Pseudo Similar Neutrosophic Fuzzy Matrices

S.Princy Christinal Esther[1], M.M.Shanmugapriya[2]
[1,2]*Department of Mathematics, Karpagam Academy of Higher Education, Coimbatore, Tamilnadu, India, 641021*

***Abstract*- In this paper, we will characterize Pseudo Similarity and Semi Similarity for Neutrosophic Fuzzy Matrix (NSFM) and demonstrate that the Pseudo Similarity relation on a couple of NSFMs is acquired by the entirety of its powers and their transposes are comparable. Additionally, we display that the Pseudo similarity relation preserves regularity and impotency of their matrices.**

***Index Terms*- Fuzzy Matrix, Neutrosophic Fuzzy Matrix, Pseudo Similar, Semi Similar.**

I. INTRODUCTION

Scholastics in financial aspects, social science, clinical science, modern, climate science and numerous other various fields concur with the dubious, loose and rarely inadequate with regards to data of showing estimated information. Subsequently, fuzzy set hypothesis was introduced by L. A. Zadeh [13]. Then, at that point, the intuitionistic fuzzy sets was created by K. A. Atanassov [1, 2]. Assessment of non-membership values is additionally not continually feasible for the indistinguishable explanation as in the event of participation esteems thus, there exists an indeterministic part whereupon delay endures. Therefore, Smarandache et al. [5, 11, 12] has presented the idea of Neutrosophic Set (NS) which is a speculation of customary sets, fuzzy set, intuitionistic fuzzy set and so on.

The Intuitionistic fuzzy matrices (IFMs) presented first time by Khan, Shyamal and Pal [8]. Kim and Roush have fostered a hypothesis for fuzzy matrices closely resembling that for Boolean Matrices [4]. In [7], Meenakshi have determined different sorts of requesting on regular fuzzy matrices. Fuzzy matrices concurred with just participation values. In [9,10], Poongodi et.al have given the new portrayal on Neutrosophic fuzzy matrices. The Pseudo Similarity in semi groups of fuzzy matrices were discussed by Chen and Meenakshi[3,6,7]

In this paper, we characterized Pseudo Similarity and Semi Similarity for Neutrosophic Fuzzy Matrix (NSFM) and demonstrate that the Pseudo Similarity relation on a couple of NSFMs is acquired by the entirety of its powers and their transposes are comparable. Additionally, we display that the Pseudo similarity relation preserves regularity and impotency of their matrices.

II. PRELIMINARIES

In this part, a couple of major definitions and results required are given. Let $\mathcal{N}_n$ implies the course of action of all nxn NSFM.

Definition 2.1

A matrix C ∈ F_n is known as to be regular that there exist a matrix U ∈ F_n, to such an extent that CUC =C. Then, at that point, U is called g-reverse of C. Let C{1} = {U/CUC = C}. F_n signifies the arrangement of all fuzzy matrices of order nxn.

Definition 2.2

An neutrosophic fuzzy matrix (NSFM) C of order m × n is defined as C= $[X_{ab,} <c_{(ab)T,}\ c_{(ab)F,}\ c_{(ab)I}>]_{mxn}$, where $c_{(ab)T,}\ c_{(ab)F}\ c_{(ab)I}$ are called enrollment work (T), the non-participation (F) work and the indeterminancy work (I) of U_{ab} in C, which sustaining the condition $0 \leq (c_{(ab)T,} + c_{(ab)F,}\ + c_{(ab)I}\ ) \leq 3$. For simplicity, we write C = $[c_{ab}]_{mxn}$ where $c_{ab} = <c_{(ab)T,}\ c_{(ab)F,}\ c_{(ab)I}>$. Let $\mathcal{N}_n$ symbolizes the arrangement of all nxn NSFM.

Definition 2.3.

A Matrix A∈ $(NSFM)_n$ is said to be invertible if and only if there exists X∈ $(NSFM)_n$ such that AX = XA = In = $[I_T, I_F, I_I]$, where I_n is the identity matrix in $(NSFM)_n$

Definition 2.4.

A square neutrosophic fuzzy matrix is called neutrosophic fuzzy permutation matrix, if every row and column contain exactly one [1,0] and all other entries are [0,1]. Let P_n be the set of all nxn permutation matrices in $(NSFM)_n$.

