Intuitionistic Polygonal Fuzzy Numbers

Dr. Savitha M T

Asst. Professor, Department of Mathematics, KSMDB College, Sasthamcotta, Kollam, Kerala, India

***Abstract*-Nowadays, intuitionistic fuzzy sets and numbers are studied and applied to different real-life problems. In this paper, intuitionistic fuzzy numbers with different number of vertices called intuitionistic polygonal fuzzy numbers were considered. The formulas derived for finding membership function associated with intuitionistic fuzzy number with n number of vertices and corresponding alpha cut is very useful in using fuzzy logic in decision-making which help us to compile the experts' judgements while developing decision-making models. A membership function for a intuitionistic fuzzy number of the form $(a_1, a_2, a_3, \dots, a_n, a_1', a_2', a_3', \dots, a_n')$ was developed which helps to define membership and non-membership function for any value of n. Different formulas have been developed for odd and even values of n.**

***Key words:* Intuitionistic polygonal fuzzy numbers, membership functions, non-membership functions, α-cuts, arithmetical operations**

1. INTRODUCTION

To represent fuzzy sets, its membership functions are to be defined. An intuitionistic polygonal fuzzy number and its operations are very useful in Fuzzy Multi-Criteria Decision-Making Process. To express human thinking more clearly, we can use intuitionistic fuzzy numbers with more vertices and it provides more flexibility to handle ambiguity. When we express linguistic terms as intuitionistic fuzzy numbers while developing Fuzzy Multi-Criteria Decision-Making models, we can use any type of intuitionistic fuzzy number as the case may be. Intuitionistic Fuzzy numbers with more vertices provide higher flexibility to the decision maker to express his own linguistics which ensure better handling of the Fuzzy Multi-Criteria Decision-Making Problem.

2. PRELIMINARIES

Definition 2.1 (Klir and Yuan, 2001)

A fuzzy set A in a universe of discourse X is defined as the set of pairs,
$A = \{(x, \mu_A(x)) : x \in X\}$, where $\mu_A(x): X \to [0,1]$ is called the membership value of $x \in X$ in the fuzzy set A.

Definition 2.2 (Klir and Yuan, 2001)

The α-cut, $\alpha \in (0, 1]$ of a fuzzy number A is a crisp set defined as
$A(\alpha) = \{x \epsilon R : A(x) \geq \alpha\}$. Every A_α is a closed interval of the form $[A_L(\alpha), A_U(\alpha)]$.

Definition 2.3 (Ban and Lucian, 2014)

A fuzzy number A is a fuzzy subset of the real line; $A: R \to [0,1]$ satisfying the following properties:

(i) A is normal (i.e., there exists $x_0 \in R$ such that $A(x_0) = 1$);

(ii) A is fuzzy convex;

(iii) A is upper semi continuous on R. ie; $\forall \varepsilon > 0$, $\exists\, \delta > 0$ such that

(iv) $A(x) - A(x_0) < \varepsilon$ whenever $|x - x_0| < \delta$;

(v) The closure , $cl(supp(A))$ is compact.

Definition 2.4

Let A_1 and A_2 are two fuzzy numbers and an arithmetic operation $\circ$ is defined, where $\circ \in \{+, -, *, /\}$, in terms of α-cuts as follows

$$(A_1 \circ A_2)(\alpha) = A_1(\alpha) \circ A_2(\alpha) = \{x \circ y \,/\, x \in A_1(\alpha), x \in A_2(\alpha)\};\ \alpha \in [0,1]$$

The result of an arithmetic operation $(A_1 \circ A_2) = \bigcup_{\alpha\in[0,1]} (A_1 \circ A_2)(\alpha)$ is always a fuzzy number (Skalna et al., 2015). For any two fuzzy numbers A and B, addition and subtraction operations result again in fuzzy numbers.

