Multiplicative Labeling Based on Maximum Degree for Some Simple Connected Graph

S. RAGULSANKAR[1], V. RAJESWARI[2]
[1] *Research Scholar Department of Mathematics, Don Bosco College, Dharmapuri*
[2] *Assistant Professor, Department of Mathematics, Don Bosco College, Dharmapuri.*

Abstract— A Graph G with p vertices and q edges is said to be multiplicative labeling based on maximum degree graph if the vertices are assigned distinct number 1,2, 3……, p such that the labels induced on the edges by the product of the end vertices divided by its maximum degree are distinct. We prove some of the graphs such as Crown Graph, Path Graph, Star Graph, Equal Bi-Star Graph are multiplicative labeling based on maximum degree graph.

Indexed Terms— Maximum degree, Crown graph, Path graph, Star graph, Equal Bistar graph

I. INTRODUCTION

In this paper only finite, simple, connected and undirected graphs are considered . A graph labeling of G[2] is an assignment of labels to vertices or edges or both by following certain rules. Labeling of graph plays an important role in application of graph theory in neural, coding, circuit analysis etc.

II. DEFINITIONS

2.1 MULTIPLICATIVE LABELING BASED ON MAXIMUM DEGREE GRAPH (MLBMD GRAPH)

Let G (V,E) be a graph with p vertices is said to be multiplicative labeling based on maximum degree if we define a bijective mapping
$f: V(G) \rightarrow \{1,2, \dots\dots, p\}$ such that label induced on the edges is given by $g: E(G) \rightarrow N$ such that $g(uv) = \left[\frac{f(u)f(v)}{\Delta}\right]$, where [ ] denoted the integer part, Δ denotes the maximum degree of G.A graph which admits above labeling is called multiplicative labeling based on maximum degree graph (MLBMD graph)

2.2 GRAPH

A Graph G is a pair $G = (V, E)$ consisting of a finite set V and a set E (infinites graphs are also studied, but we consider only finite graphs). The elements of V are called vertices (points, nodes, junctions or 0-simplixes) and elements of E are called edges (line, arcs, branches or 1- simplexes). The set V is known as the vertex set of G and E as edge set of G[3].

2.3 CROWN GRAPH

The crown graph [2][4] $C_n^+ = C_n \odot K_1$ is obtained by joining a pendant edge to each vertex of cycle C_n.

2.4 PATH GRAPH

A path [1][4] is a simple graph whose vertices can be arranged in a linear sequence in such a way that two vertices are adjacent if they are consecutive in the sequence and are nonadjacent otherwise.

2.5 STAR GRAPH

Star graph [5][6] is a special type of graph in which n-1 vertices have degree 1 and a single vertex have degree n-1 this looks n-1 vertex is connected to a single central vertex. A star graph with total n- vertex is termed as S_n.

2.6 EQUAL BI-STAR GRAPH

The Equal Bi-star $EB_{m,m}$is the graph obtained by joining the apex vertices of two copies of star $K_{1,m}$ and $K_{1,m}$ by an edge.

III. MAIN RESULTS

3.1 THEOREM:
The Crown Graph C_n^+ is a MLBMD Graph.

PROOF:
Let $\{ p_1, p_2, \dots, p_n, p_{n+1}, p_{n+2}, \dots p_{2n} \}$ be the points of C_n^+
The Crown Graph C_n^+ has 2n points and 2n edges.
The points labeling is constructed as
$f: P(C_n^+) \rightarrow \{1,2, \dots \dots, 2n\}$ given by
$f(p_i) = i$, $i = 1,2, \dots, (n+1)$
$f(p_{n+2}) = 2n$
$f(p_{i+n+2}) = f(p_{i+n+1}) - 1$, $i = 1,2, \dots \dots (n-2)$

From the above labeling pattern on points, the edge labeling is given by

$g(p_i p_j) = \left[\frac{f(p_i)f(p_j)}{3}\right]$ where 3 is maximum degree of C_n^+ and [ ] denotes the integer part.

Hence Crown Graph C_n^+ is a MLBMD graph.