Definition 2.5.

Let A∈ $(NSFM)_n$. Then

(i) A is reflexive $A \geq I_n$

(ii) A is symmetric $A = A^T$.

(iii) A is transitive $A^2 \leq A$.

(iv) A is idempotent $A = A^2$

Lemma: 2.6

For $C=[C_T, C_F, C_I] \in \mathcal{N}_{mn}$ and $D=[D_T, D_F, D_I] \in \mathcal{N}_{nm}$, the following hold.

(i) $C^T = [C_T^T, C_F^T, C_I^T]$

(ii) $CD = [C_TD_T, C_FD_{F,} C_ID_I]$

III. PSEUDO SIMILAR INTERVAL VALUED FUZZY MATRICES

In this section, the pseudo similarity and semi similarity of a neutrosophic fuzzy matrix are discussed.

Definition 3.1

$C\in\mathcal{N}_m$ and $D\in\mathcal{N}_n$ are said to be Pseudo-Similar and denote it by $C \backsimeq D$ if there exist $U\in \mathcal{N}_{mxn}$, and $V\in \mathcal{N}_{nxm}$ such that C = UDV, D = VCU and U = UVU.

Definition 3.2

$C\in\mathcal{N}_m$ and $D\in\mathcal{N}_n$ are said to be Semi-Similar and denote it by $C\approx D$ if there exist $U\in\mathcal{N}_{mxn}$ and $V\in\mathcal{N}_{nxm}$ such that C=UDV and D=VCU.

Definition 3.3

For $C \in \mathcal{N}_n$, the least d >0 such that $C^{k+d} = C^k$ for some integer k, is called the period of C.

Lemma 3.1

Let $C = [C_T, C_F, C_I] \in \mathcal{N}_m$ and $D = [D_T, D_F, D_I] \in \mathcal{N}_n$ then $C \backsimeq D \Leftrightarrow C_T \backsimeq D_T$, $C_F \backsimeq D_F$ and $C_I \backsimeq D_I$ for C_T, C_F, C_I , D_T, D_F , D_I ∈ F_{mn}.

Proof:

Let $C = [C_T, C_F, C_I] \in \mathcal{N}_m$ and $D = [D_T, D_F, D_I] \in \mathcal{N}_n$

$C \backsimeq D \Rightarrow$ C=UDV, D=VCU and U=UVU for some $U\in \mathcal{N}_{mxn}$ and $V\in \mathcal{N}_{nxm}$. (By Definition (3.1))

C=UDV $\Leftrightarrow [C_T, C_F, C_I] = [U_T, U_F, U_I] [D_T, D_F, D_I] [V_T, V_F, V_I]$

$\Leftrightarrow [C_T, C_F, C_I]= [U_T D_T V_T, U_F D_F V_F, U_I D_I V_I]$ (By Lemma (2.6)(ii))

$\Leftrightarrow C_T = U_T D_T V_T, C_F = U_F D_F V_F$ and $C_I = U_I D_I V_I$

Similarly, we have,

D=VCU $\Leftrightarrow D_T = V_TC_TU_T,$ $D_F = V_FC_FU_F$ and $D_I = V_IC_IU_I$

U=UVU $\Leftrightarrow U_T= U_TV_TU_T,$ $U_F= U_FV_FU_F$, and $U_I= U_IV_IU_I$

Hence $C\backsimeq D$ $\Leftrightarrow C_T\backsimeq D_T,$ $C_F\backsimeq D_F$ and $C_I \backsimeq D_I$ for C_T, C_F, C_I, D_T, D_F, D_I ∈ Fmn.

Lemma 3.2

Let $C = [C_T, C_F, C_I] \in \mathcal{N}_m$ and $D = [D_T, D_F, D_I] \in \mathcal{N}_n$ then $C \approx D \Leftrightarrow C_T \approx D_T$, $C_F \approx D_F$ and $C_I \approx D_I$ for C_T, C_F, C_I , D_T, D_F , D_I ∈ F_{mn}.