3. INTUITIONISTIC POLYGONAL FUZZY NUMBERS

3.1 Intuitionistic Polygonal Fuzzy Number with n vertices, n is an odd number

An intuitionistic polygonal fuzzy number with n vertices (n is odd) is a fuzzy number of the form $P = (a_1, a_2, a_3, \ldots, a_n, a_1', a_2', a_3', \ldots, a_n')$ with membership function $\mu_P(x)$ (Savitha M.T., 2021) and non-membership function $\vartheta_P(x)$ as follows:

$$\mu_P(x) = \begin{cases} 0 & \text{if } x < a_1 \\ P_1(x) = \frac{2}{n-1}\left(\frac{x-a_1}{a_2-a_1}\right) & \text{if } a_1 \le x \le a_2 \\ P_2(x) = \frac{2}{n-1} + \frac{2}{n-1}\left(\frac{x-a_2}{a_3-a_2}\right) & \text{if } a_2 \le x \le a_3 \\ P_3(x) = \frac{4}{n-1} + \frac{2}{n-1}\left(\frac{x-a_3}{a_4-a_3}\right) & \text{if } a_3 \le x \le a_4 \\ \ldots\ldots\ldots & \\ \ldots\ldots\ldots & \\ P_{\frac{n-1}{2}}(x) = \frac{n-3}{n-1} + \frac{2}{n-1}\left(\frac{x-a_{\frac{n-1}{2}}}{a_{\frac{n+1}{2}}-a_{\frac{n-1}{2}}}\right) & \text{if } a_{\frac{n-1}{2}} \le x \le a_{\frac{n+1}{2}} \\ P_{\frac{n-1}{2}}'(x) = 1 - \frac{2}{n-1}\left(\frac{x-a_{\frac{n+1}{2}}}{a_{\frac{n+3}{2}}-a_{\frac{n+1}{2}}}\right) & \text{if } a_{\frac{n+1}{2}} \le x \le a_{\frac{n+3}{2}} \\ \ldots\ldots\ldots & \\ \ldots\ldots\ldots & \\ P_3'(x) = \frac{6}{n-1} - \frac{2}{n-1}\left(\frac{x-a_{n-3}}{a_{n-2}-a_{n-3}}\right) & \text{if } a_{n-3} \le x \le a_{n-2} \\ P_2'(x) = \frac{4}{n-1} - \frac{2}{n-1}\left(\frac{x-a_{n-2}}{a_{n-1}-a_{n-2}}\right) & \text{if } a_{n-2} \le x \le a_{n-1} \\ P_1'(x) = \frac{2}{n-1}\left(\frac{a_n-x}{a_n-a_{n-1}}\right) & \text{if } a_{n-1} \le x \le a_n \\ 0 & \text{if } x > a_n \end{cases}$$

$$\vartheta_P(x) = \begin{cases} 1 & \text{if } x < a_1' \\ P'_1(x) = \frac{(n-1)a'_2+(3-n)a'_1-2x}{(n-1)(a'_2-a'_1)} & \text{if } a_1' \le x \le a_2' \\ P_2'(x) = \frac{n-3}{n-1} - \frac{2}{n-1}\left(\frac{x-a_2'}{a_3'-a_2'}\right) & \text{if } a_2' \le x \le a_3' \\ P_3'(x) = \frac{n-5}{n-1} - \frac{2}{n-1}\left(\frac{x-a_3'}{a_4'-a_3'}\right) & \text{if } a_3' \le x \le a_4' \\ \ldots\ldots\ldots & \\ \ldots\ldots\ldots & \\ P_{\frac{n-1}{2}}'(x) = \frac{2}{n-1} - \frac{2}{n-1}\left(\frac{x-a'_{\frac{n-1}{2}}}{a'_{\frac{n+1}{2}}-a'_{\frac{n-1}{2}}}\right) & \text{if } a'_{\frac{n-1}{2}} \le x \le a'_{\frac{n+1}{2}} \\ P_{\frac{n-1}{2}}(x) = \frac{2}{n-1}\left(\frac{x-a'_{\frac{n+1}{2}}}{a'_{\frac{n+3}{2}}-a'_{\frac{n+1}{2}}}\right) & \text{if } a'_{\frac{n+1}{2}} \le x \le a'_{\frac{n+3}{2}} \\ \ldots\ldots\ldots & \\ \ldots\ldots\ldots & \\ P_3(x) = \frac{n-7}{n-1} + \frac{2}{n-1}\left(\frac{x-a'_{n-3}}{a'_{n-2}-a'_{n-3}}\right) & \text{if } a_{n-3}' \le x \le a_{n-2}' \\ P_2(x) = \frac{n-5}{n-1} + \frac{2}{n-1}\left(\frac{x-a'_{n-2}}{a'_{n-1}-a'_{n-2}}\right) & \text{if } a_{n-2}' \le x \le a_{n-1}' \\ P_1(x) = \frac{(n-3)a'_n+(1-n)a'_{n-1}+2x}{(n-1)(a'_n-a'_{n-1})} & \text{if } a_{n-1}' \le x \le a_n' \\ 1 & \text{if } x > a_n' \end{cases}$$

