Bayesian Estimation of Change Point of Exponentiated Inverted Weibull Distribution under Precautionary Loss Function

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Abstract- Quick detection of common changes is critical in sequential monitoring of multistream data where a common change is a change that only occurs in a portion of panels. After a common change is detected by using a combined cumulative sum (CUSUM) procedure, we first study the joint distribution for values of the CUSUM process and the estimated delay detection time for the unchanged panels. Change-points divide statistical models into homogeneous segments. Inference about change-points is discussed in many research in the context of testing the hypothesis of 'no change', point and interval estimation of a change-point, changes in nonparametric models, changes in regression, and detection of change in distribution of sequentially observed data. In this paper we consider the problem of single change-point estimation in the mean of a Exponentiated Inverted Weibull Distribution under Precautionary Loss Function. We propose a robust estimator of parameter. Then, we propose to follow the classical inference approach, by plugging this estimator in the criteria used for change-points estimation. We show that the asymptotic properties of these estimators are the same as those of the classical estimators in the independent framework. This method is implemented in the R package for Comprehensive numerical study. This package is used in the simulation section in which we show that for finite sample sizes taking into account the dependence structure improves the statistical performance of the change-point estimators and of the selection criterion.

Keywords: Exponentiated Inverted Weibull Distribution, Precautionary Loss Function, Bayesian Estimation, R-simulation.

1 INTRODUCTION

In this estimation approach, the parameter θ in the model distributions $p_\theta(x)$is treated as a random variable with some prior distribution $\pi(\theta)$. The estimator for θ is defined as a value depending on the data and minimizing the expected loss function or the maximal loss function, where the loss function is denoted as $l\left(\theta, \hat{\theta}(X)\right)$. The usual loss function includes the quadratic loss $\left(\theta-\hat{\theta}(X)\right)^2$, the absolute loss $\left|\theta-\hat{\theta}(X)\right|$ etc. It often turns out that $\hat{\theta}(X)$ can be determined from the posterior distribution of $P(\theta|X)=P(X|\theta)\,P(\theta)/P(X)$.

In decision theory the loss criterion is specified in order to obtain best estimator. The simplest form of loss function is squared error loss function (SELF) which assigns equal magnitudes to both positive and negative errors. However, this assumption may be inappropriate in most of the estimation problems. Sometime overestimation leads to many serious consequences. In such situation many authors found the asymmetric loss functions, appropriate. There are several loss functions which are used to deal such type of problem. In this research work we have considered some of the asymmetric loss function named precautionary loss functions (PLF) suggested by Norstorm (1996). Such asymmetric loss functions are also studied by Basu, A.P. and Ebrahimi, N. (1991), Goldstein, M. (1998), Perlman, M., & Balug, M. (Eds) (1997), Pandya et. al. (1994), Shah, J.B. & Patel, M.N. (2007).

2 PRECAUTIONARY LOSS

Norstrom (1996) introduced an alternative asymmetric precautionary loss function and also presented a general class of precautionary loss functions with quadratic loss function as a special case. These loss function approach infinitely near the origin to prevent underestimation and thus giving a conservative estimators, especially when, low failure rates are being estimated. These estimators are very useful and simple asymmetric precautionary loss function is

$$L\left(\hat{\theta},\theta\right)=\frac{\left(\hat{\theta}-\theta\right)^2}{\hat{\theta}} \qquad (1.2.1)$$

In a Bayesian setup, the unknown parameter is viewed as random variable. The uncertainty about the true value of parameter is expressed by a prior distribution. The parametric inference is made using the posterior distribution which is obtained by incorporating the observed data into the prior distribution using the Bayes theorem, The first theorem of inference. Hence we update the prior distribution in the light of observed data. Thus, the uncertainty about the parameter prior to the experiment is represented by the prior distribution and the same, after the experiment, is represented by the posterior distribution.

The various statistical models are considered are as;

3 NATURAL CONJUGATE PRIOR (NCP)

The various prior distributions are considered for different situations, like non-informative, when no information about the parameter is available, Natural Conjugate Prior (NCP), when post and prior distribution of parameter belong to same distribution family, etc. Hence the appropriate distribution chosen is Natural Conjugate Prior. If there is no inherent reason to prefer one prior probability distribution over another, a conjugate prior is sometimes chosen for simplicity. A conjugate prior is defined as a prior distribution belonging to some parametric family, for which the resulting posterior distribution also belongs to the same family. This is an important property. Since the Bayes estimator, as well as its statistical properties (variance, confidence interval, etc.), can all be derived from the posterior distribution.