3.1.1 EXAMPLE:

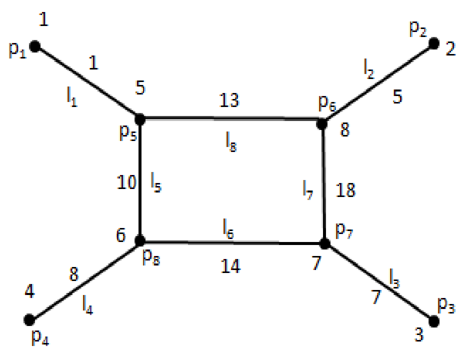

Fig :1 - C_4^+

3.1.2 EXAMPLE:

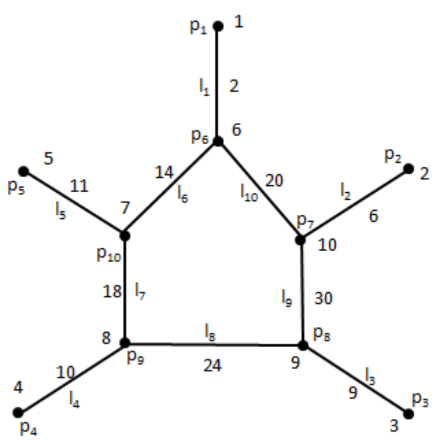

Fig: 2- C_5^+

3.2 THEOREM:

The Path Graph P_n is a MLBMD graph.

PROOF:

Let $\{ p_1, p_2, \ldots, p_n\}$ be the points of P_n

The Path Graph P_n has n points and (n-1) edges

The point labeling is constructed as

$f: P(P_n) \to \{1,2, \ldots\ldots, n\}$ given by

CASE: 1 (when n is even)

$$f(p_{2i-1}) = i \quad , i = 1,2, \ldots\ldots, \left[\frac{n}{2}\right]$$

$$f(p_{2i}) = \frac{n}{2} + i \qquad i = 1,2, \ldots\ldots, \left[\frac{n}{2}\right]$$

CASE: 2 (When n is odd)

$$f(p_{2i-1}) = i \ , \ i = 1,2, \ldots\ldots, \left(\frac{n+1}{2}\right)$$

$$f(p_{2i}) = \left(\frac{n+1}{2}\right) + i \ , \quad i = 1,2, \ldots\ldots, \left(\frac{n-1}{2}\right)$$

From the above labeling pattern on points ,the edges are labeled by

$g(p_i p_j) = \left[\frac{f(p_i)f(p_j)}{2}\right]$ where 2 is maximum degree of P_n and [ ] denotes the integer part.

Hence Path Graph P_n is a MLBMD Graph.

3.2.1 EXAMPLE:

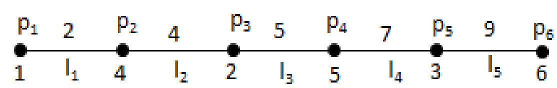

Fig: 3- P_6

3.2.2 EXAMPLE :

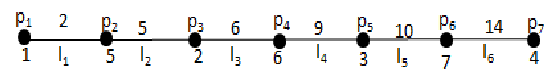

Fig:4 -P_7

3.3 Theorem:

The Star Graph S_n is a MLBMD graph.

PROOF:

Let $\{p, p_1, p_2, \ldots, p_n\}$ be the points of S_n

The star Graph S_n has n+1points and n edges

The Point labeling is constructed as

$f: P(s_n) \to \{1,2, \ldots, (n+1)\}$given by

$$f(p) = n$$

$$f(p_n) = n + 1$$

$$f(p_i) = i \quad i = 1,2, \ldots, (n-1)$$

From the above labeling pattern on points the edge labeling is given by

$g(p_i p_j) = \left[\frac{f(p_i)f(p_j)}{n}\right]$where n is the maximum degree of Sn and [] denotes the integer part.

Hence star Graph S_n is a MLBMD graph.

