Bianchi Universe in $F(R, T)$ Gravity with Time Varying Deceleration Parameter

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***Abstract*—The main purpose of this manuscript is to investigate LRS Bianchi type I metric in the presence of perfect fluid in the context of $f(R, T)$ gravity. In order to obtain deterministic solution of the field equations we determine the LRS Bianchi-I space-time by considering hybrid expansion law (HEL) for the average scale factor that yields power-law and exponential-law cosmologies. We find that the HEL Universe within the framework of LRS Bianchi type-I space-time is anisotropic at the early stages of evolution and becomes isotropic at late times. A Cosmological model, in this theory, is presented and some kinematical properties of the model are also studied.**

***Keywords*—LRS Bianchi model, perfect fluid, $f(R, T)$ Gravity, Deceleration Parameter, Hubble Parameter, Hybrid Expansion Law, Energy Conditions.**

I. INTRODUCTION

The theory of dark energy have taken special status in recent times. The dark energy is an exotic energy component with negative pressure, which explain many observations well and solves some major problems of standard cosmology. The second possibility is by assuming that the general relativity breaks down at large scales and the gravitational field can be described by a more general action. Observation plays a major role in modern cosmology. The advent of new technologies in observations enforces the theorists to rethink on the formulation of the gravitational theories time to time. The concept of decelerating expansion of the Universe had to drop by the theorists with the observation of type Ia supernovae in 1998. Since then CMB, BAO, SDSS and many more observations provide evidences in support of the accelerating expansion of the Universe. So, it is very important to take care of the observational results while building a theoretical model of the Universe. The accelerating expansion of the Universe is an important feature of present day cosmology. The Einstein field equations (EFEs) always lead to a decelerating expansion with the normal matter component in the Universe. The accelerating expansion can be described either by supplying some extra component in the energy momentum tensor part in the field equations or by doing some modifications in the geometrical part. With these principles, the past few years of research produced a plethora of cosmological models of the Universe explaining the accelerating expansion.

One of the interesting and prospective versions of modified gravity theories is the $f(R, T)$ gravity proposed by Harko et al. [1]. The $f(R, T)$ gravity models can explain the late time cosmic accelerated expansion of the Universe. Recently, Sahoo et al. [2-4] have studied cosmological models in $f(R, T)$ gravity in different Bianchi type space-times. The extraordinary phenomena of f(R, T) gravity may provide some significance signatures and effects which could distinguish and discriminate between various gravitational models. Therefore, this theory has attracted many researchers to explore different aspects of cosmology and astrophysics in isotropic and as well as in anisotropic space-times (See for example Jamil et al. [5]; Reddy et al. [6] ; Azizi [7]; Alvarenga et al. [8]; Sharif et al. [9]; Chakraborty [10]; Houndjo et al. [11]; Pasqua et al.[12]; Ram and Priyanka [13]; Singh and Singh [14]; Baffou et al. [15]; Santos and Ferst [16]; Noureen et al. [17]; Shamir [18]; Singh and Singh [19]; Alhamzawi and Alhamzawi[20]; Yousaf et al. [21]; Alves et al. [22]; Zubair et al. [23]; Sofuoglu [24]; Momeni et al. [25]; Das et al. [26]; Salehi and Aftabi [27]; Singh and Beesham [28]; Srivastava and Singh [29]; Sharif and Anwar [30]; Tiwari and Beesham [31]; Shabani et al. [32]; Rajabi and Nozari [33]; Baffou et al. [34]; Lobato et al. [35]; Tretyakov [36]; Elizalde and Khurshudyan [37]; Ordines and Carlson [38]; Maurya and Tello-Ortizb[39]; Esmaeili [40] and references therein). In recent years Bianchi universes are playing important role in observational cosmology. In cosmology, the late-time accelerated expansion of the universe has been a major subject of investigation. It seems attractive to explain the phenomena of dark energy and late time acceleration. Hence, modified theories of gravity is attracting currently several researchers to investigate dark energy (DE) model. Among these geometrically modified theories, $f(R,T)$ theory has attracted a lot of attention of many cosmologists and astrophysicists in recent times because of its ability to explain several issues in cosmology and astrophysics [41, 42].

