Reorientation of Mathematical Analysis

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***Abstract* - It is argued in this paper that the mathematical analysis must be based on rules of right reason and not on mysticism.**

***Index Terms* - Fluxion, derivative, logic, velocity acceleration.**

INTRODUCTION

George Berkely (1685-1753) critically analysed the method of fluxions of Sir Isaac Newton(1642-1727),and *Calculus differentialis* of G W Leibnitz (1646-1716) questioned the validity of principles and found some improper reasonings in both.As a result he arrived at the conclusion that both methods are logically defective, and therefore the result derived from those methods cannot be valid or accurate. As for as the procedure of differential calculus is concerned, Berkely's views deserve merit and I subscribe to them, but his treatment of The Greatest Master of Mathematical and Philosophical knowledge, The Genius, Sir Isaac Newton and his 'method of fluxions', however, seems to be based on gross misunderstanding and Misinterpretation of facts.

There is a serious misrepresentation of facts of Newton's Method. Berkely and all scientists, philosophers, logicians and mathematicians failed to observe the clear objectivity of Newton's method. Due to this un understandability, the problem of rigorous foundation remained to be solved.

Providence has given me this opportunity to explore the hidden, inbuilt logical accuracy and justness of Method of fluxions, which once understood clearly and reestablished, renders the vague, obscure, inaccurate, inadequate, cumbersome and ambiguous doctrine of 'infinitesimals' in its various forms and notations, obsolete.

Newton, The Greatest Master of Reason, devised a method to find the rule for fluxion of any power of a variable quantity. His method for consideration is taken up from his "introduction to quadrature of curves". to quote, "Let the quantity x flow uniformly, and let the fluxion of x^n be to be found. In the same time that the Quantity x by flowing becomes x+o, the Quantity x^n will become $(x+o)^n$, that is, by the method of Infinite Series's

$X^n+nox^{n-1}+nn\text{-}n/2\ oox^{n-2}+\&c.$

and the Augments o and $nox^{n-1}+nn\text{-}n/2\ oox^{n-2}+\&c.$

are to one another as

1 and $nx^{n-1}+nn\text{-}n/2\ ox^{n-2}+\&c.$

Now let those Augments vanish and their ultimate ratio will be the ratio of 1 to nx^{n-1}. And therefore the fluxion of the Quantity x is to the fluxion of the quantity x^n as 1 to nx^{n-1}.-----(1). No common man, not even an intelligent could be able to decode this mysterious method. Berkely is a person who represents all those who do not understand Newton. some of them defended with a presumption that Great Newton cannot commit logical mistakes. But Berkely raised his voice against the illogical point appeared in the method. He found logical error in the method of Newton. He concludes finally with the help of application of his sole lemma appeared in 12th section of 'The Analyst': "If with a view to demonstrate any proposition, a certain point is supposed, by virtue of which certain other points are attained; and such supposed point be itself afterwards destroyed or rejected by a contrary supposition; in that case, all the other points, attained thereby and consequent thereupon, must also be destroyed and rejected, so as from thence forward to be no more supposed or applied in the demonstration. 'This is so plain as to need no Proof". i.e the principles of the method are wrong.

According to Berkely, the second supposition of Newton destroys the first supposition of increment and from these contradictory suppositions, destroying each other, nothing new will be followed. He observes "when we suppose the increments to vanish, we must suppose their proportions, their expressions, and everything derived from the supposition of their existence to vanish with them". -----(2). James Jurin, Jacob walton, John Colson and many other staunch defenders of Newton defended Newton but by misinterpreting Newton's second supposition of 'evanescent augments', which are defined in the line keeping with Lebnitzian conceptual tradition, and embraced the fallacious doctrine of infinitesimals, which was heavily criticised by Berkely. This is an irony in the history of mathematics. To know the method which accurately derives the fluxion of f(x), for instancence $y=f(x)=x^2$, may be taken as a leading example from which we can. defend Newton and refute both Leibnitzians and Berkelys, Leibnitzians for their poor conjectures of infinitesimals and Berkely for treating evanescent increments and infinitesimals alike. The procedure of Leibnitzians is based on the axiom of synchronous increments, and its blind application. It is supposed that when y=f(x), then the increments of both sides of the equation must be equal. y=f(x), then y+dy=f(x+dx). Therefore dy, an increment in y may be had from a simple operation of subtraction, dy=f(x+dx)-y i.e $dy=2xdx+dx^2$. Then dy/dx is likewise$2xdx+dx^2/dx$ i.e 2x+dx. This dx out of 2x+dx needs to be destroyed. For this purpose a strange doctrine of infinitesimals is invoked, in symbolic form $dx\rightarrow 0$. An inconceivable symbol dx->o, dx tending to zero has not a unique meaning. First it was a finite quantity and correspondingly produced finite quantities $2xdx+dx^2$. Then dividing by a common divisor dx, dy/dx will be equal to 2x+dx. This conclusion is false, and certainly is not accurate and precise. for the truth is only 2x, as was derived by Newton. Therefore in order to get rid of this excess Quantity dx, many tricks and knacks are used and one of them is treating dx as an infinitely small, less than sensible quantity, so that it may be safely rejected. Even this trial is not free from anomalies. There again arises a logical difficulty. If dy/dx=2x+dx by Substituting the value of dx ($dx\rightarrow 0$, read as dx tending to zero, has double meanings viz; dx=some real quantity and dx=0, i.e no quantity, which ought not to be entertained in science.) The difference quotient $f(x+dx)-f(x)/dx\rightarrow f'(x)$, means the difference between difference quotient and f`(x) diminished *in infinitum*. So the difference quotient is not exactly the derivative or, f'(x). If the difference quotient is not f'(x), then what f'(x) really is? Nobody knows how exactly to derive the derivative. It is said sometimes, to be understood intuitively. Or it is said that some functions defy predictions or evade predictions. Now the delta-epsilon definition of limits is said to have resolved this difficulty. But on closer inspection it is the same fallacious idea expressed but in different symbols. f'(x)=f(x+h)-f(x)/h is cleverly altered to f'(x)= lim as h->0 f'(x+h)-f(x)/h. This means f'(x) is always greater than f(x+h)-f(x)/h -ε and less than f(x+h)-f(x)/h+ε. The reality is that in science, always the generality of truth is taken in to consideration but not an information arbitrarily adapted for our purposes.It Any formula ,if it is called to be a formula at all, must deliver general truth precisely and accurately.The difference quotient must yeild the same result, be the increments ever so little or ever so great. Science ,it can't be called the difference quotient equals derivative only when the increments are diminished in infinitum.

