Sequence of Established Corona Graph *seq* $G \odot G^*$ is Presents Anti-magic Graph

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Abstract-Hartsfield and Ringel conjectured that every tree other than the tree on two vertices has an anti-magic labeling. In spite of several research papers published, the conjecture remains open. A very few results deal with proving some classes of trees are anti-magic. In this paper, we prove that trees of diameter 4 are anti-magic. Mathematics Subject Classification: 05C78; 05C05

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INTRODUCTION

In the year 2015, Miller, and Phanalasy were proved various graphs with families repeatedly characterized as a series of graphs such that is present generalizations of the coronagraph is presented in anti-magic labelling. In the year 1990, Hartsfield and Ringel .were developed in the ant magic graph along *q* edges is known as anti-magic, if this edge has been labelled along using *1,2,…,q* on the outside replication so that the total summation of the labels is an occurrence of the edge on the way to every vertex be present are distinct. Amongst the graphs, it has been proved anti-magic are $P_n(n \geq 3)$, wheels, cycles, as well as $K_n(n \geq 3)$.

Hartsfield and Ringel demonstrated the anti-magic nature of P_n, S_n, C_n, K_m, W_m, and $K_{2,m,} m \geq 3$. They similarly hypothesized that each linked graph, excluding K_2, is ant magic, an available conjecture. Several graphs have been subsequently shown to be anti-magic.

Despite several articles were published based on anti-magic labeling, conjectures remain open still now. Even though there are many several articles published based on anti-magic labeling, we would like to mention a few for the sake of completeness of this chapter. In the year 2004, Alon et al. have presented the ideas of existing anti-magic graphs such that around occurs on absolute constant *C* that $\delta \geq C \log|V(G)|$. Cubic graphs are present as anti-magic, proved by Liang as well as Zhu [29] and Kaplan et al demonstrated some trees using degree restrictions are present as anti-magic. Intended for a comprehensive study on anti-magic graphs, we are referred to in the dynamic survey by Gallian.

In the year 2011, Lee, Lin, and Tsai have introduced the concepts of C_n^2 being present anti-magic as well as the vertex total amounts form a set of following numerals while *n* is odd. In the year 2015, Lin, Shang, and Liawwere showed the idea of a star forest contains no S_1 in addition to at most one S_2 such as mechanisms are present anti-magic [33]. The authors at present proved a star forest mS_2 stands anti-magic there *m=1* and $mS_2 \cup S_n(n \geq 3)$ views anti-magic if and only if $m \leq \min\left\{2n+1, 2n-5+\sqrt{8n^2-24n+17/2}\right\}$. In the year 2011, Ryan, Miller, Rylands, and Phanalasy have presented the ideas of the generalized web and flower graphs as well as it was demonstrated that the particular graph families are anti-magic[34,35].

In the year 2011, Ryan, Rylands, Miller, and Phanalasy [34, 35] have continued the idea of widespread web graphs on the way to the particular peak multi-generalized web graphs as well as the author was demonstrated the particular

graphs on the way to be anti-magic. The same authors have been continued the idea of generalized owner toward the single apex multi-(complete) generalized owner graphs as well as created anti-magic labelling aimed at this family of graphs. Intended for more everywhere anti-magic ness of generalized web and owner graphs.
Harts field and Ringel[5] have developed the concepts of P_n(the path graph using n vertices), S_n (means star graph by way of n vertices), C_n (means it is using n number vertices in cycle graph), K_n (means it is using n number vertices in the complete graph),W_n(means it is using n number vertices in wheel graph) and $K_{2,n}$,$n \geq 3$ (it is using for partitions of vertices of size is 2 as well as $n$in the complete bipartite graph), be present ever occurrences of anti-magic graphs.
Also, the conjectured a well-known to all linked graphs, other than K2, was anti-magic. Over the next two decades, for example, several additional families of graphs were found to be anti-magic. However, there are still many gaps that exist that requirement in the direction of occupied toward settling the conjecture intended for illustration; the individual event of the cubic snark graph is unsolved.
Indeed, the weaker form of the conjecture is still open, namely, that every tree, except for K2, is anti-magic. The several outcomes regarding anti-magic graph labelling are summarized.

