# Mathematical Modelling of two-phase blood flow in arterioles during-Iron Deficiency anemia

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**Abstract –The viscosity increases in the arterioles due to formation of rouleaux along the axis by red blood cells, Also the pumping effect of heart is very low in arterioles we apply the Herschel- Bulkley Non-Newtonian model in Bio – fluid mechanical setup. We have collected a clinical data in case of anemia for haemoglobin v/s blood pressure. The graphical presentation of blood pressure drop v/s haematocrit is performed. Haematocrit is explicit in determination of blood pressure drop in case iron deficiency Anemia. The presentation is in tensorial form and solution technique is analytical as well as numerical.**

**Keywords - Iron Deficiency Anemia, Haematocrit, Blood Flow, Herschel- Bulkley Non -Newtonian Model, Circulatory System.**

## 1-INTRODUCTION

1.1 Structure and Function of Arterioles – Arterioles is a small dimeter blood vessel in the microcirculation the extends and branches out from an Artery and leads to capillaries. Arterioles have muscular walls and are the primary site of vascular resistance.

The greatest change in blood pressure and velocity of blood flow occurs at the Transition of Arterioles to capillaries. The decreased velocity of flow in the capillaries increase the blood pressure, due to Bernoulli's Principle. This induces gas and nutrients to move from the blood to the cells. due to the lower osmotic pressure outside of the capillary. The opposite process occurs when the blood leaves the receive autonomic nervous system innervation and responds to various circulating hormones in order to regulate. Their diameter retinal vessels lack of functional sympathetic innervation. The up down fluctuation of the Arterioles blood pressure is due to the pulsatile nature of the cardiac output and determined by the interaction of the stroke volume versus volume and elasticity of the major Arteries, in a healthy vascular system. Arterioles diameters decrease with age and with exposure to air pollution.

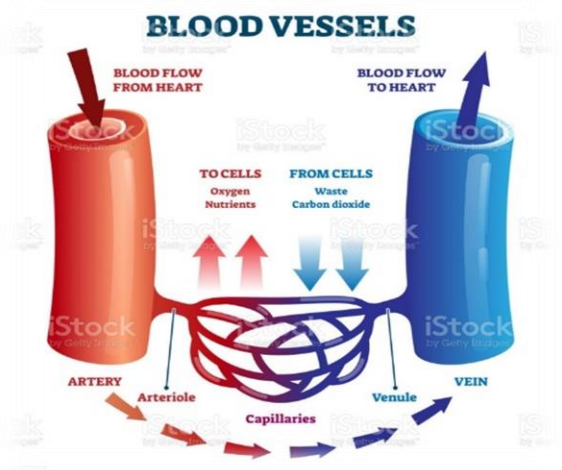



Fig.1)

1.2 Description of Iron -Deficiency Anemia –

Anemia is prevalent in most parts of the world. Iron-deficiency anemia (IDA) is one of the more common causes of anemia. Iron is very important in maintaining many body functions including the production of Haemoglobin, molecule in your body that carry oxygen. Iron is also necessary to maintain healthy cells, skin, hair and nails. Iron from the food you eat is absorbed into the body by the cells. The groups of people are risk (IDA)- if period is heavy, Are pregnant with more than one child, etc. They are many symptoms of IDA -Tiredness, Weakness, hair loss etc. These are symptoms of iron deficiency anemia (Paricha SR. et al .2010). The focus will be on IDA in children and adults. Readers are referred elsewhere for information on the presentation, symptoms, and laboratory diagnosis of iron-deficiency anemia [1] and

on issues that are specific to pregnancy [2]. According to world health organization [4], globally, anemia affects 1.62 billion people, which corresponding to 24.8% of the population.