Proof:

Let $C = [C_T, C_F, C_I] \in \mathcal{N}_m$ and $D = [D_T, D_F, D_I] \in \mathcal{N}_n$

$C\approx D \Rightarrow$ C=UDV, D=VCU for some $U\in \mathcal{N}_{mxn}$ and $V\in\mathcal{N}_{nxm}$. (By Definition (3.2))

C=UDV $\Leftrightarrow [C_T, C_F, C_I]= [U_T, U_F, U_I] [D_T, D_F, D_I] [V_T, V_F, V_I]$

$\Leftrightarrow [C_T, C_F, C_I]= [U_T D_T V_T, U_F D_F V_F, U_I D_I V_I]$ (By Lemma (2.6)(ii))

$\Leftrightarrow C_T = U_T D_T V_T, C_F = U_F D_F V_F$ and $C_I = U_I D_I V_I$

Similarly, we have,

D=VCU $\Leftrightarrow D_T = V_TC_TU_T,$ $D_F = V_FC_FU_F$ and $D_I = V_IC_IU_I$

Hence $C \approx D \Leftrightarrow C_T \approx D_T$, $C_F \approx D_F$ and $C_I \approx D_I$ for C_T, C_F, C_I , D_T, D_F , D_I ∈ F_{mn}.

Theorem 3.1

Let $C\in \mathcal{N}_m$ and $D \in \mathcal{N}$n. Then the following are equivalent.

(i) $C \backsimeq D$

(ii) There exist $U \in \mathcal{N}_{mxn}$ and $V \in \mathcal{N}_{nxm}$ such that C=UDV and D=VCU and $UV\in \mathcal{N}_m$ is idempotent.

(iii) There exist $U \in \mathcal{N}_{mxn}$ and $V \in \mathcal{N}_{nxm}$ such that C=UDV and D=VCU and $VU\in \mathcal{N}_n$ is idempotent.

Proof:

(i) ⇒(ii) and (i) ⇒(iii) are trivial, since U=UVU implies $UV\in \mathcal{N}_m$ and $VU\in \mathcal{N}_n$ are idempotent matrices.

(ii) ⇒(i): C=UDV = (UV)C(UV)= (UVU)D(VUV).

Similarly, D=VCU =(VUV)C(UVU)

Put U‟=UVU and V‟= VUV. Then C=U‟DV‟ and D=V‟CU‟.

Further using UV is idempotent, we get
$U''V''= (UVU)(VUV) = UV$ and $(U''V'')(U''V'') = (UV)(UV) = U''V''$.
Thus $U''V''$ is idempotent.
Set $U''''=U''V''U''$ and $V''''= V''U''V''$. Then $C = U''DV'' = U''(V''CU'')V''= U''V''(U''DV'')U''V''= (U''V''U'')D(V''U''V'') = U''''DV''''$
Similarly, $D = V''C U''$.
By using $U''V''$ is idempotent, we have
$U''V''U'' = (U''V''U'')(V''U''V'')(U''V''U'') = U''V''U'' = U''$.
Therefore $C \simeq D$.Thus (i) holds.
(iii) ⇒ (i): This can be proved in the same manner and hence omitted.

Theorem 3.2:
Let $C \in \mathcal{N}_m$ and $D \in \mathcal{N}_n$. Then the following are equivalent.
(i) $C \simeq D$
(ii) There exist $U \in \mathcal{N}_{mxn}$ and $V \in \mathcal{N}_{nxm}$ such that $C=UDV$ and $D=VCU$ and $(UV)^k \in \mathcal{N}_m$ is idempotent for some odd $k \in N$.
(iii) There exist $U \in \mathcal{N}_{mxn}$ and $V \in \mathcal{N}_{nxm}$ such that $C=UDV$ and $D=VCU$ and $(VU)^k \in \mathcal{N}_n$ is idempotent for some odd $k \in \mathcal{N}$.
Proof
(i) ⇒(ii): Follows from Theorem (3.1).
(ii) ⇒(i): Let $k = 2r+1$ with $r \in N$, Since $C=UDV$ and $D=VCU$, we have
$C = U(VCU)V = (UV)\,UDV(UV)= (UV)U(VCU)V(UV) = (UV)^2C(UV)^2 = (UV)^2\,UDV\,(UV)^2$
Thus proceeding we get, $C = (UV)^r\,UDV\,(UV)^r$.
Similarly, $D = V(UV)r\,C\,(UV)rU$.
Set $U''= (UV)^rU$ and $V''=V(UV)^r$.
Then $C=U''DV''$, $D=V''CU''$ and $U''V'' = (UV)^rUV(UV)^r = (UV)^{2r+1} = (UV)^k$ is idempotent.
Hence by Theorem (3.1), $C \simeq D$. Thus (i) holds.
(i) ⇒(iii): Follows from Theorem (3.1)
(iii) ⇒(i): Let $k = 2r+1$ with $r \in N$,Since $C=UDV$ and $D=VCU$, we have
$C= U(VU)^rD(VU)^rV$ and $D = (VU)^rVCU(VU)^r$