RELATED WORK

We describe the generalized coronagraph as well as snowflake graphs, along with new notations used, to illustrate our result. Let G = (V, E) whether connected or disconnected graph using p vertices as well as let $H = (V^{'}, E^{'})$ whether connected or disconnected graph. This coronagraph of the graph G using a graph H represented utilizing $G \odot H$, is known as the graph achieved as a result of taking an individual copy of G, and p copies of H, as well as linking the j^{th} vertex of G using an edge toward each vertex in the j^{th} copy of H to each vertex.
Regarding the explanation of coronagraph is present-day generalized to a new common one in whichever the p cases of graph H aren't estimated to be equal.

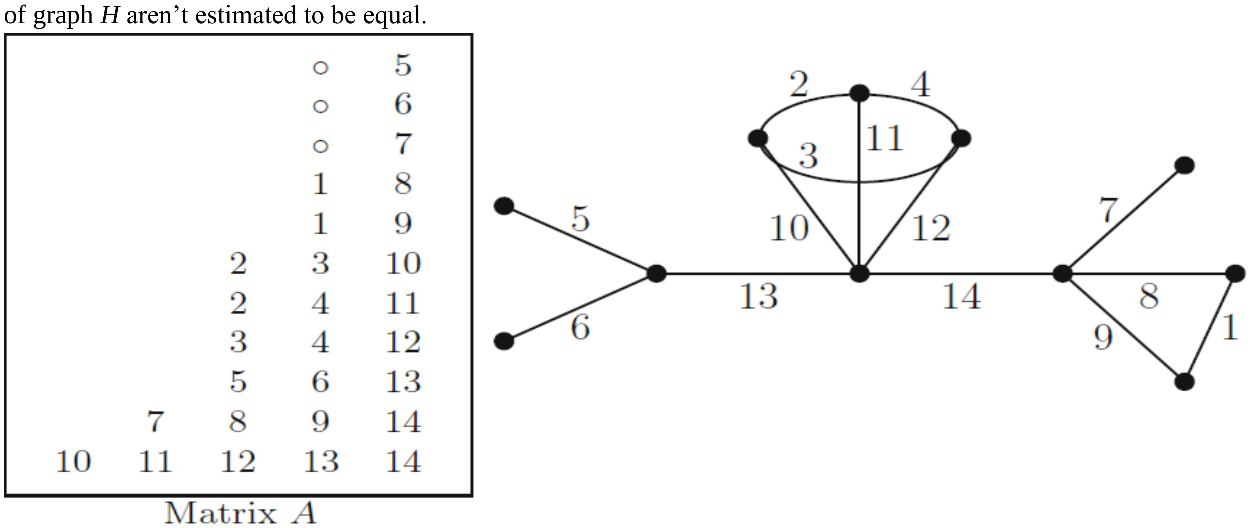

Figure-2.1 Anti-magic labelling of the generalized coronagraph

A generalized coronagraph $G \odot \mathfrak{H}$, anywhere $|V(G)|=p$, as well as $\mathfrak{H}$ be present a set of p connected or disconnected graphs. The graph achieved as a result of connecting a vertex of G, about v_j, using an edge in the direction of all vertex of a graph in $\mathfrak{H}$, say H_j. Such that H_1, H_2. . .H_p in $\mathfrak{H}$ can provide increase to several generalized coronagraphs dependent on the instruction of the graphs in $\mathfrak{H}$ as well as the instruction of the vertices in G. The corona $G \odot H$ is present a different event of $G \odot \mathfrak{H}$ while every graph in $\mathfrak{H}$ are H. while G is disconnected, then $G \odot \mathfrak{H}$ remains also disconnected. Figure-2.1 Referred a generalized coronagraph.