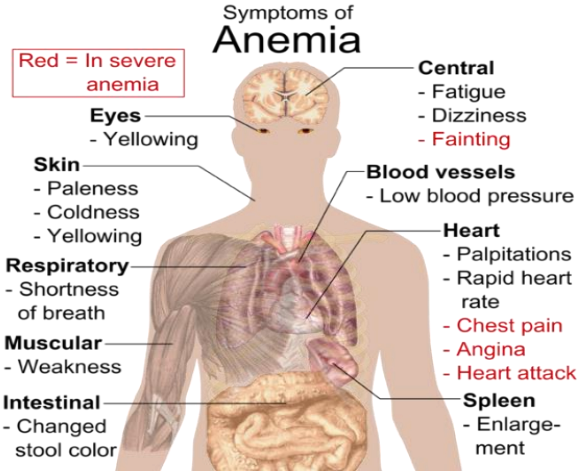



Fig. 2)

1.3 Structure and Function of Circulatory System-

And help in fighting disease, stabilizing temperature and PH, and the circulatory system also called the cardiovascular system or vascular system , is an organs system that permits blood to circulate and transport nutrients , oxygen and carbon dioxide hormones and blood cells to and from the cells in the body to provide nourishment maintenance homeostasis

- The circulatory system includes the lymphatic system which circulate lymph .
- Blood is a fluid consisting of plasma, red blood cells, white blood cells and platelets that is circulate by the heart through the vertiports vascular system, Carrying oxygen and nutrients and waste material away from all body tissue.
- The circulatory system of the blood is seen as having two components ,or systemic circulation and pulmonary circulation

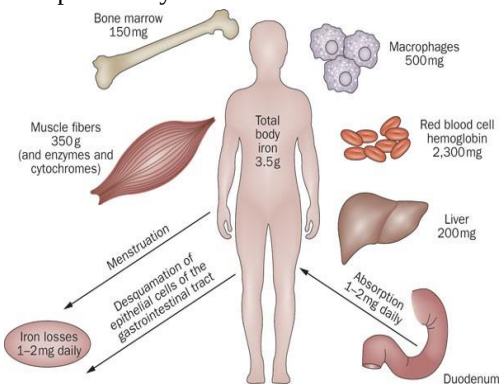



Fig. 3) Circulatory System

1.4 Blood and its Composition –

Blood is composed of plasma and formed elements. The plasma contains water 91.5, and Proteins 7% and other 1.5%.

## 2-REAL MODEL

2.1-Choice of Frame and Reference –

This Mathematical Model we have Select Orthogonal Curvilinear generalized three-dimensional Co-Ordinate System denoted by $E^{3\ called}$ three-dimensional Euclidean Space of the moving blood. All quantities related to blood flow in tensorial form which is comparatively more realistic. Let the P Co-ordinate axis be $PX^i$ where P is origin and I =1,2,3. At time t and which give the distribution blood velocity $V^k = V^k(X^i, t)$.

2.2 Choice of parameters –

Blood is Non-Newtonian fluids. If the relation between stress and strain rate are linear the flow is Newtonian otherwise non-Newtonian.

$\tau = \eta e^n$ =Constitution Equation of Fluids

If n≠ 1 $= Then\ the\ Nature\ of\ fluids\ is\ Non - Newtonian$. Fluids $\tau$ is denoted by stress, e is denoted by strain rate, n is denoted by parameter, p is denoted by blood pressure,$\rho$ is denoted by density. The constitutions equation is called Herschel- Bulkley Non- Newtonian.

$\eta_c$= Viscosity Coefficient of blood cells.

$\eta_m$ = Viscosity Coefficient of Mixture of two phase.

$\eta_p$ = Viscosity of Coefficient Plasma.

$\Delta P$= Pressure gradients.

Q = flow flux.

2.3 Boundary Condition –

1) The Velocity of blood flow on the axis of blood vessel at r=0 will be Maximum and finite. say $v_0$ = Maximum Velocity.

2) The Velocity of blood flow on the blood vessel wall at r=R when R is the radius of blood vessel. This condition is Known as no Slip.