THE RIGOUROUS METHOD

Now it is time to replace this inadequate formula with a more general one and it is possible only when we define the concepts of fluxion and difference accurately. Difference is always regarded as the difference between two successive quantities, whereas fluxion is used by Newton as a ratio of 'nascent and evanescent augments'. It is very important to know what exactly these nascent and evanescent ratios are and, prime and ultimate ratios of nascent and evanescent increments.

The greatest turn of Newtonian thought in the field of mathematics is his paradigm shift, a shift from a concept of difference to fluxion, but his method is found to be defective in reasoning. This defect may be due to the poverty of logical thinking of all others and is also partly due to the unclear presentation of the matter by Newton himself.

METHOD OF FLUXION

According to Newton, Fluxions are accurately the prime and ultimate ratios of nascent and evanescent augments. While finding the rule for fluxion of any power of a variable quantity Sir Isaac carried on somewhat like a mental experiment. He supposed an increment to the variable quantity and accordingly obtained the corresponding increments in the power of that variable quantity. Then he rejected the increments and retained the fluxions of powers. This conclusion appears as if it is derived illogically**. Berkely based** on his sole lemma premised in 'The Analyst holds that the increments of power will too disappear as soon as the increment of variable quantity is nullified or rejected and there remains nothing like fluxions of powers. This is the mystery hidden in the logical derivation, Sir Isaac Newton supposed first an increment to x, which becomes (x+o). At the same time the power of x becomes $(x+o)^n$. On expansion, we have, $x^n+nox^{n-1}+nn-n/2\ oox^{n-2}$ and so on. Then Sir isaac Newton makes his second supposition of vanishing increments. To the common sense of Berkely it appeared that this second supposition destroys the first supposition of increment. And all we have to start afresh again. How in this stage Newton is able to derive the fluxion of x^n?..In answer to this, James Jurin - an admirer of both Newton and L'hospital twists the meaning of second supposition "*evanesecent jam augmenta illa, & vorum ratio ultima erit"* as the ratios of increment at the point of ceasing to exist**.----(3)**. This is nothing but the expression of infinitesimal concept in different words. The difficulty of this ratio at the point of beginning to exist unfolds when we try to derive the derivative of derivative or the second derivative. When the first derivative itself is indivisible, how can anyone is able to find or conceive second, third, fourth.... derivatives?.

A True Interpretation of second supposition of Newton's Method of Fluxions:

The second supposition of vanishing increments is grossly misinterpreted and misrepresented. It really is not infinitesimal increment as we are, all these three hundred and more years, kept to believe but it is certainly a finte increment. In the binomial of power n the prime quantity x and the ultimate quantity o have their own expressions and ratios. For instance, when n=2, we have $x^2+2xo+o^2$. x, the original quantity is expressed by power 2 and so also the increment o. Then the additional quantity 2xo is the product of those two quantities, x and o. On differentiation 2xo, we find the ratio of increment of prime quantity x=2xo/o=2x. Likewise the ratio of increment of ultimate quantity can be obtained by taking away the proportions of prime quantity x. i.e 2xo/x=o+o=2o. By second supposition of vanishing increments, we must understand that the expression of increment and its proportions to be vanished, leaving behind only the expression of x and its proportions., i.e x^2 and 2x. It is not logical to Vanish the proportions of x when we vanish an expressin and proportions of increment o.In addition to the first fluxion or prime ratio of x, there emerge second ,third......orders of fluxions. For instance, when we derive the derivative of x^4, which is $3x^2$, the first derivative, and will have $6x^2$ as the second derivative. $(x+o)^4=x^4+4x^3o+6x^2o^2+4o^3x+o^4$. The first derivative or fluxion or velocity is $4x^3$ and the second derivative or the derivative of derivative, or fluxion of fluxion or acceleration is $6x^2$.

So, this is how we should lay rigorous foundation for calculus.

CONCLUSION

Sir Isaac was the real inventer of calculus.i.e he instituted the principles to analyse in finite ones,not in infinitesimals.

REFERENCE

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