Definition 3.1

The α-cut of the intuitionistic polygonal fuzzy number

$P = (a_1, a_2, a_3, \dots, a_n, a_1', a_2', a_3', \dots, a_n')$ (n is an odd number) of the universe of discourse X is defined as $P(\alpha) = \{x \in X,\ \mu_P(x) \geq \alpha,\ \vartheta_P(x) \geq \alpha\}$ where $\alpha \in [0,1]$

3.2 Intuitionistic Polygonal Fuzzy Number with n vertices, n is an even number

An intuitionistic polygonal fuzzy number with n vertices (n is even) is a fuzzy number of the form $Q = (a_1, a_2, a_3, \dots, a_n, a_1', a_2', a_3', \dots, a_n')$ with membership function $\mu_Q(x)$ (Savitha M.T., 2021) and non-membership function $\vartheta_Q(x)$ as follows:

$$\mu_Q(x) = \begin{cases} 0 & \text{if } x < a_1 \\ Q_1(x) = \frac{2}{n-2}\left(\frac{x-a_1}{a_2-a_1}\right) & \text{if } a_1 \leq x \leq a_2 \\ Q_2(x) = \frac{2}{n-2} + \frac{2}{n-2}\left(\frac{x-a_2}{a_3-a_2}\right) & \text{if } a_2 \leq x \leq a_3 \\ Q_3(x) = \frac{4}{n-2} + \frac{2}{n-2}\left(\frac{x-a_3}{a_4-a_3}\right) & \text{if } a_3 \leq x \leq a_4 \\ \dots\dots\dots & \\ Q_{\frac{n-2}{2}}(x) = \frac{n-4}{n-2} + \frac{2}{n-2}\left(\frac{x-a_{\frac{n-2}{2}}}{a_{\frac{n}{2}}-a_{\frac{n-2}{2}}}\right) & \text{if } a_{\frac{n-2}{2}} \leq x \leq a_{\frac{n}{2}} \\ 1 & \text{if } a_{\frac{n}{2}} \leq x \leq a_{\frac{n+2}{2}} \\ Q_{\frac{n-2}{2}}'(x) = 1 - \frac{2}{n-2}\left(\frac{x-a_{\frac{n+2}{2}}}{a_{\frac{n+4}{2}}-a_{\frac{n+2}{2}}}\right) & \text{if } a_{\frac{n+2}{2}} \leq x \leq a_{\frac{n+4}{2}} \\ \dots\dots\dots & \\ Q_3'(x) = \frac{6}{n-2} - \frac{2}{n-2}\left(\frac{x-a_{n-3}}{a_{n-2}-a_{n-3}}\right) & \text{if } a_{n-3} \leq x \leq a_{n-2} \\ Q_2'(x) = \frac{4}{n-2} - \frac{2}{n-2}\left(\frac{x-a_{n-2}}{a_{n-1}-a_{n-2}}\right) & \text{if } a_{n-2} \leq x \leq a_{n-1} \\ Q_1'(x) = \frac{2}{n-2}\left(\frac{a_n-x}{a_n-a_{n-1}}\right) & \text{if } a_{n-1} \leq x \leq a_n \\ 0 & \text{if } x > a_n \end{cases}$$