2.4 Equation of continuity for two phase blood flow-

The flow of blood was affected by the presence of blood cells. Let the volume portion covered by blood cells in unit volume be X, this X is replaced by H/100, where H is the Haematocrit the volume percentage of blood cells. Then the volume portion covered by the

plasma will be 1-X. If mass ratio of blood cells to plasma is r then clearly.

$$r = \frac{X\rho_c}{(1-X)\rho_p} \quad (2.1)$$

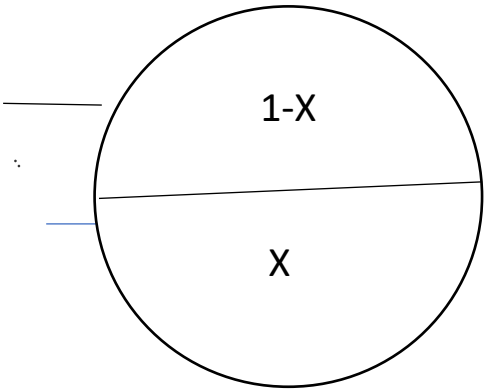



Fig.4) UNIT VOLUME

The two phases of blood, plasma and blood cells move with the common Velocity. V. Upadhyay and P.N. Pandey have already discussed two phase model (Upadhyay et al.2012). The principle of conservation of mass in circulatory System, equation of continuity for two phases.

$$\frac{\partial X\rho_c}{\partial t} + (X\rho_c v^i), i = 0 \quad (2.2)$$

$$\text{And } \frac{\partial (1-X)\rho_p}{\partial z} + ((1-X)\rho_p v^i), j = 0 \quad (2.3)$$

Where, v is the common velocity of two-phase plasma and blood cells

If we define the uniform density of the blood $\rho_m$ as follow

$$X\rho_c \frac{\partial v^i}{\partial t} + (X\rho_c v^j)v_{,j} = -X_{p,j}g^{ij} + X\eta_c(g^{ij}v^i,k),j \quad (2.6)$$

Similarly, taking the equation of motion for plasma will be as follow-

$$(1-X)\rho_p \frac{\partial v^i}{\partial t} + \{(1-X)\rho_p v^i\}v^i_{,j} = -(1-X)p_{,j}\, g^{ij} + (1-X)\eta_c(g^{ij}v^i_{,k})_{,j} \quad (2.7)$$

Noe adding equation (2.6) and (2.7) and using this relation (2.4), the equation of motion for blood flow with the two phases will be as follows –

$$\rho_m \frac{\partial v^i}{\partial t} + (\rho_m v^i)v^i_{,j} = -p_{,j}g^{ij} + \eta_m + (g^{ij}v^i_{,j})_{,j} \quad (2.8)$$

Where $\eta_m = X\eta_{c+(1-X)\eta_p}$ is the viscosity coefficient of blood as a mixture of two phases.

## 3- MATHEMATICAL MODELING

The Herschel -Bulkley low holds good on the two-phase blood flow through arterioles,

Where $\rho_c$ *and* $\rho_p$ are denoted by blood cells and blood plasma. The mass ratio is a parameter, but it may be treated as a constant in arterioles (1986).

$$\frac{1+r}{\rho_m} = \frac{r}{\rho_c} + \frac{1}{\rho_p} \quad (2.4)$$

Then the equation (2.2) & (2.3) can be combined,

$$\frac{\partial \rho_m}{\partial t} + (\rho_m v^i), i = 0 \quad (2.5)$$

2.5) Equation of motion for two phase blood flow

Two phase of blood cells and plasma are equilibrium sate in blood. Taking viscosity coefficient of blood cells to be $\eta_c$ and applying the principle of conservation of momentum, we get the equation for phase of blood cells.

Stress rate of strain rate between relation of constitutive equation

T' = $\eta_m e^n + T_p$ (T'≥ $T_p$) and

e= 0 (T'<$T_p$)

Where, $T_p$ is the yield stress.

Stain rate e = 0 (T'< $T_p$) a core region is formed a plug. Let the radius of the plug $r_p$ . The stress acting on the surface of plug will be $T_p$. Equation the forces on the plug, we get,

P$\pi$ $r^2{}_p = T_p\ 2\pi r_p$

$r_p = 2\frac{T_p}{P}$ (radius of plug) (3.1)

$T_p = \frac{Pr_p}{2}$ ( yield stress)