In each case we observe that the statistical analysis based on the sufficient statistic will be effective as the one based on the entire data set $\underline{x}$.

As in frequentist framework, sufficient statistic plays an important role in Bayesian inference in constructing a family of prior distributions known as Natural Conjugate Prior (NCP) . The family of prior distributions $g(\theta)$, $\theta \epsilon \Omega$, is called a natural conjugate family if the corresponding posterior distribution belongs to the same family as $g(\theta)$. De Groot (1970) has outlined a simple and elegant method of constructing a conjugate prior for a family of distributions $f(x|\theta)$ which admits a sufficient statistic.

One of the fundamental problems in Bayesian analysis is that of the choice of prior distribution $g(\theta)$ of θ. The non informative and natural conjugate prior distributions are which in practice, Box and Tiao (1973) and Jeffrey (1961) provide a thorough discussion on non informative priors.

Both De Groot (1970) and Raffia & Schlaifer (1961) provide proof that when sufficient statistics exist a family of conjugate prior distributions exists.

The most widely used prior distribution of θ is the inverted Gamma distribution with the parameters 'a' and 'b' (> 0) with p.d.f. given by

$$g(\theta) = \begin{cases} \frac{b^a}{\Gamma a}\theta^{-(\alpha+1)}e^{-b/\theta}\ ;\ \theta > 0\ ;\ (a,b) > 0, \\ 0 \qquad\qquad ,\text{otherwise.} \end{cases}$$

(1.3.1)

The main reason for general acceptability is the mathematical tractability resulting from the fact that the inverted Gamma distribution is conjugate prior of θ Raffia & Schlaifer (1961), Bhattacharya (1967) and others have found that the inverted Gamma can also be used for practical reliability applications.

In this paper the Bayesian estimation of change point 'm' and scale parameter $'\theta'$ of Exponentiated Inverted Weibull distribution is obtained by using Precautionary Loss Function (PLF) and Natural conjugate Prior distribution as Inverted Gamma prior. The sensitivity analysis of Bayesian estimates of change point and the parameters of the distributions have been done by using R-programming.

4 EXPONENTIATED INVERTED WEIBULL DISTRIBUTION

The Inverted Weibull distribution is one of the most popular probability distributions to analyze the lifetime data with some monotone failure rates. G.S. mudholkar et al (1995) introduced the Exponentiated Weibull Distribution as generalization of the statndard Weibull Distribution. The two parameter EIW distribution has the following probability density function

$$f(x) = \theta\beta x^{-(\beta+1)}\left(e^{-\theta x}\right)^{-\beta}; \quad x > 0, (\beta > 0, \theta > o)$$

(1.4.1)

And the distribution function

$$F(x) = \left(e^{-x^{-\beta}}\right)^{\theta}; \qquad x > 0 \qquad (1.4.2)$$

Also, the reliability function of the EIW distribution with two shape parameters θ and β are given by

$$R(t) = 1 - \left(e^{-t^{-\beta}}\right)^{\theta}; \qquad t > 0 \qquad (1.4.3)$$

5 BAYESIAN ESTIMATION OF CHANGE POINT IN EXPONENTIATED INVERTED WEIBULL DISTRIBUTION UNDER PRECAUTIONARY LOSS FUNCTION (PLF)

A scequence of independent lifetime $x_{1\,,}x_{2,}\ \dots\dots x_{m,}x_{(m+1)},\dots x_n$ $(n \geq 3)$ were observed from Exponentiated Inverted Weibull Distribution with parameter β,θ . But it was found that there was a change in the system at some point of time 'm' and it is reflected in the sequence after 'x_m' which results change in a sequence as well as parameter value. The Bayes estimate of θ and 'm'are derived for symmetric and asymmetric loss function under inverted gamma prior as natural conjugate prior.