In Ref.[43], we studied power-law (a(t) $\propto t^{\alpha}$) and exponential-law (a(t) $\propto e^{\beta t}$) cosmologies within the framework of Bianchi-V models with non-interacting matter fluid and DE components. Thereafter in another study [44], we investigated various features of power-law cosmology by constraining it with a host of observational data and found that such a cosmology is not a complete package for cosmological purposes. In fact, power-law and exponential law cosmologies can be used only to describe epoch based evolution of the Universe because of the constancy of deceleration parameter. For instance, these cosmologies do not exhibit the transition of the Universe from deceleration to acceleration. In a recent paper [45], we considered the following anastz for the scale factor of the Universe:

$$a(t) = t^{\alpha} e^{\beta t}$$

Various researchers have studied locally rotational symmetric (LRS) Bianchi type models. An inhomogeneous LRS Bianchi type models investigated by [46-49].

The main goal of this paper is to investigate the perfect fluid anisotropic universe in the framework f(R, T) theory of gravity. The article is methodized as follows: In Section II we provide a brief review of f(R, T) gravity. In Section III we present the Metric and field equations. In Section IV, we apply energy conditions in our solution. Finally, in section V we further discuss our results.

II. A BRIEF REVIEW OF $f(R,T)$ GRAVITY FORMALISM

The $f(R,T)$ theory of gravity is one of the important modifications of General Relativity (GR). In $f(R,T)$theory, the gravitational Lagrangian is given by an arbitrary function of the Ricci scalar R and of the trace of T of the stress energy tensor. The action for $f(R,T)$ theory of gravity is given by [1]

$$S = \int \left[\frac{1}{2\kappa} f(R,T) + L_m\right] \sqrt{-g}\, d^4x, \qquad (1)$$

where $f(R,T)$is an arbitrary function of the Ricci scalar R, T is the stress energy tensor T_{ij} of matter andL_m is the matter Lagrangian density. It would be worthwhile to mention that if we replace $f(R,T)$ with $f(R)$, we get the action for $f(R)$ gravity and the displacement of $f(R,T)$with R leads to the action of GR. g is the determinant of the metric tensor g_{ij} .The $f(R,T)$ gravity field equations are obtained by varying the action S in equation (1) with respect to the metric tensor

$$f_R(R,T)R_{ij} - \frac{1}{2} f(R,T) g_{ij} - \left(\nabla_i \nabla_j - g_{ij}\Pi\right) f_R(R,T) = \kappa T_{ij} - f_T(R,T)\left(T_{ij} - \frac{1}{3}\Theta_{ij}\right). \qquad (2)$$

where ∇_i being the covariant derivative and $\Pi = \nabla^i \nabla_i, f_R = \frac{\partial f(R,T)}{\partial R}$and

$$f_T = \frac{\partial f(R,T)}{\partial T} \Theta_{ij} = g^{\alpha\beta} \frac{\delta T_{\alpha\beta}}{\delta g^{ij}}$$

Contraction of Eq. (2) yields

$$f_R(R,T)R + 3\Pi f_R(R,T) - 2f(R,T) = \kappa T - f_T(R,T)(T + \Theta) \qquad (3)$$

where $\Theta = g^{ij}\Theta_{ij}$, Equation (3) gives a relation between Ricci scalar R and the trace T of energy momentum tensor. Using the matter Lagrangian L_m, the standard matter energy-momentum tensor is derived as

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij} \qquad (4)$$

Satisfying the EoS

$$p = \omega\rho \qquad (5)$$

where $u^i = (0,0,0,1)$ is the four velocity vector in co-moving coordinate system. ρ, p are energy density and pressure of the fluid, respectively, and the matter Lagrangian can be taken as $L_m = -p$ since there is no unique definition of the matter Lagrangian. Which gives

$$\Theta_{ij} = -2T_{ij} - p g_{ij} \qquad (6)$$

It is evident from equation (2) that the physical nature of the matter field decides the behavior of the field equations of$f(R,T)$theory of gravity. Therefore different choice of the matter source will lead to different cosmological models in$f(R,T)$gravity. In other words, one can construct viable cosmological models with different choices of the functional $f(R,T)$. However, Harko et al. [1] have constructed three possible models by considering the functional $f(R,T)$ to be either of

$f(R,T) = R + 2f(T)$ or $f(R,T) = f_1(R) + f_2(T)$ or $f(R,T) = f_1(R) + f_2(R)f_3(T)$

In this paper, we consider the first class only to explore the exact LRS Bianchi I solutions.