Set $U''= U(VU)^r$ and $V''=(VU)^rV$. Then $C=U''DV''$, $D=V''CU''$ and $V''U'' = (VU)^r(VU)(VU)^r = (VU)^{2r+1} = (VU)^k \in N_n$ is idempotent.
Hence by Theorem (3.1), $C \simeq D$. Thus (i) holds.

Theorem 3.3:
Let $C \in \mathcal{N}_m$ and $D \in \mathcal{N}_n$. Then the following are equivalent
(i) $C \simeq D$
(ii) There exist $U \in \mathcal{N}_{mxn}$ and $V \in \mathcal{N}_{nxm}$ such that $C=UDV$ and $D=VCU$ and $(UV) \in \mathcal{N}_m$ is periodic with even order.
(iii) There exist $U \in \mathcal{N}_{mxn}$ and $V \in \mathcal{N}_{nxm}$ such that $C=UDV$ and $D=VCU$ and $(VU) \in \mathcal{N}_n$ is periodic with even order.
Proof:
(i)⇒(ii): Follows from Theorem (3.1), since $UV \in N_{mxn}$ is idempotent, it follows that UV is periodic with even order.
(ii) ⇒(i): Suppose $C=UDV$ and $D=VCU$ and UV is periodic with even order $2k(k \in N)$ say, then $(UV)^{2k}=UV$.
Hence $(UV)^{2k-1}(UV)=UV$
Further $(UV)^{2k-1}(UV)^{2k-1} = (UV)^{2k-1}UV(UV)^{2k-1} = (UV)^{2k-1}$.
Thus $(UV)^{2k-1}$ is idempotent.
Therefore $C \simeq D$, by Theorem (3.2). Thus (i) holds.
(i)⟺ (iii): This can be proved in the same manner and hence omitted.

Theorem 3.4:
Let $C \in \mathcal{N}_m$ and $D \in \mathcal{N}_n$. Then the following are equivalent.
(i) $C \simeq D$
(ii) There exist $U \in \mathcal{N}_{mxn}$ and $V \in \mathcal{N}_{nxm}$ such that $C=UDV$, $D=VCU$, $U=UVU$ and $V=VUV$
(iii)There exist $U \in \mathcal{N}_{mxn}$ and $V, W \in \mathcal{N}_{nxm}$ such that $C=UDV$, $D=WCU$, $U=UVU = UWU$
Proof:
(i)⇒(iii): Since $C \simeq D$, By Definition (3.1), there exist $U \in \mathcal{N}_{mxn}$ and $V \in \mathcal{N}_{nxm}$ such that
$C=UDV$, $D=VCU$, $U=UVU$.
Let $V=W$ then $D=WCU$ and $U=UWU$ as required. Thus(iii) holds.
(iii)⇒(ii): Suppose there exist $U \in \mathcal{N}_{mxn}$ and $V, W \in \mathcal{N}_{nxm}$ such that $C=UDV$,
$D=WCU$, $U= UVU= UWU$,
then $C = UDV = U(WCU)V = (UWU)D(VUV) = UD(VUV)$ and
$D = WCU = W(UDV)U = (WUW)C(UVU) = (WUW)CU$.
Set $V''=VUV$ and $W''=WUW$. Then $U = UVU = UV(UVU) = UV''U$ and