1.5.1 Likelihood, Prior, Posterior and Marginal

Let $x_1, \dots\dots\dots\dots . x_n$, (n≥ 3) be a sequence of observed discrete life times . First let observations $x_1, \dots\dots\dots\dots . x_n$ have come from Exponentiated Inverted Weibull Distribution with probability density function as

$\mathrm{f}(\mathrm{x}, \beta, \theta) = \theta\beta x^{-(\beta+1)}\left(e^{-\theta x}\right)^{-\beta}$; $(\mathrm{x}, \beta, \theta > 0)$ (1.5.1.1)

Let 'm' is change point in the observation which breaks the distribution in two sequences as $(x_1, x_2, \dots\dots\dots . . x_m)$ & $(x_{m+1}, x_{m+2}, \dots\dots\dots\dots x_n)$

The probability density function of the above sequences are

$f_1(x) = \theta_1\, \beta_1\, x^{-(\beta_1+1)}\left(e^{-\theta_1 x}\right)^{-\beta_1}$ (1.5.1.2)

Where $x_1, x_2, \dots\dots\dots . . x_m, \theta_1, \beta_1 > o$

$f_2(x) = \theta_2\, \beta_2\, x^{-(\beta_2+1)}\left(e^{-\theta_2 x}\right)^{-\beta_2}$ (1.5.1.3)

$x_{m+1}, \dots\dots\dots . . x_n, \theta_2, \beta_2 > o$

The likelihood functions of probability density function of the sequence are

$$L_1(x, \theta_1, \beta_1) = \prod_{j=1}^{m} f(x_j, \theta_1, \beta_1)$$

$$= {\theta_1}^m {\beta_1}^m \prod_{j=1}^{m} x_j^{-(\beta_1+1)} e^{-\theta_1 \sum_{j=1}^{m} x_j^{-\beta_1}}$$

$$L_1(x, \theta_1, \beta_1) = (\theta_1\beta_1)^m U_1 e^{-\theta_1 T_{2m}} \quad (1.5.1.4)$$

Where $U_1 = \prod_{j=1}^{m} x_j^{-(\beta_1+1)}$ and $T_{2m} = \sum_{j=1}^{m} x_j^{-\beta_1}$

$$L_2(x, \theta_2, \beta_2) = \prod_{j=m+1}^{n} f(x_j, \theta_2, \beta_2) = {\theta_2}^{n-m}{\beta_2}^{n-m} \prod_{j=m+1}^{n} x_j^{-(\beta_2+1)} e^{-\theta_2 \sum_{j=1}^{m} x_j^{-\beta_2}}$$

$$L_2(x, \theta_2, \beta_2) = (\theta_2\beta_2)^{n-m} U_2 e^{-\theta_2(T_{2n}-T_{2m})}$$

Where $U_2 = \prod_{j=m+1}^{n} x_j^{-(\beta_2+1)}$

$T_{2n} - T_{2m} = \sum_{j=m+1}^{n} x_j^{-\beta_2}$

And the joint Likelihood function is given by

$$\mathrm{L}(\theta_1, \theta_2 | \underline{x}) \propto (\theta_1\beta_1)^m U_1 e^{-\theta_1 T_{2m}} (\theta_2\beta_2)^{n-m} U_2 e^{-\theta_2(T_{2n}-T_{2m})} \quad (1.5.1.6)$$

Suppose the marginal prior distributions 0f θ_1, θ_2 are natural conjugate prior

$$\pi_1(\theta_1, \underline{\mathrm{x}}) = \frac{b_1^{a_1}}{\Gamma a_1}\theta_1^{(a_1-1)} e^{-b_1\theta_1}; \; a_1, b_1 > 0, \theta_1 > 0 \quad (1.5.1.7)$$

$$\pi_2(\theta_2, \underline{\mathrm{x}}) = \frac{b_2^{a_2}}{\Gamma a_2}\theta_2^{(a_2-1)} e^{-b_2\theta_2}; \; a_2, b_2 > 0, \theta_2 > 0 \quad (1.5.1.8)$$

The joint prior distribution of θ_1,θ_2 and change point 'm' is

$$\pi(\theta_1, \theta_2, m) \propto \frac{b_1^{a_1}}{\Gamma a_1}\frac{b_2^{a_2}}{\Gamma a_2}\theta_1^{(a_1-1)} e^{-b_1\theta_1}\theta_2^{(a_2-1)} e^{-b_2\theta_2} \quad (1.5.1.9)$$

where $\theta_1, \theta_2 > 0$ & m = 1,2.........(n-1)