For the model $f(R,T) = R + 2f(T)$, the field equations become

$$R_{ij} - \frac{1}{2}Rg_{ij} = \kappa T_{ij} + 2f_T(T)T_{ij} + [f(T) + 2pf_T(T)]g_{ij}\,. \quad (7)$$

For the sake of simplicity, we use a natural system of units $(G = 1, c = 1)$ and $f(T) = \lambda T$, where λ is arbitrary constant. In this case, the gravitational field equations take a form similar to GR,

$$R_{ij} - \frac{1}{2}Rg_{ij} - \lambda(T + 2p)g_{ij} = (8\pi + 2\lambda)T_{ij}\,. \quad (8)$$

III.METRIC AND FIELD EQUATIONS FOR

$f(R,T) = R + 2f(T)$

The Locally Rotationally Symmetric (LRS) Bianchi Type I line element can be written as

$$ds^2 = dt^2 - A^2dx^2 - B^2(dy^2 + dz^2) \quad (9)$$

where A, B are the metric functions of cosmic time t only

The average scale factor a, spatial volume V, scalar expansion θ for metric (9) are

$$a = (AB^2)^{1/3}, V = a^3 = AB^2, \theta = u^i_{;i} = \frac{\dot{A}}{A} + \frac{2\dot{B}}{B} \quad (10)$$

The average Hubble parameter H is given in the form

$$3H = \frac{\dot{A}}{A} + \frac{2\dot{B}}{B} \quad (11)$$

In view of equation (4) for LRS Bianchi Type I space time (eq.(9))

The field equation (8) lead to

$$\frac{2\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} = (8\pi + 3\lambda)\rho - \lambda p \quad (12)$$

$$-\frac{2\ddot{B}}{B} - \frac{\dot{B}^2}{B^2} = (8\pi + 3\lambda)p - \lambda\rho \quad (13)$$

$$-\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} - \frac{\dot{A}\dot{B}}{AB} = (8\pi + 3\lambda)p - \lambda\rho \quad (14)$$

From equations (13) and (14), we have

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{c_1}{AB^2} \quad (15)$$

where c_1 is constant of integration. Again integrating

$$\frac{A}{B} = c_2 exp\left[c_1 \int \frac{\mathrm{dt}}{a^3}\right] \quad (16)$$

where c_2 is integration constant.

Using the above value in equation (10), we can get

$$A = ac_2^{\frac{2}{3}} exp\left[\frac{2c_1}{3}\int a^{-3}dt\right] \quad (17)$$

$$B = ac_2^{-\frac{1}{3}} exp\left[\frac{-c_1}{3}\int a^{-3}dt\right] \quad (18)$$

IV. SOLUTION OF THE FIELD EQUATION

In order to obtain an explicit solution to the field equations, a great number of parametrization schemes have been investigated in the literature with the requirement of their theoretical consistency and observational viability. In particular, we can quote the power-law expansion $(a(t)\, \alpha\, t^{\alpha})$ and exponential law$(a(t)\, \alpha\, e^{\beta t})$, with α,β being nonnegative constants. We require a supplementary constrain equation for the consistency of the system. This one extra constraint can be chosen by assuming linear relationship between two variables in the field equations or we can parametrize any particular variable. In a recent paper [48], here we consider a simple anastz which is obtained by multiplying the power and exponential laws, called hybrid expansion law (HEL). It provides an elegant description of the transition from deceleration to accelerated cosmic expansion. We referred this generalized form of scale factor to as the Hybrid Expansion Law (HEL) being the mixture of power-law and exponential-law cosmologies. One may immediately observe that the HEL leads to the power-law cosmology for β = 0 and to the exponential-law cosmology for α = 0. In other words, the power-law and exponential-law cosmologies are the special cases of the HEL cosmology. Therefore, the case α > 0 and β > 0 leads to a new cosmology arising from the HEL.