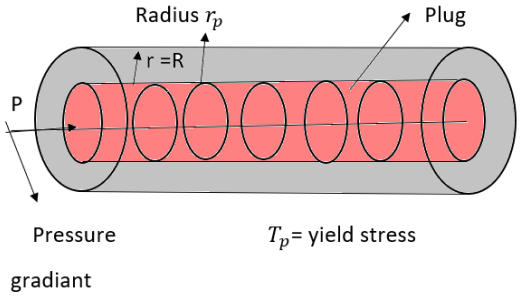



Fig .2- Herschel Bulkley blood flow

The Constitutive equation for test part of the blood vessel is

T' = $\eta_m e^n + T_p$

T'- $T_p$ = $\eta_m e^n = T_e$

Where, $T_e$ = effective stress

The generalized form will be as follows

$T^{ij}$ = -P$g^{ij} + T_e^{ij}$ (3.2)

$Where\ , T_e^{ij} = \eta_m (g^{ij})^n$While $[e^{ij} = g^{jk} v_k^i]$

Where, the symbols have their usual meanings.

We describe the basic equation for Herschel-Bulkley blood flow follow:

Equation of Continuity:

$\frac{1}{\sqrt{g\sqrt{(gv^i)_{,i}}}} = 0$ (3.3)

Equation of motion :

$\rho_m \frac{\partial v^i}{\partial t} + \rho_m v^j v^i_{,j} = -T^{ij}_{e,j}$ (3.4)

where are the symbols have their usual meanings .

Newton Raphson Method:

The General Newton Raphson Method Formula is

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ (3.5)

The above formula is repeated until a sufficiently precise value is obtained.

4-ANALYSIS

The blood vessels are cylindrical; the above equation have to be transformed into cylindrical Co- Ordinate. As we know earlier.

$X^1$ =r, $X^2 = \theta$ ,$X^3$ = z be cylindrical Co- ordinates and square length of small element $ds^2 = dr^2 + r^2 d\theta^2 + dz^2$

Christoffel's symbols of first kind and second kind are as follows:

$[ij,k] = \frac{1}{2}\left[\frac{\partial g_{jk}}{\partial x^i} + \frac{\partial g_{ik}}{\partial x^j} + \frac{\partial g_{ij}}{\partial x^k}\right]$ and $\left\{{k \atop ij}\right\} = g^{k\alpha}[ij,\alpha]$

$[g_{ij}]$ be matrix of matrix tensor and $[g^{ij}]$ matrix of conjugate matrix

tensor where $g_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $g^{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{r^2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Metric elements $g_{rr}$=1 $g_{\theta\theta} = r^2, g_{zz}$=1

Or $g_{11}$= 1, $g_{22}$ =, $g_{33}$=1

Christoffel's symbols of second kind for cylindrical co-ordinate

$\left\{{1 \atop 2\ 2}\right\}$ =-r $\left\{{1 \atop 2\ 2}\right\}$ =$\left\{{1 \atop 2\ 2}\right\} = \frac{1}{r}$ Remaining others are zero.

Physical components

Since $\sqrt{g_{11}}\ \ v^1 = v_r\ \ or\ \ v_r = v^1$

$\sqrt{g_{22}}\ \ v^2 = v_\theta\ \ or\ \ v_\theta = rv^2$

$\sqrt{g_{33}}\ \ v^3 = v_z\ \ or\ \ v_z = v^3$

Matrix of physical components of shearing stress tensor

$T'^{ij} = \eta_m (e^{ij})^n = \eta_m (g^{ik} v^i_{,k} + g^{jk} v^j_{,k})^n$

$T'^{ij} = \begin{bmatrix} 0 & 0 & \eta_m (\frac{dv}{dr})^n \\ 0 & 0 & 0 \\ \eta_m (\frac{dv}{dr})^n & 0 & 0 \end{bmatrix}$

$T'^{ij}_{\ ,j} = \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^j} (\sqrt{g} T^{ij}) + \left\{{i \atop jk}\right\} T'^{kj}$

The governing tensorial equation can be transformed into cylindrical forms which are follows:

The equation of continuity: $\frac{\partial v}{\partial z} = 0$

The equation of motion:

R- component: - $\frac{\partial p}{\partial z} = 0$

$\theta$ − components : 0 = 0

z- components: - $\frac{\partial p}{\partial z} + \frac{\eta_m}{r}$ [r$(\frac{\partial v_z}{\partial r})^n$ ]

Here, this fact has been taken in view that the blood flow is axially symmetric in arteries concerned, $V_\theta=0$ and $V_r$ , $V_z$ and p do not depend upon θ.