The joint posterior density of θ_1, θ_2 and m say $\rho(\theta_1, \theta_2, m/\underline{x})$ is obtaited by using equations (1.5.1.6) & (1.5.1.9)

$$\rho(\theta_1, \theta_2, m|\underline{x}) = \frac{\mathrm{L}(\theta_1, \theta_2/\underline{x})\pi(\theta_1, \theta_2, m)}{\sum_m \iint_{\theta_1\theta_2} \mathrm{L}(\theta_1, \theta_2/\underline{x})\pi(\theta_1, \theta_2, m)\, d\theta_1 d\theta_2} \quad (1.5.1.10)$$

$$= \frac{{\theta_1}^{(m+a_1-1)} e^{-\theta_1(T_{2m}+b_1)} {\theta_2}^{(n-m+a_2-1)} e^{-\theta_2(T_{2n}-T_{2m}+b_2)}}{\sum_m \int_0^\infty e^{-\theta_1(T_{2m}+b_1)} {\theta_1}^{(m+a_1-1)} d\theta_1 \int_0^\infty {\theta_2}^{(n-m+a_2-1)} e^{-\theta_2(T_{2n}-T_{2m}+b_2)} d\theta_2}$$

Assuming $\theta_1(T_{2m} + b_1) = x$ & $\theta_2(T_{2n} - T_{2m} + b_2) = y$

$\theta_1 = \frac{x}{(T_{2m}+b_1)}$ & $\theta_2 = \frac{y}{T_{2n}-T_{2m}+b_2}$

$d\theta_1 = \frac{dx}{(T_{2m}+b_1)}$ & $\mathrm{d}\theta_2 = \frac{dy}{T_{2n}-T_{2m}+b_2}$

$$= \frac{e^{-\theta_1(T_{2m}+b_1)} {\theta_1}^{(m+a_1-1)} e^{-\theta_2(T_{2n}-T_{2m}+b_2)} {\theta_2}^{(n-m+a_2-1)}}{\sum_m \frac{\Gamma(m+a_1)}{(T_{2m}+b_1)^{(m+a_1)}} \frac{\Gamma(n-m+a_2)}{(T_{2n}-T_{2m}+b_2)^{(n-m+a_2)}}}$$

$$\rho(\theta_1, \theta_2, m|\underline{x}) = \frac{e^{-\theta_1(T_{2m}+b_1)} {\theta_1}^{(m+a_1-1)} e^{-\theta_2(T_{2n}-T_{2m}+b_2)} {\theta_2}^{(n-m+a_2-1)}}{\xi(a_1, a_2, b_1, b_2, m, n)} \quad (1.5.1.11)$$

Where $\xi(a_1, a_2, b_1, b_2, m, n) = \sum_{m=1}^{n-1}\left[\frac{\Gamma(m+a_1)}{(T_{2m}+b_1)^{m+a_1}} \frac{\Gamma(n-m+a_2)}{(T_{2n}-T_{2m}+b_2)^{(n-m+a_2)}}\right]$

The Marginal posterior distribution of change point 'm' using the equations (1.5.1.6), (1.5.1.7) & (1.5.18)

$$\rho(m|\underline{x}) = \frac{\mathrm{L}(\theta_1, \theta_2/\underline{x})\, \pi(\theta_1)\, \pi(\theta_2)}{\sum_m \mathrm{L}(\theta_1, \theta_2/\underline{x})\, \pi(\theta_1)\, \pi(\theta_2)}$$

On solving which gives

$$\rho(m|\underline{x}) = \frac{{\theta_1}^{(m+a_1-1)} e^{-\theta_1(T_{2m}+b_1)} {\theta_2}^{(n-m+a_2-1)} e^{-\theta_2(T_{2n}-T_{2m}+b_2)}}{\sum_m {\theta_1}^{(m+a_1-1)} e^{-\theta_1(T_{2m}+b_1)} {\theta_2}^{(n-m+a_2-1)} e^{-\theta_2(T_{2n}-T_{2m}+b_2)}}$$

$$\rho(m|\underline{x}) = \frac{\int_0^\infty e^{-\theta_1(T_{2m}+b_1)} {\theta_1}^{(m+a_1-1)} d\theta_1 \int_0^\infty e^{-\theta_2(T_{2n}-T_{2m}+b_2)} {\theta_2}^{(n-m+a_2-1)} d\theta_2}{\sum_m \int_0^\infty e^{-\theta_1(T_{2m}+b_1)} {\theta_1}^{(m+a_1-1)} d\theta_1 \int_0^\infty e^{-\theta_2(T_{2n}-T_{2m}+b_2)} {\theta_2}^{(n-m+a_2-1)} d\theta_2}$$