		○	1
		○	2
		○	3
		○	4
		○	5
		○	6
	1	7	10
	2	7	11
	3	8	12
	4	8	13
	5	9	14
	6	9	15
10	11	16	17
12	13	16	18
14	15	17	18

Matrix A

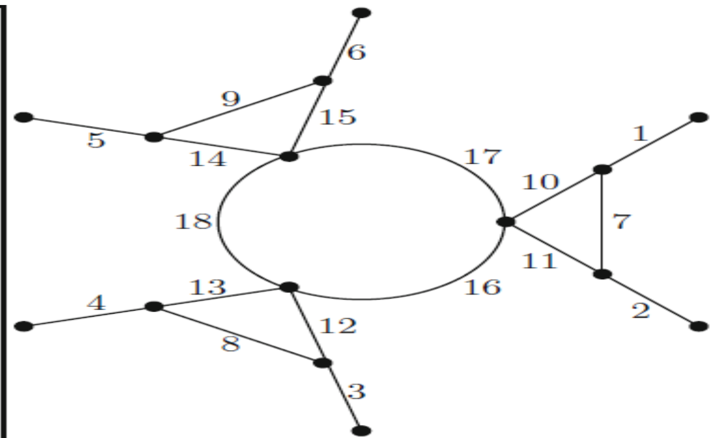

Figure 2.2 Anti-magic labelling of the generalized snowflake graph

By addition copies of a graph in the direction of the situation neighbouring graph with series, a snowflake graph is presently built repeatedly as of a structure of graphs, in an identical technique on the way to the corona structure; the generalized method doesn't want every graph to be similar. Therefore, consider the series of graphs H_1, $H_{2\ldots}$,H_m, as well as let H_j, $1 \le j \le m$, hold n_j vertices.

The generalized snowflake graph is G, represented as a result of $S\,f(H_1,H_2,\ldots, H_m, G)$, is well-defined on the way to be located the graph achieved as of G and H_j, $1 \leqslant j \leqslant m$, by way of linking the f^{th} vertex of G on the way to every vertex of the f^{th} copy ofH_m, for $1 \leqslant f \leqslant p$, as well as at that time linking the g^{th} vertex of everything p copies of H_m to every vertex of the g^{th} copy of $H_m{-}1$, for $1 \leqslant g \leqslant pn_m$; continue repeating up to the process of linking the h-th vertex of every copy of H_2on the way to every vertex of the h-th copy of H_1, for $1 \leqslant h \leqslant n_2n_3\ldots n_m\, p$, is extended. Such as while $m=1$, $S\,f\,(H, G)$ is the coronagraph $G\Theta H$. Refer the figure 2. 2 for the generalized snowflake graph.

Theorem 2.1: Let G is a connected or disconnected graph using p vertices. Now, in the sequence of established coronagraph *seq* $G\Theta\mathfrak{H}$ is present anti-magic.

Corollary 2.2: Let G is a connected or disconnected graph using no K_1 such as a sub graph.

1. If H_1=K_0, then *seq* $G\Theta\mathfrak{H}$ is anti-magic.
2. If H_i=K_0, *for every i*, $1 \le i < j \le p$, as well as G, is anti-magic by labelling on $\{a+\sum_{r=1}^{p}(q_r+n_r), a=1,2,\ldots,q\}$, at that moment *seq* $G\Theta\mathfrak{H}$ is present anti-magic.

Theorem 2.3: Let G is a graph is connected or disconnected using p vertices as well as let H_j, $1 \leqslant j \leqslant m$, used as m $\geqslant$ 2, is present the connected or disconnected $k_j$is a regular graph by way of n_j vertices such as m = 2, $\delta(G) + n_2 > k_2 + n_1$, as well as used for m $\geqslant$ 3, $\delta(G) + n_m > k_j + n_{j-1} \geqslant k_h + n_{h-1}$, $2 \leqslant h < j \leqslant m$. At that point the universal snowflake graph $Sf\,(H_1, H_2,\ldots,H_m, G)$ is anti-magic.

In the year 2014, S. Arumugam and Millerwere proved a novel family of graphs, generalized pyramid graphs, as well as build anti-magic used for the family of graphs. We as well additionally expand the outcomes to universal cases, the particular peak multi-generalized pyramid graphs.