The overall average scale factor $a(t)$, the HEL in the form

$$a(t) = t^{\alpha} e^{\beta t} \quad (19)$$

The average scale factor rise monotonically with respect to cosmic time t and the universe expands with acceleration for large values of the average scale factor. The average scale factor diverges $t \to \infty$. It is further observed that average scale factor is zero at the initial epoch $t = 0$. hence the model has point type singularity

So that the Hubble and deceleration parameters are

$$H = \frac{(\beta t + \alpha)}{t} \quad (20)$$

$$q = -1 + \frac{\alpha}{(\beta t+\alpha)^2} \quad (21)$$

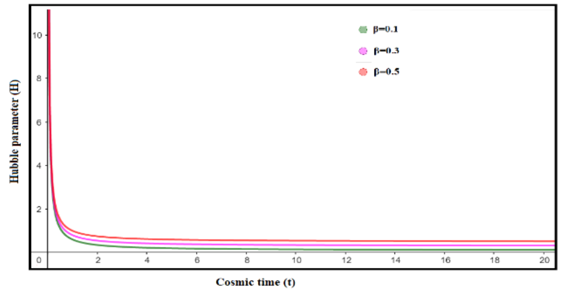


Fig.1 Behavior of Hubble parameter Hversus cosmic time (t)

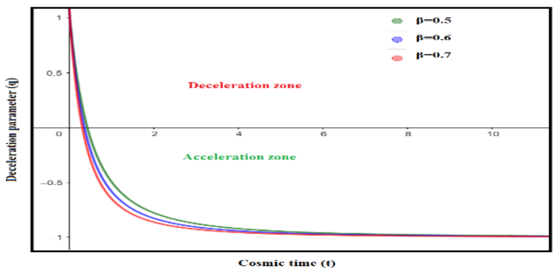


Fig.2 Behavior of Deceleration parameter (q)versus cosmic time (t)

Here we noticed from the Figure 1 that, Hubble parameter is a decreasing function of time and it approaches towards zero with the evolution of time.

The behavior of the model is determined by the sign of deceleration parameter q. The positive value of deceleration parameter indicates a decelerating model while the negative value gives inflation. From Eq. (21) it is clear that there is a transition phase from deceleration to acceleration at $t = -\frac{\alpha}{\beta} \pm \frac{\sqrt{\alpha}}{\beta}$ with $0 < \alpha < 1$. Since the negativity of the second term leads to a negative time, which indicates an unphysical context of the Big Bang cosmology. We conclude that the cosmic transition may have occurred at $t = \frac{\sqrt{\alpha}-\alpha}{\beta}$.

Using the equation (19) in (17) and (18) the metric potentials are obtained as functions of time as

$$A = c_2^{\frac{2}{3}} t^{\alpha} e^{\beta t} exp\left[-\frac{2c_1}{3}(3\beta)^{3\alpha-1}\Gamma(1-3\alpha, 3\beta t)\right] \quad (22)$$

$$B = c_2^{\frac{1}{3}} t^{\alpha} e^{\beta t} exp\left[-\frac{c_1}{3}(3\beta)^{3\alpha-1}\Gamma(1-3\alpha, 3\beta t)\right] (23)$$

where$\Gamma(\alpha, t)$ is the lower incomplete Gamma function.

We find the condition α ≤ 1/3 for the metric functions *B(t)* and *C(t)* to be realistic

4.1 Geometric Behavior Of The Model

The directional Hubble parameters for the model along x, y and z axis are respectively given by

$$H_1 = \frac{2c_1}{3(t^{\alpha}e^{\beta t})^3} + \frac{(\beta t+\alpha)}{t} \quad (24)$$

$$H_2 = H_3 = \frac{-c_1}{3(t^{\alpha}e^{\beta t})^3} + \frac{(\beta t+\alpha)}{t} \quad (25)$$

The Spatial Volume (V), Shear scalar and expansion scalar, respectively are given by

of the required model is obtained as

$$V = (t^{\alpha} e^{\beta t})^3 \quad (26)$$

$$\sigma^2 = \frac{c_1^2}{3(t^{\alpha}e^{\beta t})^6} \quad (27)$$

$$\theta = 3H = 3\frac{(\beta t+\alpha)}{t} \quad (28)$$

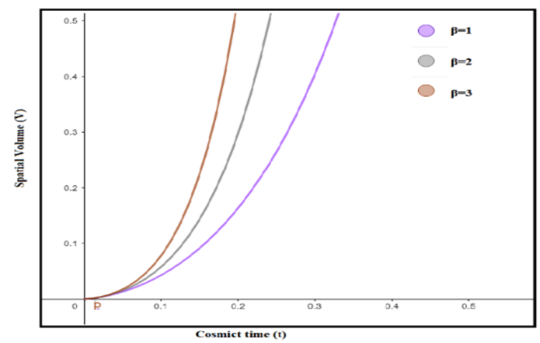


Fig.3 Behavior of Spatial Volume Vversus cosmic time (t)