Assuming $\theta_1(T_{2m} + b_1) = y$ & $\theta_2(T_{2n} - T_{2m} + b_2) = z$

$\theta_1 = \frac{y}{(T_{2m}+b_1)}$ & $\theta_2 = \frac{z}{T_{2n}-T_{2m}+b_2}$

$d\theta_1 = \frac{dy}{(T_{2m}+b_1)}$ & $d\theta_2 = \frac{z}{T_{2n}-T_{2m}+b_2}$

$$\rho(m|\underline{x}) = \frac{\frac{\Gamma(m+a_1)}{(T_{2m}+b_1)^{(m+a_1)}} \frac{\Gamma(n-m+a_2)}{(T_{2n}-T_{2m}+b_2)^{(n-m+a_2)}}}{\sum_m \frac{\Gamma(m+a_1)}{(T_{2m}+b_1)^{(m+a_1)}} \frac{\Gamma(n-m+a_2)}{(T_{2n}-T_{2m}+b_2)^{(n-m+a_2)}}}$$

$$\rho(m|\underline{x}) = \frac{\frac{\Gamma(m+a_1)}{(T_{2m}+b_1)^{(m+a_1)}} \frac{\Gamma(n-m+a_2)}{(T_{2n}-T_{2m}+b_2)^{(n-m+a_2)}}}{\xi(a_1, a_2, b_1, b_2, m, n)} \quad (1.5.1.12)$$

The marginal posterior distribution of θ_1, using equations (1.5.1.6)& (5.5.1.7)

$$\rho(\theta_1|\underline{x}) = \frac{\mathrm{L}(\theta_1, \theta_2/\underline{\mathrm{x}})\, \pi(\theta_1)}{\int_0^\infty \mathrm{L}(\theta_1, \theta_2/\underline{\mathrm{x}})\, \pi(\theta_1) \mathrm{d}\theta_1}$$

$$\rho(\theta_1|\underline{x}) = \frac{\sum_m \int_0^\infty \mathrm{L}(\theta_1, \theta_2/\underline{\mathrm{x}})\, \pi(\theta_1)\, \pi(\theta_2)\, \mathrm{d}\theta_2}{\sum_m \iint_0^\infty \mathrm{L}(\theta_1, \theta_2/\underline{\mathrm{x}})\, \pi(\theta_1)\, \pi(\theta_2)\, \mathrm{d}\theta_1\, \mathrm{d}\theta_2}$$

On solving which gives

$$\rho(\theta_1|\underline{x}) = \frac{\sum_m e^{-\theta_1(T_{2m}+b_1)} {\theta_1}^{(m+a_1-1)} \int_0^\infty e^{-\theta_2(T_{2n}-T_{2m}+b_2)} {\theta_2}^{(n-m+a_2-1)} d\theta_2}{\sum_m \int_0^\infty e^{-\theta_1(T_{2m}+b_1)} {\theta_1}^{(m+a_1-1)} d\theta_1 \int_0^\infty e^{-\theta_2(T_{2n}-T_{2m}+b_2)} {\theta_2}^{(n-m+a_2-1)} d\theta_2}$$

Assuming $\theta_1(T_{2m}+b_1)=y$ & $\theta_2(T_{2n}-T_{2m}+b_2)=z$

$\theta_1=\frac{y}{(T_{2m}+b_1)}$ & $\theta_2=\frac{z}{T_{2n}-T_{2m}+b_2}$

$d\theta_1=\frac{dy}{(T_{2m}+b_1)}$ & $d\theta_2=\frac{dz}{T_{2n}-T_{2m}+b_2}$

$$\rho(\theta_1|\underline{x})=\frac{\sum_m e^{-\theta_1(T_{2m}+b_1)}\,{\theta_1}^{(m+a_1-1)}\int_0^\infty e^{-z}\frac{z^{(n-m+a_2-1)}}{(T_{2n}-T_{2m}+b_2)^{(n-m+a_2-1)}}\frac{dz}{(T_{2n}-T_{2m}+b_2)}}{\sum_m\int_0^\infty e^{-y}\frac{y^{(m+a_1-1)}}{(T_{2m}+b_1)^{(m+a_1-1)}}\frac{dy}{(T_{2m}+b_1)}\int_0^\infty e^{-z}\frac{z^{(n-m+a_2-1)}}{(T_{2n}-T_{2m}+b_2)^{(n-m+a_2-1)}}\frac{dz}{(T_{2n}-T_{2m}+b_2)}}$$