We get $V_z$ = v(r) and dp =p(z),

$0= -\frac{\partial p}{\partial z} +\frac{\eta_m}{r}[r(\frac{\partial v_z}{\partial r})^{n}]$ (4.1)

Pressure gradient $-\frac{dp}{dz}$ = P $r(\frac{dv}{dz})^{n} = -\frac{pr2}{2\eta_m}$ + A

We apply boundary condition: r= 0, V = $V_0$

Then $= -\frac{dv}{dr} =(\frac{pr}{2\eta_m})^{\frac{1}{n}}$ replace r from r-$r_p$

$-\frac{dv}{dr} = \left(\frac{\frac{1}{2}pr-\frac{1}{2}pr_p}{\eta_m}\right)^{\frac{1}{n}}$

$-\frac{dv}{dr} = \left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}}$ (r- $r_p$)$^{\frac{1}{n}}$ (4.2)

Integrating above equation (3.7) under the no slip boundary condition V= 0 at r=R so we get –

$V=\left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}}\frac{n}{n+1}(R-r_p)^{\frac{1}{n+1}} - (r-r_p)^{\frac{1}{n}+1}$ (4.3)

This is the formula for velocity of blood flow in arterioles, Venules and veins. putting r =$r_p$ to get the velocity $v_p$ of plug flow as follows:

$V_p = \frac{n}{n+1}\left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}}(R-r_p)^{\frac{1}{n+1}}$ (4.4)

Where the value of $r_p$ taken from equation of motion (4.4)

## 5- RESULTS (BIO -PHYSICAL ITERPRETATION)

Observation: Haematocrit V/S Blood Pressure from an LG Hospital Ahmedabad by Dr. Bhanu Pratap Patel

Patient name – Mrs. Saroj pandey**.** Age -29 years

Diagnosis: - Iron Deficiency Anemia (pregnancy Report)

| Date | HB (Haemoglobin) in (gram /dl) | Blood pressure (BP)in (mmhg) | Haematocrit in ($3HB$) (kg/m$^3$) | Arterioles Blood Pressure Drop - $\nabla P = \left(\frac{\frac{S+D}{2}+D}{3}\right) - \frac{S+D}{2}$ | Blood Pressure in Pascal P=133.32 |
|---|---|---|---|---|---|
| 01/01/22 | 8.3 | 100/80 | 24.9 | -34 | 4532.96 |
| 15/02/22 | 9.4 | 110/70 | 28.2 | -37 | 4932.93 |
| 20/03/22 | 9.6 | 120/80 | 28.8 | -40 | 5332.9 |
| 12/04/22 | 10.2 | 130/70 | 30.6 | -44 | 5866.19 |
| 01/05/22 | 12.2 | 120/70 | 36.6 | -40 | 5332.9 |

According to Berkow, Robert the haematocrit is normally about three times haemoglobin concertation (g/dl).

The Flow Flux of two Phased blood flow in arterioles, venioles and veins is

$Q=\int_0^{r_p} 2\pi r V_p \, dr + \int_{r_p}^{R} 2\pi r V dr$

$\int_0^{r_p} 2\pi r \frac{n}{n+1}\left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}}(R-r_p)^{\frac{1}{n+1}} dr + \int_0^{r_p} 2\pi r \frac{n}{n+1}\left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}}[(R-r_p)^{\frac{1}{n+1}} -(r-r_p)^{\frac{1}{n+1}}]dr$

Using (4.3) and (4.4) then

$Q= \frac{2\pi n}{n+1}\left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}}(R-r_p)^{\frac{1}{n+1}}\left[\frac{r^2}{2}\right]_0^{r_p} + \frac{2\pi n}{n+1}\left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}}[\frac{r^2}{2}(R-r_p)^{\frac{1}{n+1}} - \frac{r(r-r_p)^{\frac{1}{n}}}{\frac{1}{n}+2} + \frac{r(r-r_p)^{\frac{1}{n}}}{(\frac{1}{n}+2)(\frac{1}{n}+3)}]_{r_p}^{R}$