and

$$\vartheta_Q(x) = \begin{cases} 1 & \text{if } x < a_1' \\ Q_1'(x) = \frac{(n-2)a_2' + (4-n)\,a_1' - 2x}{(n-2)(a_2'-a_1')} & \text{if } a_1' \leq x \leq a_2' \\ Q_2'(x) = \frac{n-4}{n-2} - \frac{2}{n-2}\left(\frac{x-a_2'}{a_3'-a_2'}\right) & \text{if } a_2' \leq x \leq a_3' \\ Q_3'(x) = \frac{n-6}{n-2} - \frac{2}{n-2}\left(\frac{x-a_3'}{a_4'-a_3'}\right) & \text{if } a_3 \leq x \leq a_4 \\ \dots\dots\dots & \\ Q'_{\frac{n-2}{2}}(x) = \frac{2}{n-2} - \frac{2}{n-2}\left(\frac{x-a'_{\frac{n-2}{2}}}{a'_{\frac{n}{2}}-a'_{\frac{n-2}{2}}}\right) & \text{if } a'_{\frac{n-2}{2}} \leq x \leq a'_{\frac{n}{2}} \\ 0 & \text{if } a_{\frac{n}{2}} \leq x \leq a_{\frac{n+2}{2}} \\ Q_{\frac{n-2}{2}}(x) = \frac{2}{n-2}\left(\frac{x-a'_{\frac{n+2}{2}}}{a'_{\frac{n+4}{2}}-a'_{\frac{n+2}{2}}}\right) & \text{if } a'_{\frac{n+2}{2}} \leq x \leq a'_{\frac{n+4}{2}} \\ \dots\dots\dots & \\ Q_3(x) = \frac{n-8}{n-2} + \frac{2}{n-2}\left(\frac{x-a'_{n-3}}{a'_{n-2}-a'_{n-3}}\right) & \text{if } a'_{n-3} \leq x \leq a'_{n-2} \\ Q_2(x) = \frac{n-6}{n-2} + \frac{2}{n-2}\left(\frac{x-a'_{n-2}}{a'_{n-1}-a'_{n-2}}\right) & \text{if } a'_{n-2} \leq x \leq a'_{n-1} \\ Q_1(x) = \frac{(n-4)a_n' + (2-n)\,a_{n-1}' + 2x}{(n-2)(a_n'-a_{n-1}')} & \text{if } a_{n-1}' \leq x \leq a_n' \\ 1 & \text{if } x > a_n' \end{cases}$$

Definition 3.2:

The α-cut of the polygonal fuzzy number $Q = (a_1, a_2, \dots, a_n, a'_1, a'_2, \dots, a'_n)$ (n is an even number) of the universe of discourse X is defined as

$Q(\alpha) = \{x \in X / \mu_Q(x) \geq \alpha, \vartheta_Q(x) \geq \alpha\}$ where $\alpha \in [0,1]$

Example 3.1: Intuitionistic Pentagonal fuzzy number

Let $P = (a_1, a_2, a_3, a_4, a_5, a_1', a_2', a_3', a_4', a_5')$ be a pentagonal fuzzy number. Then its membership function $\mu_P(x)$ and non-membership function $\vartheta_P(x)$ are given by,

$$\mu_P(x) = \begin{cases} 0 & \text{if} \quad x < a_1 \\ P_1(x) = \frac{1}{2}\left(\frac{x - a_1}{a_2 - a_1}\right) & \text{if } a_1 \le x \le a_2 \\ P_2(x) = \frac{1}{2} + \frac{1}{2}\left(\frac{x - a_2}{a_3 - a_2}\right) & \text{if } a_2 \le x \le a_3 \\ P_2'(x) = 1 - \frac{1}{2}\left(\frac{x - a_3}{a_4 - a_3}\right) & \text{if } a_3 \le x \le a_4 \\ P_1'(x) = \frac{1}{2}\left(\frac{a_5 - x}{a_5 - a_4}\right) & \text{if } a_4 \le x \le a_5 \\ 0 & \text{if} \quad x > a_5 \end{cases}$$

$$\vartheta_P(x) = \begin{cases} 1 & \text{if} \quad x < a_1' \\ P'_1(x) = \frac{4a'_2 - 2a'_1 - 2x}{4(a'_2 - a'_1)} & \text{if } a_1' \le x \le a_2' \\ P_2'(x) = \frac{1}{2} - \frac{1}{2}\left(\frac{x - a_2'}{a_3' - a_2'}\right) & \text{if } a_2' \le x \le a_3' \\ P_2(x) = \frac{1}{2}\left(\frac{x - a'_3}{a'_4 - a'_3}\right) & \text{if } a_3' \le x \le a_4' \\ P_1(x) = \frac{2a'_5 - 4a'_4 + 2x}{4(a'_5 - a'_4)} & \text{if } a_4' \le x \le a_5' \\ 1 & \text{if} \quad x > a_5' \end{cases}$$

The α-cut of the intuitionistic pentagonal fuzzy number $P = (a_1, a_2, a_3, a_4, a_5, a_1', a_2', a_3', a_4', a_5')$ is given by,