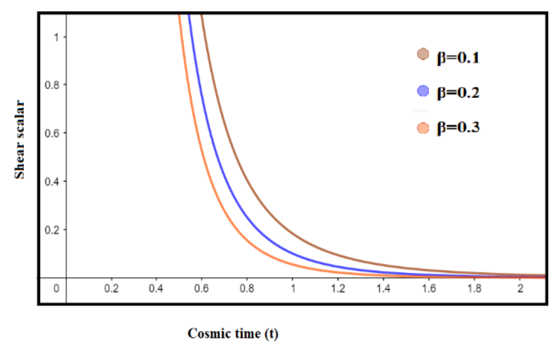


Fig.4 Behavior of Shear scalar σ^2versus cosmic time (t)

From fig. 3, The spatial volume rises monotonically with respect to cosmic time t. When $t \to 0$ spatial volume is zero. Spatial volume diverges when $t \to \infty$.

The expansion scalar is infinite at $t = 0$, which suggests that the universe starts evolving with zero volume at $t = 0$.i.e. we have the big bang scenario. Shear scalar is infinite when $t = 0$ and it is zero when $t = \infty$.

The mean anisotropy parameter Δ is defined and takes the value

Anisotropy Parameter $\varDelta = \frac{2c_1^2 t^2}{9\left(t^{\alpha}e^{\beta t}\right)^6(\beta t+\alpha)^2}$ (29)

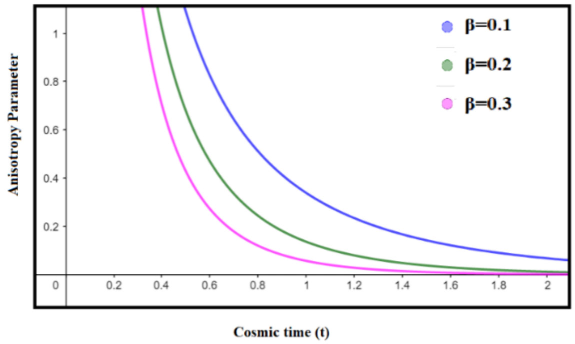


Fig.5 Behavior of Anisotropy parameter $\varDelta$versus cosmic time (t)

The mean anisotropy parameter is represented by following graph in fig.5. The dynamics of the mean anisotropy parameter depends on three constants $c_1, \alpha\ and\ \beta$. The mean anisotropy parameter decreases monotonically with respect to cosmic time and tends to zero value in the large time limit. When $t \to 0$, the mean anisotropy parameter is infinite. At late time when $t \to \infty, \varDelta \to 0$. Thus our model has transition from initial anisotropy to isotropy at present epoch.

$$p = \frac{1}{[(8\pi+3\lambda)^2-\lambda^2]}\left[\frac{2(8\pi+3\lambda)\alpha}{t^2} - \frac{c_1^2(8\pi+4\lambda)}{3\left(t^{\alpha}e^{\beta t}\right)^6} - \frac{8\pi(\beta t+\alpha)^2}{t^2}\right]$$

(30)

$$\rho = \frac{1}{[(8\pi+3\lambda)^2-\lambda^2]}\left[\frac{(24\pi+8\lambda)(\beta t+\alpha)^2}{t^2} + \frac{2\alpha\lambda}{t^2} - \frac{c_1^2(8\pi+4\lambda)}{3\left(t^{\alpha}e^{\beta t}\right)^6}\right]$$

(31)

The equation of state parameter can be obtained in a straightforward manner from equations (27) and (28) as

$$\omega = \frac{6(8\pi+3\lambda)\alpha\left(t^{\alpha}e^{\beta t}\right)^6 - c_1^2(8\pi+4\lambda)t^2 - 24\pi(\beta t+\alpha)^2\left(t^{\alpha}e^{\beta t}\right)^6}{(24\pi+8\lambda)(\beta t+\alpha)^2 3(t^{\alpha}e^{\beta t})^6 + 6\alpha\lambda(t^{\alpha}e^{\beta t})^6 - c_1^2(8\pi+4\lambda)t^2}$$