$= \frac{\pi n}{n+1}\left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}}(R)^{\frac{1}{n}+3}\left[\frac{r_p{}^2}{R^2}(1-\frac{r_p}{R})^{\frac{1}{n}+1} + (1+\frac{r_p}{R})(1-\frac{r_p}{R})^{\frac{1}{n}+2} + \frac{2(1-\frac{r_p}{R})^{\frac{1}{n}+3}}{(\frac{1}{n}+2)(\frac{1}{n}+3)} - \frac{2(1-\frac{r_p}{R})^{\frac{1}{n}+2}}{(\frac{1}{n}+2)}\right]$ (4.5)

Let Haematocrit (H) = 24.9 and BP drop in pascal = 4532.96 pa

Q=900 ml/min = 0.015 litre /sec, R=1,$r_p = \frac{1}{3}$

According to Gustafson Daniel R. (1980)

$\eta_p$ = 0.0015 (pascal -sec) According to Glenn Elert (2010)

$\eta_m$=0.035(pascal-sec)

Length of Arterioles ($\nabla P$) = 50$\mu m$ =5× $10^{-5}$ meter =0.0005 m

We Know that -

$\eta_m=\eta_c X + \eta_p(1-X)$ (5.1)

where X=$\frac{H}{100}$

$\eta_m=\eta_c\frac{H}{100} + \eta_p(1-\frac{H}{100})$

Now putting the value of $\eta_m$,$\eta_p$ and H in above relation we get $\eta_c$ =

$0.035=\eta_c\frac{24.9}{100} + 0.0015(1-\frac{24.9}{100})$

$\eta_c$ = 0.136038 (pascal -sec) =Velocity of cells
And again, using same relation.
$\eta_m$= 0.001345H+ 0.0015 (5.2)
Now from equation (4.5) flow flux is given a

$$Q= \frac{2\pi}{27}\left(\frac{P}{3\eta_m}\right)^{\frac{1}{n}}\left[\frac{26n^3+33n^2+9n}{6n^3+11n^2+6n+1}\right]$$

Where $P= -\frac{dP}{dz}$

$$\frac{Q\times 27}{2\pi}=\left(\frac{\Delta P}{3\eta_m}\right)^{\frac{1}{n}}\left[\frac{26n^3+33n^2+9n}{6n^3+11n^2+6n+1}\right]$$

$$\frac{27\times 0.015}{2\times 3.14} = \left(\frac{4532.96}{3\times 0.035}\right)^{\frac{1}{n}}\left[\frac{26n^3+33n^2+9n}{6n^3+11n^2+6n+1}\right]$$

$$0.06449= (43171.047)^{\frac{1}{n}}\left[\frac{26n^3+33n^2+9n}{6n^3+11n^2+6n+1}\right]$$

Substituting the of Q, $\Delta P$, and $\eta_m$ in above equation and solve by numerical method (by Newton's Raphson Method) we get n=-3.8
Now again

$$P= 3\eta_m\left(\frac{Q\times 27}{2\pi}\right)^{\frac{1}{n}}\left[\frac{26n^3+33n^2+9n}{6n^3+11n^2+6n+1}\right]$$

