\alpha^2 t^2}{(1+2\lambda){t_*}^4} + \frac{(1+4\lambda)M^2}{8\pi(1+2\lambda)D^4} exp\left(-4\alpha \frac{t^2}{{t_*}^2}\right) + \frac{4\alpha(1+3\lambda)}{(1+2\lambda)^2 {t_*}^2} \qquad (31)$$

$$\rho = \frac{M^2}{8\pi(1+2\lambda)D^4} exp\left(-4\alpha \frac{t^2}{{t_*}^2}\right) - \frac{12\alpha^2 t^2}{(1+2\lambda){t_*}^4} - \frac{4\alpha\lambda}{(1+2\lambda)^2 {t_*}^2} \qquad (32)$$

$$\omega = \frac{8\pi D^4[\alpha^2 t^2(1+2\lambda) + 4\alpha(1+3\lambda){t_*}^2] exp\left(4\alpha\frac{t^2}{{t_*}^2}\right) + M^2 {t_*}^4(1+2\lambda)(1+4\lambda)}{8\pi D^4[4\alpha\lambda {t_*}^2 - 12\alpha^2 t^2(1+2\lambda)] exp\left(4\alpha\frac{t^2}{{t_*}^2}\right) + M^2 {t_*}^4(1+2\lambda)} \qquad (33)$$

EoS parameter ω can be obtained from Eqn.(33) at $t = 0$,

$$\omega|_{t=0} = \frac{d_1(1+3\lambda) + d_2(1+2\lambda)(1+4\lambda)}{d_1\lambda + d_2(1+2\lambda)} \qquad (34)$$

Where, $d_1 = 32\pi\alpha D^4 {t_*}^2$ and $d_2 = M^2 {t_*}^2$.

For $\omega < -1$, that is for phantom phase, λ must have restrictions. We have restricted $\lambda < -1.185$ for $\omega < -1$.

IV. OBSERVATION AND DISCUSSION

- We have considered exponential cosmic scale factor for the solution of the field equations, which is increasing function of cosmic time t can be observed from Figure 1.
- We have obtained the hubble parameter is positive and increasing gradually with respect to cosmic time t from Figure 2. Positive value of Hubble parameter indicates the universe is expanding.

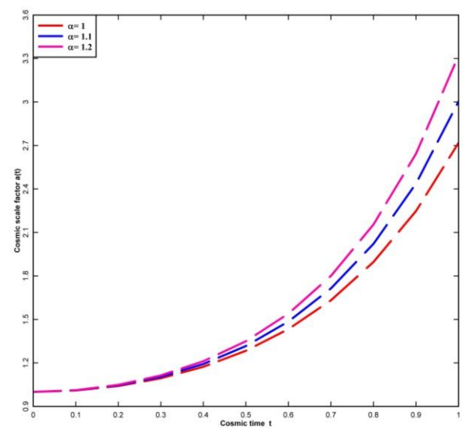

Figure 1: Cosmic scale factor Vs cosmic time t for $D = t_* = 1$

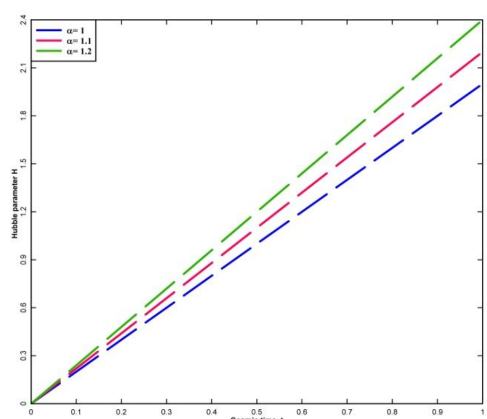

Figure 2: Hubble parameter Vs cosmic time t for $D = t_* = 1$

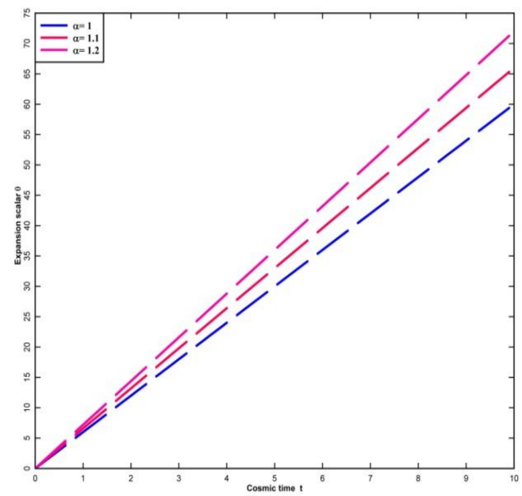

Figure 3: Expansion scalar Vs cosmic time t for $D = t_* = 1$

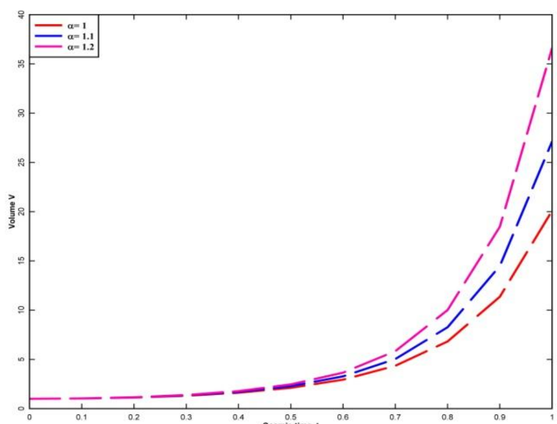

Figure 4: Volume Vs cosmic time t for $D = t_* = 1$

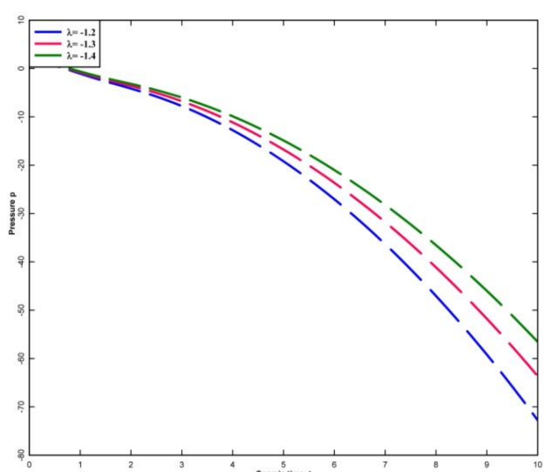

Figure 5: Pressure Vs cosmic time t with $\lambda < -1.185$.