3.3.1EXAMPLE:

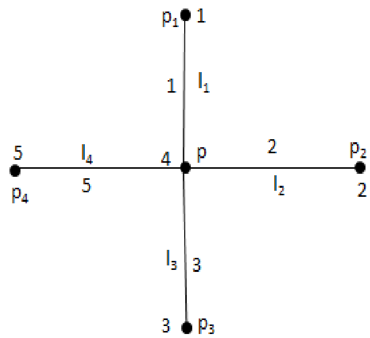

Fig: 5 - S_4

3.3.2EXAMPLE :

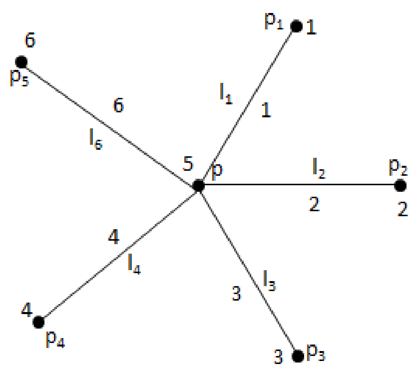

Fig: 6 - S_5

3.4 THEOREM:

The Equal Bi-Star $EB_{m,m}$ Graph admits a MLBMD graph

PROOF:

Let $\{p_0,p_1,p_2,\ldots,p_m\}$ and $\{s_0,s_1,s_2,\ldots,s_m\}$ be the points of $EB_{m,m}$

The Equal Bi-Star Graph $EB_{m,m}$ has $2(m+1)$ points and $(2m+1)$ edges

The point labeling is constructed as

$f: P(EB_{m,m}) \rightarrow \{1,2,\ldots,2(m+1)\}$ given by

$f(p_0) = m+1$

$f(p_i) = i \quad i = 1,2,\ldots,m$

$f(s_0) = m+2$

$f(s_j) = 2m+(j-1) \quad j = 1,2,\ldots,m$

From the above labeling pattern on points, the edge labeling is given by

$g(p_i p_j) = \left[\frac{f(p_i)f(p_j)}{(m+1)}\right]$ where (m+1) is maximum degree of Equal Bi-Star $EB_{m,m}$ and

[] denotes the integer part.

Hence Equal Bi-Star $EB_{m,m}$ Graph is a MLBMD graph.

3.4.1 EXAMPLE:

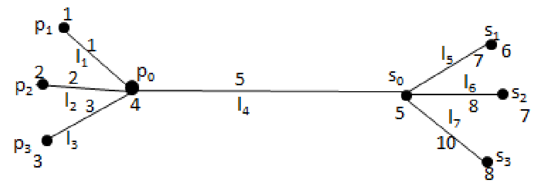

Fig:7- $B_{3,3}$

3.4.2 EXAMPLE:

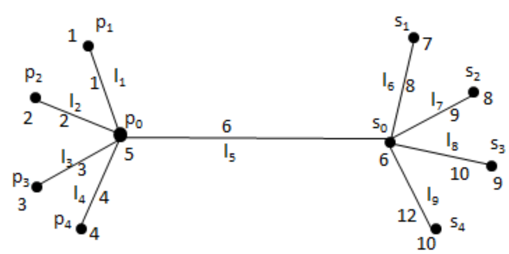

Fig:8-$B_{4,4}$

REFERENCES

[1] Bondy J.A and Murty U.S.R, "Graph Theory", Springer, ISBN: 978-1-84628-969-9.

[2] Gallian, J.A., "A Dynamic Survey of Graph Labeling", electronic journal of combinatorics [2010].

[3] Suresh Singh G, "Graph Theory", PHI Learning Private Limited New Delhi, ISBN-978-81-203-4105-0.

[4] V. Rajeswari, K. Thiagarajan, Ponnammal Natarajan "Maximum Degree based Vertex Graceful Lableing Graph with Odd Labeling on Edges", Smart Innovation, Systems and Technologies Volume 159 ISSN 2190-3018, ISSN 2190-3026(electronic), ISBN 978-981-13-9281-8, ISBN 978-981-13-9282-5(eBook), https://doi.org/10.1007/978-981-13-9282-5 Page No: :261-267

[5] www.javatpoint.com/graph- theory- types -of -graphs.

[6] www.geeksforgeeks.org/check-star-graph.