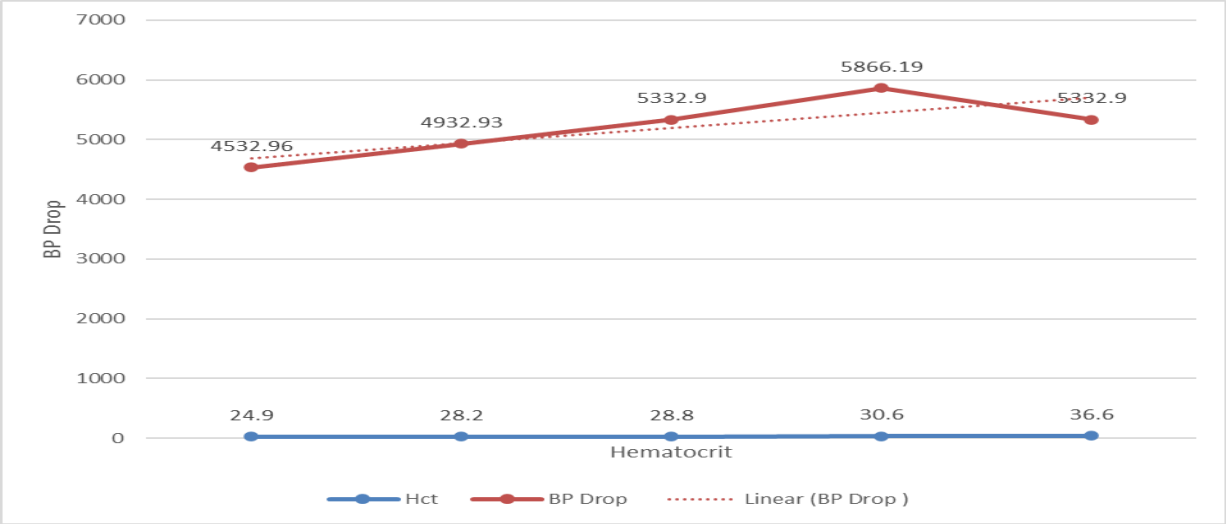
$$P= 3\eta_m\left(\frac{27\times 0.015}{2\times 3.14}\right)^{\frac{1}{3.8}}\left[\frac{26(3.8)^3+33(3.8)^2+9(3.8)}{6(3.8)^3+11(3.8)^2+6(3.8)+1}\right]$$

$$p = 3.7303H + 4.1602 \quad (5.3)$$

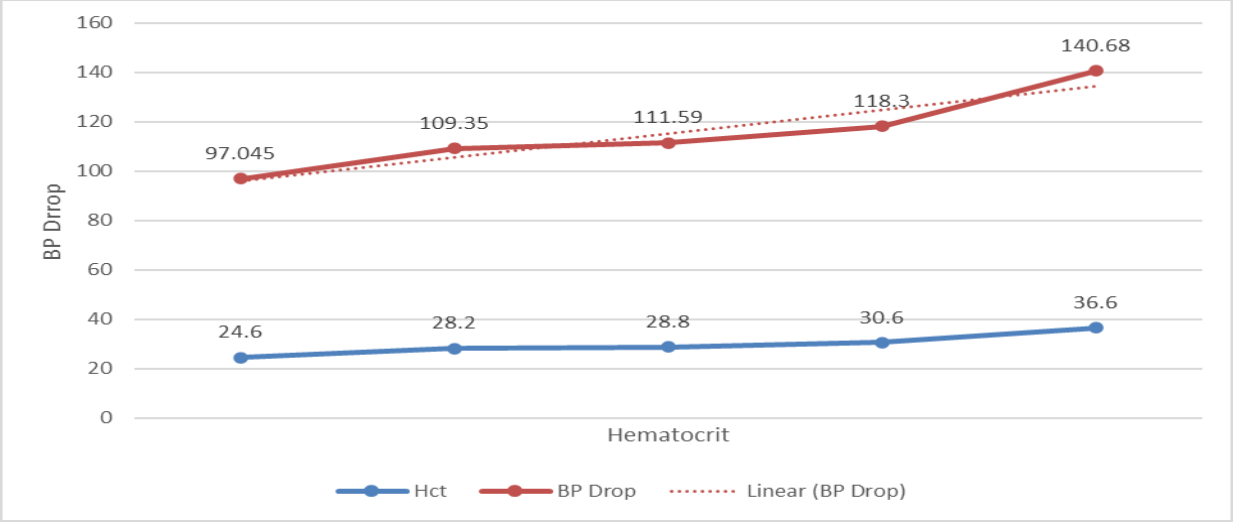
Putting the values of H in above equation (5.3), we get the Following table of blood pressure drop

| Haematocrit | 24.9 | 28.2 | 28.8 | 30.6 | 36.6 |
|---|---|---|---|---|---|
| BP Drop | 97.045 | 109.35 | 111.59 | 118.30 | 140.68 |

1)- Graphical Presentation of Pathological Data




2)- Graphical Presentation of Mathematical Modulate Data



## 6- CONCLUSION

In Pathological Data fig.3), fig. 4) The graph between haematocrit and blood pressure drops in iron - deficiency anemia patient we have conclude that when haematocrit increased then the blood pressure drop is also increased.

## 7- ACKNOWLEDGEMENT

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