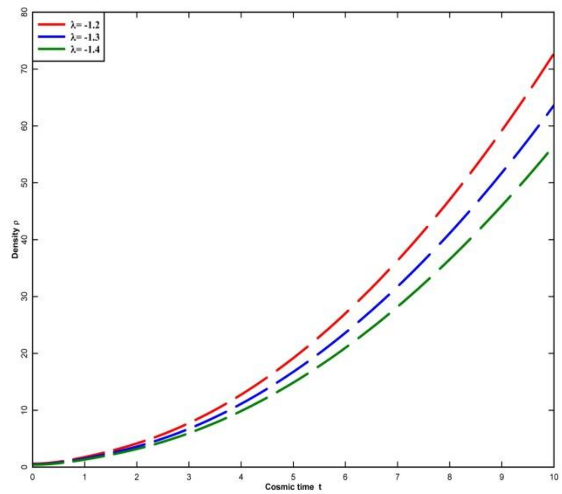

Figure 6: Density Vs cosmic time t with $\lambda < -1.185$.

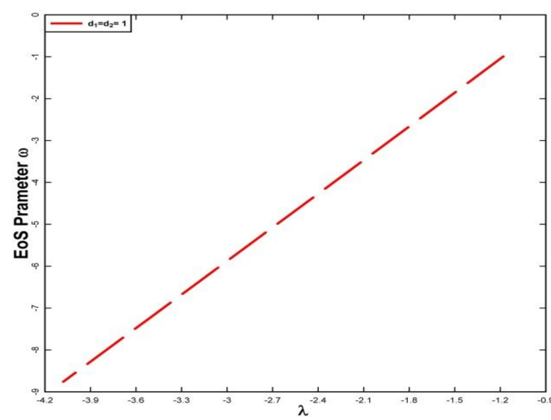

Figure 7: EoS parameter Vs λ at $t = 0$.

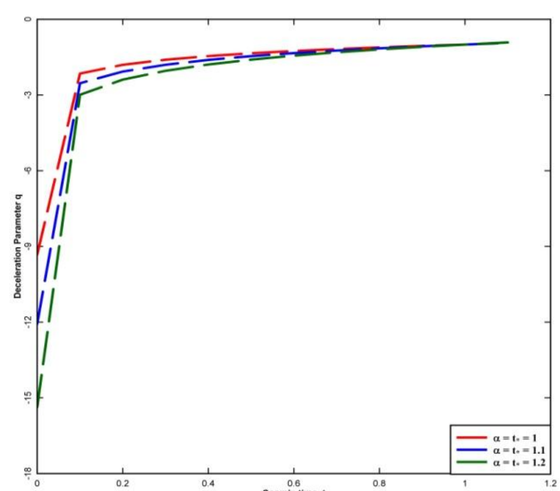

Figure 8: Deceleration parameter Vs cosmic time t.

- From Figure 3, we have observed the rate of expansion is increasing as time increases and from Figure 4, volume of the universe is also increasing with the cosmic time t.
- We have described the profile of the pressure and density. From Figure 5, it is noted that the pressure is negative, which indicates the presence of the dark energy. It goes to large negative value for large time t. Density is positive and increases with increase in time t Figure 6.
- Lastly, we have observed the deceleration parameter and EoS parameter. We have analysed the EoS parameter ω with constraining $\lambda < -1.185$ for the phantom phase ($\omega < -1$). From Figure 7, it can be clear that for $\lambda < -1.185$, we have $\omega < -1$. The presence of phantom energy can indeed cause the universe's expansion to accelerate to the point where the Big Rip occurs. We found the deceleration parameter is negative (Figure 8) which shows that the universe is accelerating.
- For $\frac{t_*}{2\alpha} = 1$, the deceleration parameter $q < 0$ as $t < 1$ and $q > 0$ as $t > 1$. Hence, we found the universe is accelerating for $t < 1$ and then decelerating for $t > 1$. It indicates the transition phase of the universe.

V. CONCLUSION

We have explored the solution of the FLRW space-time with perfect fluid coupled with electromagnetic field in the frame of $f(R,T)$ gravity. For that we have considered exponential scale factor which is increasing exponentially with respect to time t. The Hubble parameter, which indicates Expansion for $H > 0$ and contraction for $H < 0$. We have found expanding universe as positive value of Hubble parameter. Also the rate of expansion is increasing in nature. As a results, a volume of the universe is increasing with increase in cosmic time t which is good agreement with [50]. As we know modified theories are one of the candidate for the dark energy. We have found the existence of the dark energy because the pressure is negative in nature. Density is increasing as time increases. The deceleration parameter which indicates acceleration for $q < 0$ and deceleration for $q > 0$. We have observed the the universe is accelerating as well as decelerating in particular time interval. Hence, we have found the transition phase of the universe. Also, we have analyzed the EoS parameter ω particulary for phantom phase and constrained λ accordingly. We have restricted $\lambda < -1.185$ for $\omega < -1$.

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