$$\rho(\theta_1|\underline{x})=\frac{\sum_m e^{-\theta_1(T_{2m}+b_1)}\,{\theta_1}^{(m+a_1-1)}\frac{\Gamma(n-m+a_2)}{(T_{2n}-T_{2m}+b_2)^{(n-m+a_2)}}}{\sum_m\frac{\Gamma(m+a_1)}{(T_{2m}+b_1)^{(m+a_1)}}\frac{\Gamma(n-m+a_2)}{(T_{2n}-T_{2m}+b_2)^{(n-m+a_2)}}}$$

$$\rho(\theta_1|\underline{x})=\frac{\sum_m e^{-\theta_1(T_{2m}+b_1)}\,{\theta_1}^{(m+a_1-1)}\frac{\Gamma(n-m+a_2)}{(T_{2n}-T_{2m}+b_2)^{(n-m+a_2)}}}{\xi(a_1,a_2,b_1,b_2,m,n)}$$

(1.5.1.13)

The marginal posterior distribution of θ_2, using the equation (1.5.1.6) & (1.5.1.8) is

$$\rho(\theta_2|\underline{x})=\frac{L(\theta_1,\theta_2/\underline{x})\,\pi(\theta_2)}{\int_0^\infty L(\theta_1,\theta_2/\underline{x})\,\pi(\theta_2)\,d\theta_2}$$

$$\rho(\theta_2|\underline{x})=\frac{\sum_m\int_0^\infty L(\theta_1,\theta_2/\underline{x})\,\pi(\theta_1)\,\pi(\theta_2)\,d\theta_1}{\sum_m\iint_0^\infty L(\theta_1,\theta_2/\underline{x})\,\pi(\theta_1)\,\pi(\theta_2)\,d\theta_1\,d\theta_2}$$

Assuming $\theta_1(T_{2m}+b_1)=y$ & $\theta_1=\frac{y}{(T_{2m}+b_1)}$ ρ

$$\rho(\theta_2|\underline{x})=\frac{\sum_m\frac{\Gamma(m+a_1)}{(T_{2m}+b_1)^{(m+a_1)}}e^{-\theta_2(T_{2n}-T_{2m}+b_2)}\,{\theta_2}^{(n-m+a_2-1)}}{\sum_m\frac{\Gamma(m+a_1)}{(T_{2m}+b_1)^{(m+a_1)}}\frac{\Gamma(n-m+a_2)}{(T_{2n}-T_{2m}+b_2)^{(n-m+a_2)}}}$$

$$\rho(\theta_2|\underline{x})=\frac{\sum_m\frac{\Gamma(m+a_1)}{(T_{2m}+b_1)^{(m+a_1)}}e^{-\theta_2(T_{2n}-T_{2m}+b_2)}\,{\theta_2}^{(n-m+a_2-1)}}{\xi(a_1,a_2,b_1,b_2,m,n)}$$

(1.5.1.14)

1.5.2 Bayes Estimators under Precautionary Loss Function (PLF)

The Precautionary loss function is given by

$$L_3(\hat{\theta},\theta)=\frac{(\hat{\theta}-\theta)^2}{\hat{\theta}} \qquad (1.5.2.1)$$

The Bayes estimator of θ under precautionary Loss Function is obtain by solving the equation;

$$\frac{\partial}{\partial\hat{\theta}}E_\rho[\,L_3(\hat{\theta},\theta)]=0$$

$$\Rightarrow\ \hat{\theta}_{BP}=[E_\rho(\theta^2)]^{1/2} \qquad (1.5.2.2)$$

The Bayes estimate $\widehat{m}_{BP}$ of m using the marginal posterior from equation (1.5.1.14) is

$$\widehat{m}_{BP}=[E_\rho(m^2)]^{1/2}$$

$$\widehat{m}_{BP}=\left[\frac{\sum_m m^2\frac{\Gamma(m+a_1)}{(T_{2m}+b_1)^{(m+a_1)}}\frac{\Gamma(n-m+a_2)}{(T_{2n}-T_{2m}+b_2)^{(n-m+a_2)}}}{\xi(a_1,a_2,b_1,b_2,m,n)}\right]^{1/2}$$

(1.5.2.3)