U = UWU = UW(UWU) = UW‟‟U.
In addition, we have C = UDV‟‟ and D = W‟‟CU.
Set V‟‟ = W‟‟UV‟‟. Then UV‟‟‟‟U = UW‟‟(UV‟‟U) = UW‟‟U = U and
V‟‟UV‟‟ = W‟‟(UV‟‟U) V‟‟ = W‟‟UV ‟‟ = V‟‟‟‟.
We check that UDV‟‟ = UDW‟‟UV‟‟ = UW‟‟CUW‟‟UV‟‟ = UW‟‟CUV‟‟ = UDV‟‟ =C,
V‟‟CU = W‟‟UV‟‟UDV‟‟U = W‟‟UDV‟‟U = W‟‟CU = D.
Thus there exist U∈ $\mathcal{N}_{mxn}$, V" ∈ $\mathcal{N}_{nxm}$ such that
C = UDV‟‟, D = V‟‟CU, U = UV‟‟‟‟U and V‟‟ = V‟‟UV‟‟
as asserted and (ii) holds.
(ii)⇒(i): This is trivial.
Remark 3.1:
We observe that Theorem (3.4) infers the symmetry of Pseudo similarity relation, that is, C ≍D D ≍C.

Theorem 3.5:
Let C∈ $\mathcal{N}_m$ and D∈ $\mathcal{N}_n$. Then the following are equivalent.
(i) C ≍ D
(ii) C^T ≍D^T
(iii) C^k ≍ D^k for any integer k≥1
(iv) PCP^T ≍ QDQ^T for some permutation matrices P∈ $\mathcal{N}_m$ and Q∈ $\mathcal{N}_n$
Proof:
(i) ⟺ (ii): This is direct by taking transpose on both sides of C=UDV, D=VCU and using (C^T)=C and $(CU)^T=U^TC^T$.
(i)⟺(iii): (iii)⟹(i) is trivial. (i)⟹(iii) can be proved by induction on k. Since C ≍D, there exist U∈ $\mathcal{N}_{mxn}$ and V∈ $\mathcal{N}_{nxm}$ such that C=UDV, D=VCU, U=UVU. By Theorem (3.4), further we have V=VUV. Let us assume that $C^r=UD^rV$, $D^r=VC^r U$ for some r∈N.
C=UDV ⟹ CUV = UDVUV = UDV = C,
$UD^{r+1}V = UD^r DV = UD^r DV = UD^r (VCUV) = UD^rVC = C^{r+1}$
Similarly, it can be checked $VC^{r+1} U = D^{r+1}$.
Thus, by induction for all positive integer k ,we get

$C^k=UD^kV$, $D^k=VC^kU$ for U∈ N_{mxn} and V∈ N_{nxm} such that U=UVU. Hence C^k ≍D^k.
Thus (i) ⟹(iii) hold.
(i) ⟺ (iv): Suppose C ≍D, then there exist U∈ N_{mxn} and V∈ N_{nxm} such that C=UDV, D=VCU, U= UVU.
C=UDV ⟹ PCP^T= $PUDVP^T$ = $(PUQ^T)(QDQ^T)(QVP^T)$
D=VCU ⟹ QDQ^T= $QVCUQ^T$= $(QVP^T)(PCP^T)(PUQ^T)$.

Set U‟‟=PUQ^T, V‟‟=QVP^T then U‟‟V‟‟U‟‟=U‟‟,
PCP^T=U(QDQ^T)V‟‟ and QDQ^T=V‟‟(PCP^T)U‟‟.
Thus PCP^T ≍ QDQ^T. Conversely, if PCP^T ≍QDQ^T then as above, P^T(PCPT)P ≍ $Q^T(QDQ^T)Q$ ⟹ C ≍D
Thus (i)⇒(iv) holds.

Theorem 3.6:
Let C∈ $\mathcal{N}_m$ and D∈ $\mathcal{N}_n$. Then the following are equivalent.
(i) C ≈ D
(ii) C^T ≈ D^T
(iii) C^k ≈D^k for any integer k≥1
(iv) PCP^T ≈ QDQ^T for some permutation matrices P∈ $\mathcal{N}_m$ and Q∈ $\mathcal{N}_n$
Proof:
This can be proved along the same manner as that of Theorem (3.5) and hence omitted.
Remark 3.2:
It is clear that Similarity ⇒ Pseudo similarity ⇒ Semi similarity but converse is not true.