$P(\alpha) =$

$$\begin{cases} [a_1 + 2(a_2 - a_1)\alpha, a_5 - 2\alpha(a_5 - a_4)]; [4a'_2 - 2a'_1 - 4\alpha(a'_2 - a'_1), 4\alpha(a'_5 - a'_4) - 2a'_5 + 4a'_4]: \alpha \epsilon \left[0, \frac{1}{2}\right) \\ [a_2 + (2\alpha - 1)(a_3 - a_2), a_3 + 2(1 - \alpha)(a_4 - a_3)]; [a'_2 + (1 - 2\alpha)(a'_3 - a'_2), a'_3 + 2\alpha(a'_4 - a'_3)]: \alpha \epsilon \left[\frac{1}{2}, 1\right) \end{cases}$$

3.3 Arithmetic Operations on Intuitionistic Polygonal Fuzzy Number with n vertices

Here basic arithmetic operations; addition, subtraction, multiplication and division are discussed by means of α-cut. Then arithmetic mean is also considered.

For defining basic operations, we consider two intuitionistic fuzzy numbers $P_1 = (a_1, a_2, a_3, \dots, a_n, a_1', a_2', a_3', a_4', a_5')$ and $P_2 = (b_1, b_2, b_3, \dots, b_n, b_1', b_2', b_3', b_4', b_5')$

(i) Addition:

$P_1 + P_2 = (a_1 + b_1, a_2 + b_2, \dots, a_n + b_n, a'_1 + b'_1, a'_2 + b'_2, \dots, a'_n + b'_n)$

(ii) Subtraction:

$P_1 - P_2 = (a_1 - b_1, a_2 - b_2, \dots, a_n - b_n, a'_1 - b'_1, a'_2 - b'_2, \dots, a'_n - b'_n)$

(iii) Multiplication:

$P_1 \times P_2 = (a_1 \times b_1, a_2 \times b_2, \dots, a_n \times b_n, a'_1 \times b'_1, a'_2 \times b'_2, \dots, a'_n \times b'_n)$

(iv) Division:

$P_1/P_2 = (\frac{a_1}{b_1}, \frac{a_2}{b_2}, \frac{a_3}{b_3}, \dots, \frac{a_n}{b_n}, \frac{a'_1}{b'_1}, \frac{a'_2}{b'_2}, \frac{a'_3}{b'_3}, \dots, \frac{a'_n}{b'_n}),\ \ b_i \neq 0, b'_i \neq 0; i = 1,2 \dots, n$

(v) Arithmetic mean:

$\frac{P_1 + P_2}{2} = \left(\frac{a_1 + b_1}{2}, \frac{a_2 + b_2}{2}, \dots, \frac{a_n + b_n}{2}, \frac{a'_1 + b'_1}{2}, \frac{a'_2 + b'_2}{2}, \dots, \frac{a'_n + b'_n}{2}\right)$

As for any $\alpha \in [0,1]$, the arithmetic intervals corresponding to the α-cuts are the same, $(P_1 + P_2)(\alpha) = P_1(\alpha) + P_2(\alpha)$, $(P_1 - P_2)(\alpha) = P_1(\alpha) - P_2(\alpha)$

$(P_1 \times P_2)(\alpha) = P_1(\alpha) \times P_2(\alpha)$, $(P_1/P_2)(\alpha) = P_1(\alpha)/P_2(\alpha)$

4. CONCLUSION

The formulas derived for finding membership function associated with any kind of intuitionistic fuzzy number and corresponding arithmetic operations are very useful in using fuzzy logic in different areas such as multi-criteria decision-making models, fuzzy optimization etc

REFERENCES

[1] Ban, Adrian I., and Lucian Coroianu. "Existence, uniqueness and continuity of trapezoidal approximations of fuzzy numbers under a general condition." *Fuzzy sets and Systems* 257 (2014): 3-22.
[2] KIir, G.J.andYuan,B. "Fuzzy Sets and Fuzzy Logic: Theory and Applications", Prentice Hall, New Delhi,(2001).
[3] Savitha, M.T. “Arithmetic Operations on Polygonal Fuzzy Numbers”, Indian Journal of Applied Research, Volume II, Issue 3 (2021)
[4] Skalna, Iwona, et al. "Advances in fuzzy decision making." Studies in Fuzziness and Soft Computing. Springer, New York (2015).