The edge labels of graph G are well-defined, henceforth by way of an array L (not rectangular required), wherever every edge label occurring using a vertex is organized in a row. Every edge label needs to appear in two separate rows precisely once. $Wt(v)$ is used to denote the weightiness of a vertex v.

First, we'll describe a new graph family. Let G, using p vertices, is a k-regular graph. This *established pyramid graph*, $P(G, 1)$, presents the graph achieved as of graph G by linking a vertex named the *apex* to every vertex of graph G.

A graph G is known as the base. So that when $G=C_n$, $n \ge 3$, the wheel graph is a primary event of the universal pyramid graph P (G, 1). The *2-level universal pyramid graph*, *P*(*G,2*), is present the graph achieved by way of linking every vertex at the base on the way on the way to the consistent vertex in the additional copy of *G* as of the graph *P*(*G, 1*). The outcomes of the graph are known as the *m-level universal pyramid graph* (or basically, *universal pyramid graph*) as a result of repeating the method as well as is represented by *P*(*G, m*), m≥1.Otherwise, takings the Cartesian product $G \times P_m$ to obtain *P*(*G,m*), as well as connect a vertex on the way to every vertex of one final copy of *G*.

Let a graph *G*, using *p* vertices as well as *q* edges, is a *k*-regular graph. As a result of assigning numbers*1, 2…q* we primary pointing the edges of G randomly. The weights of vertices are present then determined and the vertices ordered such that $wt(v_i) \leq wt(v_{i+1}),\ 1 \leq i \leq p-1$. This gathering outcome in the edge is pointing on an array of G. In the generalized pyramid graph family, it is called the original labelling as well as will be used for graphs in the paper to create anti-magic labelling.

We represent utilizing T as a transpose of the array T.

Theorem 2.4: Let $G \neq K_1$ is present a *k*-regular graph. Formerly, the established pyramid graph *P*(*G,m*), *m≥1*, stands anti-magic

Corollary 2.5: Let $G \neq K_1$ be to some extent graph then and there the established pyramid graph *P*(*G,1*) is present anti-magic.

The resulting of the 2 theorems is an expansion of anti-magic established pyramid graph *P*(*G,m*),*m* ≥ *1*, toward additional complex anti-magic graphs.

Theorem 2.6: Let $G \neq K_1$ is present a k-regular graph. At that point, the established dual pyramid graph *DP*(*G,m*), $m \geq 2$, is present anti-magic.

We requirement the next concepts to extend the outcomes in the above theorems are more general cases. The graph achieved as of generalized pyramid graphs is the particular peak multi-generalized pyramid graph. Dual pyramid graphs simplified, varied pyramid graphs generalized by way of a typical apex.

REFERENCE

[1] N. Alon, G. Kaplan, A. Lev, Y. Roditty, R. Yuster, Dense graphs are antimagic, J. Graph Theory 47, (2004), 297-309.

[2] J.A. Gallian, A Dynamic Survey of Graph Labeling, The Electronic Journal of Combinatorics, 22nd Edition, (2019), #DS6.

[3] N. Hartsfield, G. Ringel, Pearls in Graph Theory, Academic Press, INC, Boston, 1990, 108-109, Revised version 1994.

[4] G. Kaplan, A. Lev, Y. Roditty, On zero-sum partitions and anti-magic trees, Discrete Math. 309, (2009), 2010-2014.

[5] Y. Liang, X. Zhu, Anti-magic labeling of cubic graphs, J. Graph Theory 75 (2014), 31-36.

[6] D.B. West, Introduction to Graph Theory, Prentice Hall of India, 2nd Edition, 2001.

[7] Yu-Chang Liang, Tsai-Lien Wong, Xuding Zhu, Anti-magic labeling of trees, Discrete Math.,331, (2014), 9-14.

[8] Y. Hou and W.C. Shiu, The spectrum of the edge corona of two graphs, Elec. Jour. Of Lin. Alge, 20 (2010), 586-594.