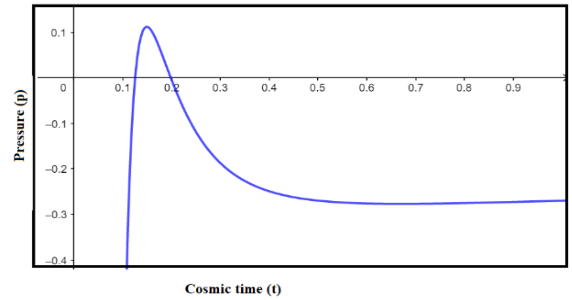
(32)

From the figure we, we observed that at the initial epoch the values of pressure(p), energy density (ρ) are very high and these values gradually decreases with the evolution of time i.e.p, ρ tend to zero as $t \to \infty$. Pressure and energy density are decreasing function of time.

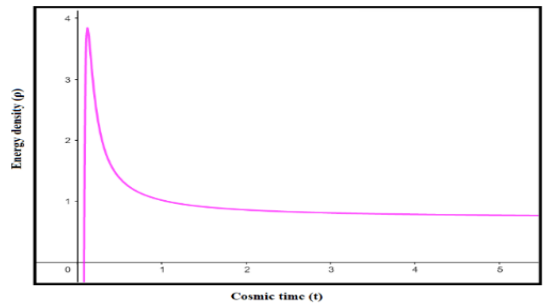
Alvarenga et al. have tested the energy conditions in $f(R,T)$ theory of gravity. We apply the energy conditions to our solutions for the effective energy density and effective pressure.

We observe that the average scale factor $a(t) \to 0\ as\ t \to 0$ and $a(t) \to \infty\ as\ t \to \infty$. This indicates that there exists inflation.

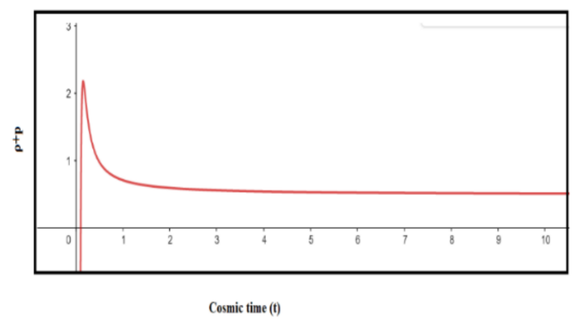
The profile of energy density and pressure is presented in the Figure 6



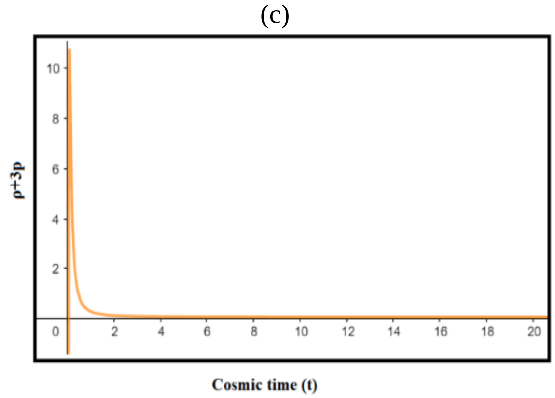
(a)



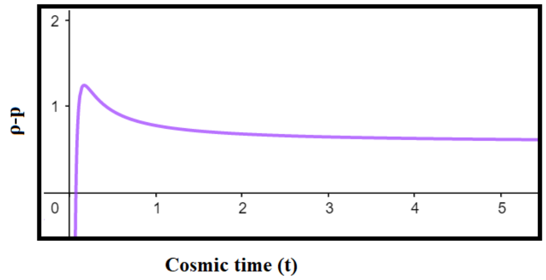
(b)



(c)



(d)



(e)

Fig.6 depicts the variation of $p,\ \rho,\ \rho+p,\ \rho+3p,\ \rho-p$versus cosmic time (t) for $\lambda=1,\ \beta=2.6, \alpha=0.5$.

ENERGY CONDITIONS

*Strong energy condition (SEC): gravity should be always attractive and in cosmology and in cosmology the relation $\rho+3p\geq 0$ must be obeyed;

*Weak energy condition (WEC): The effective energy density should always be non-negative when measured by any observer. i.e$\rho>0, \rho+p\geq 0$;

*Null energy density (NEC): It is the minimum requirement which is obtained from SEC and WEC, i.e. $\rho+p\geq 0$;

*****Dominant energy condition (DEC): The effective energy density must always be positive when measured by any observer. i.e.$\rho\geq|p|$ must be obeyed.

we can understand the energy conditions as able to provide us the validity regions of our solutions. Since they evade, for instance, the presence of space time singularities.