The Bayes estimator $\hat{\theta}_{1BP}$ of θ_1 under PLF using the marginal posterior from equation (1.5.1.15) is

$$\hat{\theta}_{1BP}=[E_\rho({\theta_1}^2)]^{1/2}$$

$$\hat{\theta}_{1BP}=\left[\frac{\int_0^\infty{\theta_1}^2\sum_m e^{-\theta_1(T_{2m}+b_1)}\,{\theta_1}^{(m+a_1-1)}\frac{\Gamma(n-m+a_2)}{(T_{2n}-T_{2m}+b_2)^{(n-m+a_2)}}d\theta_1}{\xi(a_1,a_2,b_1,b_2,m,n)}\right]^{1/2}$$

$$\hat{\theta}_{1BP}=\left[\frac{\sum_m\frac{\Gamma(n-m+a_2)}{(T_{2n}-T_{2m}+b_2)^{(n-m+a_2)}}\int_0^\infty e^{-\theta_1(T_{2m}+b_1)}\,{\theta_1}^{(m+a_1+1)}d\theta_1}{\xi(a_1,a_2,b_1,b_2,m,n)}\right]^{1/2}$$

Assuming $\theta_1(T_{2m}+b_1)=y$ & $\theta_1=\frac{y}{(T_{2m}+b_1)}$

Then

$$\hat{\theta}_{1BP}=\left[\frac{\sum_m\frac{\Gamma(n-m+a_2)}{(T_{2n}-T_{2m}+b_2)^{(n-m+a_2)}}\int_0^\infty e^{-y}\frac{y^{(m+a_1+1)}}{(T_{2m}+b_1)^{(m+a_1+1)}}\frac{dy}{(T_{2m}+b_1)}}{\xi(a_1,a_2,b_1,b_2,m,n)}\right]^{1/2}$$

$$\hat{\theta}_{1BP}=\left[\frac{\sum_m\frac{\Gamma(m+a_1+2)}{(T_{2m}+b_1)^{(m+a_1+2)}}\frac{\Gamma(n-m+a_2)}{(T_{2n}-T_{2m}+b_2)^{(n-m+a_2)}}}{\xi(a_1,a_2,b_1,b_2,m,n)}\right]^{1/2}$$

$$\hat{\theta}_{1BP}=\left[\frac{\xi[(a_1+2),a_2,b_1,b_2,m,n]}{\xi(a_1,a_2,b_1,b_2,m,n)}\right]^{1/2} \qquad (1.5.2.4)$$

The Bayes estimator $\hat{\theta}_{2BP}$ of θ_2 under PLF using the marginal posterior from equation (1.5.1.16) is

$$\hat{\theta}_{2BP}=[E_\rho({\theta_2}^2)]^{1/2}$$

$$\hat{\theta}_{2BP}=\left[\frac{\int_0^\infty{\theta_2}^2\sum_m\frac{\Gamma(m+a_1)}{(T_{2m}+b_1)^{(m+a_1)}}e^{-\theta_2(T_{2n}-T_{2m}+b_2)}\,{\theta_2}^{(n-m+a_2-1)}d\theta_2}{\xi(a_1,a_2,b_1,b_2,m,n)}\right]^{1/2}$$

$$\hat{\theta}_{2BP}=\left[\frac{\sum_m\frac{\Gamma(m+a_1)}{(T_{2m}+b_1)^{(m+a_1)}}\int_o^\infty e^{-\theta_2(T_{2n}-T_{2m}+b_2)}\,{\theta_2}^{(n-m+a_2+1)}d\theta_2}{\xi(a_1,a_2,b_1,b_2,m,n)}\right]^{1/2}$$

Assuming $\theta_2(T_{2n}-T_{2m}+b_2)=y$ & $\theta_2=\frac{y}{(T_{2n}-T_{2m}+b_2)}$

Then

$$\hat{\theta}_{2BP}=\left[\frac{\sum_m\frac{\Gamma(m+a_1)}{(T_{2m}+b_1)^{(m+a_1)}}\frac{\Gamma(n-m+a_2+2)}{(T_{2n}-T_{2m}+b_2)^{(n-m+a_2+2)}}}{\xi(a_1,a_2,b_1,b_2,m,n)}\right]^{1/2}$$

$$\hat{\theta}_{2BP}=\left[\frac{\xi[a_1,(a_2+2),b_1,b_2,m,n]}{\xi(a_1,a_2,b_1,b_2,m,n)}\right]^{1/2} \qquad (1.5.2.5)$$

Numerical Comparison for Exponentiated Inverted Weibull Distribution

We have generated 20 random observations from Exponentiated Inverted Weibull distribution with parameter θ = 2 and β = 0.5. The observed data mean is μ = 1.5616 and variance σ^2 = 0.6812. Let the change in sequence is at 11th observation, so the means and variances of both sequences ($x_1,x_2,\ldots,x_m$) and ($x_{(m+1)}$, $x_{(m+2)},\ldots$, x_n) are μ_1 = 1.5491, μ_2 = 1.5768, σ_1^2 = 1.0197 and σ_2^2 = 0.3427. If the target value of μ_1 is unknown, its estimating ($\hat{\mu}_1$) is given by the mean of first m sample observation given m=11, μ = 1.5491.

