Exactly Solvable New Quantum Potential Systems Generated from the Already Solved Power law Potential Using Transformation Method

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Abstract: **We have reported some of exact bound state solutions of the Schrödinger equation generated from the already known three term and two term fractional power law potentials using the extended transformation formalism. The generated quantum systems are found anharmonic potentials. The normalized bound state energy eigenvalues of the generated potential systems are obtained.**

Index Terms: Anharmonic Potential, Exactly Solvable Potentials, Schrödinger equation, Transformation Method

I. INTRODUCTION

Almost always Schrödinger equation doesn't yield exact analytic solutions. It is well known that exact solution of Schrödinger equation is possible only for certain central potentials. In recent years, there has been considerable interest in the study of exactly solvable quantum mechanical potentials of physical interest as it provides maximum information of the quantum system [1-5]. One of the important tasks of quantum mechanics is to solve the Schrödinger equation with a physical. Various methods are used in the calculation of exact analytic solutions (EAS) of the Schrödinger equation for quantum mechanical potentials.

In this paper we deal with the generation of exactly solvable potentials from certain central fractional powerlaw potentials. We have used Extended transformation method [6-9] to generate new exactly solvable potentials from already known exactly solved potential. The present work concerns with the arbitrary dimensional Schrödinger equation with the generated potentials from the three and two term fractional powerlaw potentials which may find applications in different branches of physics and chemistry. The extended transformation (ET) includes a coordinate transformation (CT) followed by a functional transformation (FT). In quantum multerm potential it is possible to generate finite number of different exactly solved quantum systems by selecting differently the working potential.

II. FORMALISM AND APPLICATION

(a) Generation of Exactly Solvable Potential from the Three term Fractional power law Potential:

The exactly solvable three term fractional powerlaw potential is given by [10]:

$$V_A(r)=\alpha_1 r^{\frac{2}{3}}+\alpha_2 r^{-\frac{2}{3}}+\alpha_3 r^{-\frac{4}{3}} \qquad \text{(i)}$$

Where the parameters of the potential is related by the following constraint equation:

$$\left(2l_A+\frac{5}{3}\right)\left\{\left(2l_A+\frac{7}{3}\right)\sqrt{\alpha_1}+\alpha_2\right\}^{1/2}+\alpha_3=0.$$

The exact analytic solution of the already known A-QS is given by [3, 4]:

$$\Psi_A(r)=N_A r^{l_A}\exp\left[-\frac{3}{4}\sqrt{\alpha_1}r^{\frac{4}{3}}+\frac{3}{4}\frac{E_A}{\sqrt{\alpha_1}}r^{\frac{2}{3}}\right].$$

The energy eigenvalues for the potential system is given by [3, 4]

$$E_A=\pm\sqrt{4\alpha_1}\left\{\left(2l_A+\frac{7}{3}\right)\sqrt{\alpha_1}+\alpha_2\right\}^{\frac{1}{2}}.$$

Under extended transformation (ET), which consist of coordinate transformation [6-9]:

$$r\rightarrow g_B(r)$$

and followed by functional transformation:

$$\Psi_B(r)=f^{-1}(r)\Psi_A(g_B(r)).$$

This leads to [6-9]:

$$\Psi_B(r) = g_B^{'-\frac{1}{2}}(r) g_B^{\frac{DA-1}{2}} r^{-\frac{DB-1}{r}} \Psi_A(g_B(r)).$$

In this method we restrict ourselves to taking only one term working potential.

The following ansatze [6-9] leads the Standard Schrödinger equation form by selecting the working potential as:

$V_A^W(r) = \alpha_3 r^{-\frac{4}{3}}$, We obtain the following transformation function [6-9]:

$$g_B(r) = \left[\frac{1}{3} \left(-\frac{E_B}{\alpha_3} \right)^{\frac{1}{2}} r \right]^3;$$

Where the integration constant C is assumed to be equal to 0.

Once the transformation function becomes known that leads to by ansatze [6-9]

$$g_B^{'2}\left(V_A^W(g)\right) = -E_B;$$

And

$$g_B^{'2}(r) E_A = -V_B^1(r).$$

We obtain:

$$V_B^1(r) = C_B^2 r^6$$

Where C_B^2 is the Characteristic Constant of B-QS.