The energy conditions have been graphed in figs. 6(c), (d) and (e). From these figures we observe that

- The WEC and DEC for the derived model are satisfied
- The SEC is also satisfied.

It has been shown by Wald [50] that under very general conditions all Bianchi cosmologies (except Bianchi type IX) with an energy momentum tensor satisfying the strong and dominant energy conditions, will unavoidably enter a phase of exponential expansion.

STATEFINDER DIAGNOSTIC

Sahniet al.[51] have introduced a pair of parameters {r, s}, called Statefinder parameters. In fact, trajectories in the {r, s} plane corresponding to different cosmological models demonstrate qualitatively different behavior. The Statefinder parameters can effectively differentiate between different form of dark energy and provide simple diagnosis regarding whether a particular model fits into the basic observational data> The above Statefinder diagnostic pair has been following form:

$$r=1+3\frac{\dot{H}}{H^2}+\frac{\ddot{H}}{H^3} \qquad (33)$$

$$s=\frac{r-1}{3(q-1/2)} \qquad (34)$$

to differentiate among different form of dark energy. Here H is the Hubble parameter and q is the deceleration parameter. The two parameters {r, s} are dimensionless and are geometrical since they are derived from the cosmic scale factor $a(t)$ alone, though one can reproduce them in terms of the parameters of dark energy and dark matter. This pair provides information about dark energy in a model independent way, that is, it categorizes dark energy in the context of back-ground geometry only which is not dependent on theory of gravity. Hence geometrical variables are universal.

For our model, the parameters {r, s} can be explicitly written in terms of t as:

$$r=\frac{1}{(tH)^3}\left[(tH)^3-3\alpha tH+2\alpha\right] \qquad (35)$$

$$s=\frac{\alpha[2-3tH]}{3(2q-1)(tH)^3} \qquad (36)$$

V. DISCUSSION

In modified gravity theories, dark energy emerges as a result of modified gravitational effects and is purely geometrical in nature. In this paper we studied the phenomena of late-time acceleration by considering a perfect fluid in the framework of f(R, T) modified gravity. We have determined the LRS Bianchi type-I space-time by considering HEL for the average scale factor that yields power law and exponential-law cosmologies in its special cases. We find that the HEL Universe exhibits transition from deceleration to acceleration which is an essential feature of dynamic evolution of the Universe. The Bianchi type-I HEL Universe begins with high anisotropy but becomes isotropic at the later stages of the evolution.

Some key observations of present study are as follows:

- The model is based on exact solutions of the$f(R,T)$ gravity field equations for the anisotropic Bianchi type-I space-time filled with perfect fluid.

- For suitable choice of constant, the anisotropic parameter Δ tend to zero for sufficiently large time (Fig. (5)). Hence the present model is isotropic at late time which is consistent to the current observations.
- Our LRS Bianchi type I universe was decelerated in past and accelerating at present epoch. The DP must show signature flipping. Therefore, our consideration of DP to be variable is physically justified. Our derived model is accelerating at present epoch.
- We observe that our derived solutions are physically acceptable in concordance with the fulfillment of WEC, DEC and SEC. The model in $f(R,T)$ theory of gravity has stability and has initial singularity.
- The transition phenomena can also be noticed in the evolution of the effective pressure in time, as fig.6(a). It predict the pressure of the universe to eventually assume negative values. It is well known that a negative pressure fluid is the exact mechanism able to explain a cosmic acceleration within standard cosmology. We can interpret the prediction of cosmic acceleration as a consequence of energy transference between geometry and matter. In other words, such a transferring process is able to provide an effective fluid of sufficient negative pressure, responsible for driving the cosmic speed up.
- The effective EoS parameter in eq. (32) also presents some properties for which a more profound discussion is worthy.
- From The Statefinder parameter {r, s}, the behavior of different stages of the evolution of the universe have been generated.

Thus, the solutions demonstrated in this paper may be useful for better understanding of the characteristic of Bianchi type-I cosmological models in the evolution of the universe within the framework of $f(R,T)$ theory of gravity.

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