Sensitivity Analysis of Bayes Estimates

In this section we have studied the sensitivity of the Bayes estimates with respect to changes in the parameters of prior distribution a_1, b_1, a_2 and b_2. The means and variances of the prior distribution are used as prior information in computing these parameters. Then with these parameter values we have computed the Bayes estimates of m, θ_1 and θ_2 under PLF considering different set of values of (a_1, b_1) and (a_2, b_2).We have also considered the different sample sizes n=10(10)30. The Bayes estimates of the change point 'm' and the parameters θ_1 and θ_2 are given in table-1.2 under PLF. Their respective mean squared errors (M.S.E's) are calculated by repeating this process 1000 times and presented in same table in small parenthesis under the estimated values of parameters. All these values appears to be robust with respect to correct choice of prior parameter values and appropriate sample size. All the estimators perform better with sample size n=20 and (a_1=1.8,1.9)(b_1=2.3,2.4),(a_2=1.3,1.4) and (b_2=1.55, 1.65) .Similarly the Bayes estimates of PLF are presented in table 5.2 appears to be sensitive with wrong choice of prior parameters and sample size. All the calculations are done by R-programming. From the below two table we conclude that –

The Bayes estimates of the parameters θ_1 and θ_2 of EIW obtained with loss function PLF have more or less same numerical values. The respective M.S.E's shows that the Bayes estimates uniformly smaller for $\hat{\theta}_{1BP}$ and $\hat{\theta}_{2BP}$ under PLF except of $\hat{m}_{BP}$. The Bayes estimates of the parameters are robust uniformly with all values of prior parameters as and all sample size.

Table 1.1-Bayes Estimates of m, θ_1& $\theta_2$for EIW sequences and their respective M.S.E.'s Under PLF

(a_1, b_1)	(a_2, b_2)	N	$\hat{m}_{BP}$	$\hat{\theta}_{1BP}$	$\hat{\theta}_{2BP}$
(1.25, 1.50)	(1.50 ,1.60)	10	5.4565 (9.1183)	1.6971 (0.0289)	1.5953 (0.2185)
		20	10.3973 (7.3787)	1.5949 (0.2262)	1.5869 (0.2733)
		30	13.9314 (6.7106)	1.5895 (0.2924)	2.0356 (0.2005)
(1.50, 1.75)	(1.70, 1.80)	10	5.8061 (10.4414)	1.9918 (0.0104)	1.6433 (0.2369)
		20	9.8904 (3.7561)	1.3836 (0.4143)	2.0469 (0.0212)
		30	13.9181 (5.0310)	1.2898 (0.2193)	1.6959 (0.0142)
(1.75 ,2.0)	(1.90 ,2.0)	10	5.3711 (6.5221)	1.4647 (0.2373)	1.5399 (0.1743)
		20	10.7288 (7.8659)	1.5764 (0.3482)	1.9549 (0.6091)
		30	22.4536 (5.6978)	2.4728 (0.2539)	1.2424 (0.0505)
(2.0 ,2.25)	(2.10, 2.20)	10	5.7052 (7.0533)	1.6448 (0.1646)	1.291 (0.1879)
		20	11.4104 (6.7183)	2.0650 (0.1927)	1.9739 (0.3345)
		30	17.8143 (10.8602)	1.7998 (0.0448)	1.7096 (0.4649)
(2.25, 2.50)	(2.30, 2.40)	10	6.3005 (7.3724)	1.9177 (0.0716)	1.3364 (0.2789)
		20	11.6565 (42.4434)	1.8094 (.0005)	1.9538 (.0084)
		30	18.1075 (112.9848)	1.6665 (0.0615)	1.4239 (0.0374)
(2.50, 2.75)	(2.50, 2.60)	10	5.1799 (6.3749)	1.4949 (0.3050)	1.8418 (0.0727)
		20	13.1644 (50.3987)	1.7761 (0.1452)	1.5790 (0.2233)
		30	19.3982 (97.3036)	2.2405 (0.1642)	1.7733 (0.0284)

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