Theorem 3.7
Let C∈ $\mathcal{N}_m$ and D∈ $\mathcal{N}_n$ such that C ≍ D. Then C is regular matrix D is regular matrix.
Proof:
Since C ≍ D, by Theorem (3.4), there exist U∈ $\mathcal{N}_{mxn}$ and V∈ $\mathcal{N}_{nxm}$ such that C=UDV, D=VCU, U=UVU and V=VUV.
Suppose C is regular, then there exists G ∈ $\mathcal{N}_m$
such that CGC=C.
Define U=VGU. Clearly U ∈ $\mathcal{N}_n$. Since,
CUV=UDVUV = UDV=C
=(UVU)DV
=UVC,
DUD = (VCU)(VGU)(VCU)
=V(CUV)G(UVC)U
= VCGCU
=VCU=D.
Hence D is regular. Converse can be proved in the same manner.

Theorem 3.8
Let C∈ $\mathcal{N}_m$ and D∈ $\mathcal{N}_n$ such that C ≍ D. Then C is idempotent D is idempotent.
Proof:
Since C ≍ D,by Theorem(3.4), there exist U∈ $\mathcal{N}_{mxn}$ and V ∈ $\mathcal{N}_{nxm}$
such that D=VCU And C=UDV ⇒ CUV = C = CVU.

Suppose C is idempotent, then $C^2=C$,
hence $D^2=(VCU)(VCU)=V(CUV)CU=VCCU=VC^2U=VCU=D$
and D is idempotent.
Converse can be proved in the same manner.

IV. CONCLUSION

In this paper, the Pseudo similarity and Semi similarity of NSFMs are defined .The Pseudo similarity relation on a pair of NSFMs is inherited by all its powers and the Pseudo similarity relation preserve regularity and impotency of matrices are discussed.

REFERENCE

[1] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20 (1986), 87-96.

[2] K. Atanassov, Operations over interval-valued intuitionistic fuzzy sets, Fuzzy Sets and Systems 64 (1994), 159-174.

[3] Huanyin Chen.: "Pseudo similarity in Semigroups" Semi group forum. 68, 59-63(2004).

[4] Kim, K.H; Roush, F.W. (1980). Generalized fuzzy matrices, Fuzzy sets and systems, 4, 293–315.

[5] Mamouni Dhar, Said Broumi, and Florentin Smarandache, A Note on Square Neutrosophic Fuzzy Matrices, Neutrosophic Sets and Systems, Vol. 3, 2014.

[6] AR. Meenakshi, "Pseudo similarity in semigroups of fuzzy matrices" Proc. Int. Symp. On Semigroups and Appln., 64- 73(2007).

[7] AR.Meenakshi, (2008), Fuzzy Matrix, Theory and Applications, MJP Publishers, Chennai.

[8] M. Pal, S. K. Khan and A. K. Shyamal, Intuitionistic fuzzy matrices, Notes on Intuitionistic Fuzzy Sets 8(2) (2002), 51-62.

[9] P.Poongodi, S.Princy Christinal Esther and M.M.Shanmugapriya, On Regular Neutrosophic Fuzzy Matrices, Indian Journal of Natural Sciences, 13(71), pp:40932 – 40938, 2022.

[10] S.Princy Christinal Esther and M.M.Shanmugapriya, m- Ordering on Generalised Regular Neutrosophic Fuzzy Matrices,Mathematical statistician and engineering applications (Accepted).

[11] Rakhal Das, Florentin, Smarandache, Binod Chandra Tripathy, Neutrosophic Fuzzy Matrices and Some Algebraic Operations, Neutrosophic sets and systems, Vol 32, pp:401- 409, 2020.

[12] F. Smarandache, Neutrosophic set, A generalization of the intuitionistic fuzzy sets, Inter. J. Pure Appl. Math. 24 (2005), 287-297.

[13] Zadeh, L.A. (1965). *Fuzzy sets, Information and Control*, 8: 338 – 353.