Another ansatze is [6-9]:

$$-g_B^{'2}(r)\left[\alpha_1 g^{\frac{2}{3}} + \alpha_2 g^{-\frac{2}{3}}\right] = -V_B^2(r).$$

This leads to:

$$V_B^2(r) = \beta_2 r^4 + \beta_3 r^2.$$

Therefore the potential of the newly constructed B-QS is found as:

$$V_B(r) = \beta_1 r^6 + \beta_2 r^4 + \beta_3 r^2.$$

Now the parameters of the newly generated B-QS potential are defined as:

$$C_B^2 = \beta_1 = 9\alpha_1 \left(\frac{E_B}{9\alpha_3} \right)^4;$$

$$\beta_2 == \frac{E_A}{81} \left(\frac{E_B}{\alpha_3} \right)^3;$$

$$\beta_3 == \alpha_2 \left(\frac{E_B}{3\alpha_3} \right)^2.$$

The constraint equation relating the parameters of the potential are:

$$\beta_2 = 2\sqrt{\beta_1\left((2l_B + D_B + 2)\sqrt{\beta_1} + \beta_3\right)}.$$

The energy eigenvalues of B-QS comes out to be:

$$E_{B1} = \frac{\beta_2}{\sqrt{\beta_1}} \left(l_B + \frac{D_B}{2} \right).$$

The corresponding energy eigenfunctions of B-QS in desired -dimensional spaces becomes:

$$\Psi_B(r) = N_B r^{lB} \exp\left[-\frac{\sqrt{\beta_1}}{4} r^4 - \frac{\beta_2}{4\sqrt{\beta_1}} r^2 \right].$$

(b) Generation of Exactly Solvable Potential from the Two term Fractional power law Potential:

To generate new exactly solved potential we have applied our formalism on an exactly solved two term fractional power central potential [10] given by:

$$V_A(r) = \alpha_1 r^{-\frac{1}{2}} + \alpha_2 r^{-\frac{3}{2}}. \qquad \text{(ii)}$$

Where the parameters of the potential are connected by the Constraint equation:

$$\left(l_A + \frac{3}{4} \right) [8\alpha_1 (l_A + 1)]^{1/3} + \alpha_2 = 0.$$

The energy eigenvalues of the QS is found as:

$$\Psi_A(r) = N_A r^{lA} \exp[\sqrt{-(E_A)}r - \frac{\alpha_1}{\sqrt{(-E_A)}} \sqrt{r}].$$

The corresponding energy eigenvalues of A-QS are:

$$E_A = -\left[\frac{\alpha_1^2}{8(l_A + 1)} \right]^{2/3}.$$

$$V_A^W(r) = \alpha_2 r^{-\frac{3}{2}}.$$

Now selecting the following term as working potential of the fractional powerlaw potential given by equation (ii):

The transformation function is found as:

$$g_B(r)=\left(-\frac{E_B}{16\alpha_2}\right)^2 r^4.$$

The Potential of the B-QS system is obtained as:

$$V_B(r)=\beta_1 r^6+\beta_2 r^4.$$

With

$$\beta_1=C_B^2=\left[-E_A\left(-\frac{E_B}{8\alpha_2}\right)^4\right]$$

Where C_B^2 is the characteristic constant of B-QS.

$$\beta_2==2\alpha_1\left(-\frac{E_B}{8\alpha_2}\right)^3.$$

The constraint equation relating the parameters of the potential are:

$$\beta_2=2\beta_1^{3/4}\sqrt{(2l_B+D_B+2)}.$$

The energy eigenvalues of B-QS comes out to be:

$$E_B=\frac{\beta_2}{2\sqrt{\beta_1}}(2l_B+D_B).$$

The corresponding energy eigenfunctions of B-QS in desired -dimensional spaces becomes:

$$\Psi_B(r)=N_B r^{lB}\exp\left[-\frac{1}{4}\sqrt{\beta_1}r^4-\frac{1}{2}\beta_1^{1/4}\sqrt{(2l_B+D_B+2)}r^2\right].$$

III.CONCLUSION

Using the Extended Transformation method we obtain two exactly solved anharmonic potential systems taking as the basis the three term fractional powelaw potential and the two term fractional power law potential. In this method we restrict ourselves to taking only one term working potential.

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